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TABLES FOR STATISTICIANS AND BIOMETRICIANS

EDITED BY

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PREFACE

I AM very conscious of the delay which has intervened between the announcement of the publication of these Tables and their appearance. This delay has been chiefly due to two causes. First the great labour necessary, which largely fell on those otherwise occupied, and secondly the great expense involved (*a*) in the calculation of the Tables, and (*b*) in their publication. This matter of expense is one which my somewhat urgent correspondents, I venture to think, have entirely overlooked. It is perfectly true that only one single Table in this volume has been directly paid for, but a very large part of the labour of calculation has been done by the Staff of the Biometric Laboratory, whose very existence depends on the generous grant made to that laboratory by the Worshipful Company of Drapers. Our staff is not a large one and it has many duties, so that the progress of calculation has of necessity been slow. Even now I am omitting projected Tables, which I can only hope may be incorporated in a later edition of this work, e.g. Tables of the Incomplete B- and Γ-functions, and the Table needed to complete Everitt's work on High Values of Tetrachoric r when r lies between $-\cdot 80$ and $-1\cdot 00$. It would only satisfy my ideal of what these Tables should be, had I been able to throw into one volume with the present special tables, extensive tables of squares, of square roots, of reciprocals and of the natural trigonometric functions tabled to decimals of a degree. Logarithmic tables are relatively little used by the statistician to-day, which is the age of mechanical calculators, and he is perfectly ready to throw aside the fiction that there is any gain in the cumbersome notation of minutes and seconds of angle—a system which would have disappeared long ago, but for the appalling 'scrapping' of astronomical apparatus it would involve. But the ideal of one handy book for the statistician cannot be realised until we have a body of scientific statisticians far more numerous than at present. Statisticians must for the time being carry about with them not only this volume but a copy of *Barlow's Tables*, and a set of Tables of the Trigonometrical Functions.

Beside the cost of calculating these Tables, to which I have referred, must be added the cost of printing them. I had to do this slowly as opportunity offered in my Journal *Biometrika*, and the Tables as printed were moulded, in order that stereos might be taken for reproduction. Even as it is, there are a number of Tables in this volume, either printed here for the first time (e.g. Tables of the Logarithm of the Factorial and of the Fourth Moment), or published here for the first time (e.g. Tables of the $G(r, \nu)$ Integrals), the setting up of which has naturally been very expensive.

From the beginning of this work in 1901* when the first of these Tables was published and moulded, I have had one end in view, the publication, as funds would permit, of as full a series of Tables as possible. It is needless to say that no anticipation of profit was ever made, the contributors worked for the sake of science, and the aim was to provide what was possible at the lowest rate we could. The issue may appear to many as even now costly; let me assure those inclined to cavil, that to pay its way with our existing public double or treble the present price would not have availed; we are able to publish because of the direct aid provided by initial publication in *Biometrika* and by direct assistance from the Drapers' Company Grant. Yet a few years ago when a reprint of these Tables in America was only stopped by the threat to prevent the circulation of the book in which they were to appear entering any country with which we had a reasonable copyright law, I was vigorously charged with checking the progress of science and acting solely from commercial ends! Meanwhile without any leave, large portions of these tables have been reprinted, sometimes without even citing the originals, in American psychological text-books. Two Russian subjects have reissued many of these Tables in Russian and Polish versions, and copies of their works in contravention of copyright are carried into other European countries. It does not seem to have occurred to these men of science that there was anything blameworthy in depriving *Biometrika* of such increased circulation as it obtained from being the sole *locus* of these Tables, nor did they see in their actions any injury to science as a whole resulting from lessening my power to publish other work of a similar character. It is a singular phase of modern science that it steals with a plagiaristic right hand while it stabs with a critical left.

The *Introduction* gives a brief description of each individual table; it is by no means intended to replace actual instruction in the use of the tables such as

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* When issuing their prospectus in the spring of 1901 the Editors of *Biometrika* promised to provide "numerical tables tending to reduce the labour of statistical arithmetic."

is given in a statistical laboratory, nor does it profess to provide an account of the innumerable uses to which they may be put, or to warn the reader of the many difficulties which may arise from inept handling of them. Additional aid may be found in the text which usually accompanies the original publication of the tables.

In conclusion here I wish to thank the loyal friends and colleagues—Dr W. F. Sheppard, Mr W. Palin Elderton, Dr Alice Lee, Mr P. F. Everitt, Miss Julia Bell, Miss Winifred Gibson, Mr A. Rhind, Mr H. E. Soper and others—whose unremitting exertions have enabled so much to be accomplished, if that much is indeed not the whole we need. I have further to acknowledge the courtesy of the Council of the British Association, who have permitted the republication of the Tables of the $G(r, \nu)$ Integrals, originally published in their *Transactions*.

To the Syndics of the Cambridge Press I owe a deep debt of gratitude for allowing me the services of their staff in the preparation of this work. Pages and pages of these Tables were originally set up for *Biometrika*, or were set up afresh here, without the appearance of a single error. To those who have had experience of numerical tables prepared elsewhere, the excellence of the Cambridge first proof of columns of figures is a joy, which deserves the fullest acknowledgement.

Should this work ever reach a second edition I will promise two things, rendered possible by the stereotyping of the tables: it shall not only appear at a much reduced price, but it shall be largely increased in extent.

KARL PEARSON.

BIOMETRIC LABORATORY,
February 7, 1914.

Errata

The reader is requested to make before using these Tables the following corrections on pp. 82, 83, 84 and 85:

For $1{\cdot}77\sqrt{N}\Sigma_1$ and $1{\cdot}77\sqrt{N}\Sigma_2$ at the top of the Tables *read* $1{\cdot}177\sqrt{N}\Sigma_1$ and $1{\cdot}177\sqrt{N}\Sigma_2$.

When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind.

LORD KELVIN.

La théorie des probabilités n'est au fond que le bon sens réduit au calcul; elle fait apprécier avec exactitude ce que les esprits justes sentent par une sorte d'instinct, sans qu'ils puissent souvent s'en rendre compte.

LAPLACE.

CONTENTS

PAGE

Preface v

Introduction to the Use of the Tables xiii

TABLES *

TABLE			PAGE
I.	Table of Deviates of the Normal Curve for each Permille of Frequency	xv	1
II.	Tables of the Probability Integral: Area and Ordinate of the Normal Curve in terms of the Abscissa	xvii	2–7
III.	Tables of the Probability Integral: Abscissa and Ordinate in terms of difference of Areas	xvii	9–10
IV.	Tables of the Probability Integral: Logarithms of Areas for high Values of Deviate	xxi	11
V.	Probable Errors of Means and Standard Deviations	xxii	12–18
VI.	Probable Errors of Coefficients of Variation	xxii	18
VII.	Abac for Probable Error of a Coefficient of Correlation r	xxiii	19
VIII.	Probable Error of a Coefficient of Correlation: Table to facilitate the calculation of $1-r^2$	xxiii	20–21
IX.	Values of the Incomplete Normal Moment Functions, First to Tenth Moments	xxiv	22–23
X.	Numerical Values and Graph of Generalised Probable Error	xxv	24
XI.	Values of the Functions ψ_1, ψ_2, and ψ_3 required to determine the constants of a Normal Frequency Distribution from the Moments of its Truncated Tail	xxviii	25
XII.	Tables for Testing Goodness of Fit: 3 to 30 frequency groupings	xxxi	26–28

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* The Roman figures to the pages refer to the Introduction, where the Table is discussed, the Arabic to the Table itself.

TABLE		PAGE	PAGE
XIII.	Tables for Testing Goodness of Fit: Auxiliary Table A, to assist in determining P for high values of χ^2	xxxiii	29
XIV—XVI.	Tables for Testing Goodness of Fit: Auxiliary Tables B—D. Numerical values needed in the calculation and extension of Tables of P	xxxiii	30
XVII.	Tables for Testing Goodness of Fit: Value of $(-\log P)$ for high values of χ^2 when the frequency is in a fourfold Grouping	xxxiv	31
XVIII.	Probability of Association on a Correlation Scale: Values of $(-\log P)$ for an observed value of the Tetrachoric Correlation r and the Standard Deviation of r for zero Association	xxxvi	31
XIX.	Probability of Association on a Correlation Scale: Values of χ^2 corresponding to Values of $(-\log P)$	xxxvi	32
XX.	Probability of Association on a Correlation Scale: Values of $\log \chi^2$ corresponding to Values of Tetrachoric r and ${}_0\sigma_r$	xxxvi	32
XXI.	Probability of Association on a Correlation Scale: Abac to determine ${}_0\sigma_r$ for a Population of given size, divided into a fourfold Table with given marginal Frequencies	xlii	33
XXII.	Probability of Association on a Correlation Scale: Abac to determine the Equiprobable Tetrachoric Correlation r_p from a Knowledge of $\log \chi^2$ and ${}_0\sigma_r$	xlii	34
XXIII.	Approximate Values of Probable Error of r from a fourfold Correlation Table: Values of χ_r for values of r	xl	35
XXIV.	Approximate Values of Probable Error of r from a fourfold Correlation Table: Values of χ_a for Values of marginal $\frac{1}{2}(1+\alpha)$	xl	35
XXV.	Table to determine the Probability that the mean of a very small sample n (4 to 10), drawn from a normal Population will not exceed (in algebraic sense) the Mean of the Population by more than z times the Standard Deviation of the Sample	xliii	36
XXVI.	Table to assist the Calculation of the Ordinates of the Frequency Curve $y = y_0 e^{-px/a}(1 + x/a)^p$	xlv	37
XXVII.	Tables of Powers of Natural Numbers, 1 to 100	xlvi	38–39
XXVIII.	Tables of Sums of Powers of Natural Numbers 1 to 100	xlvi	40–41
XXIX.	Tables to facilitate the Determination of Tetrachoric Correlation: Tables of the Tetrachoric Functions τ_1 to τ_6 for given marginal Frequencies	l	42–51

TABLE		PAGE	PAGE
XXX.	Tables to facilitate the Determination of Tetrachoric Correlation: Supplementary Tables for determining high Tetrachoric Correlations ($r =$ ·80 to 1·00) for given positions of the dichotomic lines . . .	liii	52–57
XXXI.	Table of the Logarithms of the Gamma Function, $\log\Gamma(p)$ from $p=1$ to $p=2$	lv	58–61
XXXII.	Table to deduce the Subtense from a knowledge of Arc and Chord in the Case of the Common Catenary	lvi	62–63
XXXIII.	A, Extension of Table XXXII for very flat Catenaries; B, Extension of Table XXXII for very narrow Catenaries	lvi	64
XXXIV.	Diagram to find the Correlation Coefficient r from the Mean Positive Contingency on the Hypothesis of a Normal Distribution	lvii	65
XXXV.	Diagram to determine the Type of a Frequency Distribution from a Knowledge of the Constants β_1 and β_2. Customary Values of β_1 and β_2	lxiii	66
XXXVI.	Diagram showing Distribution of Frequency Types for High or Unusual Values of β_1 and β_2	lxiii	67
XXXVII.	Probable Errors of Frequency Constants: Table for the Probable Error of β_1 for given Values of β_1 and β_2 .	lxii	68–69
XXXVIII.	Probable Errors of Frequency Constants: Table for the Probable Error of β_2 for given values of β_1 and β_2 .	lxii	70–71
XXXIX.	Probable Errors of Frequency Constants: Values of the Correlation of Deviations in β_1 and $\beta_2 (R_{\beta_1\beta_2})$ for given values of β_1 and β_2	lxii	72–73
XL.	Probable Errors of Frequency Constants: Probable Error of the Distance between Mean and Mode for given Values of β_1 and β_2	lxii	74–75
XLI.	Probable Errors of Frequency Constants: Probable Error of the Skewness for given Values of β_1 and β_2 .	lxii	76–77
XLII.	Values of the Frequency Constants β_3, β_4, β_5 and β_6 for given Values of β_1 and β_2 on the Assumption that the Frequency can be described by one or other of Pearson's Frequency Types	lxii	78–79
XLIII.	Probable Errors of the Frequency Constants: Probable Error of the Criterion of Type, κ_2, for given Values of β_1 and β_2	lxii	80–81
XLIV.	Probable Error of the Determination of Frequency Type: Value of Semi-minor Axis of Probability Ellipse for given Values of β_1 and β_2	lxiv	82–83

TABLE		PAGE	PAGE
XLV.	Probable Error of the Determination of Frequency Type: Value of Semi-major Axis of Probability Ellipse for given Values of β_1 and β_2	lxiv	84–85
XLVI.	Probable Error of the Determination of Frequency Type: Value of Angle between Major Axis of Probability Ellipse and Axis of β_2 for given Values of β_1 and β_2	lxiv	86–87
XLVII.	Probable Error of the Determination of Frequency Type: Diagram to determine for given Values of β_1 and β_2, a given Frequency Distribution belongs definitely to a given Type of Frequency	lxv	88
XLVIII.	Probable Occurrences in Second Small Samples. Percentage Frequency of each number of Successes in a Second Small Sample of m after p Successes in a First Small Sample of n	lxx	89–97
XLIX.	Logarithm of Factorial $\underline{\lvert n}$ from $n = 1$ to 1000	lxxiii	98–101
L.	Table of Fourth Moments of Subgroup Frequencies, i.e. $n \times x^4$, for $n = 1$ to 400, $x = 1$ to 19, for Verification of the Calculation of Raw Moments	lxxiv	102–112
LI.	General Term of Poisson's Exponential Limit to the Binomial, i.e. of the so-called "Law of Small Numbers." Value of $e^{-m} m^x / \underline{\lvert x}$ for $m = \cdot 1$ to $15 \cdot 0$, and $x = 0$ up to such figure as makes the Function significant in the sixth decimal Place	lxxvi	113–121
LII.	Table of Poisson's Exponential Limit to the Binomial to be used in the Determination of the Probable Errors of Cell Frequencies $n = 1$ to 30. Percentage of Occurrences in random Samples when the Population proportionately sampled would give actually n to the Cell	lxxvii	122–124
LIII.	Angles, Arcs and Decimals of Degrees. Table giving (a) the Arc for each Degree from 1° to 180°, (b) the Arc for each Minute and Second of Angle and (c) the Value in Decimals of a Degree of each Arc and Second of Angle	lxxix	125
LIV.	Table of the $G(r, \nu)$ Integrals. Values of $\log F(r, \nu)$ and $H(r, \nu)$ for $r = 1$ to 50 and $\phi = \tan^{-1} \nu$ from 0° to 45°	lxxxi	126–142
LV.	Miscellaneous Constants in Frequent Use by Statisticians and Biometricians	lxxxiii	143

INTRODUCTION TO THE USE OF THE TABLES

For this introduction to the use of the Tables I have largely drawn on the prefaces to the original papers in *Biometrika*, and record here my acknowledgements to the authors of the same.

INTERPOLATION.

(1) A word must first be said as to interpolation. Let a function u be tabled for the argument x proceeding by differences $\Delta x = h$. Then the scheme of such a table with the differences of u is:

$$
\begin{array}{|c|c|ccccc}
x_{-3} & u_{-3} & & & & & \\
 & & \Delta u_{-3} & & & & \\
x_{-3} & u_{-2} & & \Delta^2 u_{-3} & & & \\
 & & \Delta u_{-2} & & \Delta^3 u_{-3} & & \\
x_{-1} & u_{-1} & & \Delta^2 u_{-2} & & \Delta^4 u_{-3} & \\
 & & \Delta u_{-1} & & \Delta^3 u_{-2} & & \\
x_0 & u_0 & & \Delta^2 u_{-1} & & \Delta^4 u_{-2} & \\
 & & \Delta u_0 & & \Delta^3 u_{-1} & & \\
x_1 & u_1 & & \Delta^2 u_0 & & \Delta^4 u_{-1} & \text{etc., etc.} \\
 & & \Delta u_1 & & \Delta^3 u_0 & & \\
x_2 & u_2 & & \Delta^2 u_1 & & \Delta^4 u_0 & \\
 & & \Delta u_2 & & \Delta^3 u_1 & & \\
x_3 & u_3 & & \Delta^2 u_2 & & \Delta^4 u_1 & \\
 & & \Delta u_3 & & \Delta^3 u_2 & & \\
x_4 & u_4 & & \Delta^2 u_3 & & & \\
 & & \Delta u_4 & & & & \\
x_5 & u_5 & & & & &
\end{array}
$$

where:

$$
\begin{aligned}
\Delta u_s &= u_{s+1} - u_s, \\
\Delta^2 u_s &= \Delta u_{s+1} - \Delta u_s, \\
\Delta^3 u_s &= \Delta^2 u_{s+1} - \Delta^2 u_s \text{ etc., etc.}
\end{aligned}
$$

Now there are three interpolation formulae which it is desirable to remember. If the function be required for the value $x_0 + \theta h$ and this value be termed $u_0(\theta)$, then we have:

$$u_0(\theta) = u_0 + \theta \Delta u_0 - \frac{\theta(1-\theta)}{2\,!} \Delta^2 u_0 + \frac{\theta(1-\theta)(2-\theta)}{3\,!} \Delta^3 u_0 + \dots\dots\dots \text{(i)},$$

$$u_0(\theta) = u_0 + \theta \Delta u_0 - \frac{\theta(1-\theta^2)}{3\,!} \Delta^2 u_0 - \frac{\phi(1-\phi^2)}{3\,!} \Delta^2 u_{-1} \quad \dots\dots\dots\dots\dots \text{(ii)},$$

where $\phi = 1 - \theta$. This is Everett's formula*. And lastly:

$$u_0(\theta) = u_0 + \theta \tfrac{1}{2}(\Delta u_0 + \Delta u_{-1}) + \frac{\theta^2}{2\,!} \Delta^2 u_{-1} - \frac{\theta(1-\theta^2)}{3\,!} \tfrac{1}{2}(\Delta^3 u_{-1} + \Delta^3 u_{-2}) \dots \quad \text{(iii)},$$

where we work with the differences on or adjacent to the horizontal through x_0.

---

* *Journal of the Institute of Actuaries*, Vol. XXXV, p. 452.

It is very rarely indeed that we need go beyond second differences, often the first will suffice. Not infrequently the inverse problem arises, namely we are given $u_0(\theta)$ and have to determine θ from it. If we only go as far as second differences, either (i) or (iii) gives us a quadratic to find θ and the root will generally be obvious without ambiguity. Usually it suffices to find

$$\theta' = \{u_0(\theta) - u_0\}/\Delta u_0,$$

and then determine θ from

$$\theta = (u_0(\theta) - u_0)/\Delta u_0 + \frac{\theta'(1-\theta')}{2\,!}\Delta^2 u_0/\Delta u_0 \ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)};$$

or to find

$$\theta' = (u_0(\theta) - u_0)/\tfrac{1}{2}(\Delta u_0 + \Delta u_{-1})$$

and then

$$\theta = (u_0(\theta) - u_0)/\tfrac{1}{2}(\Delta u_0 + \Delta u_{-1}) - \frac{\theta'^2}{2\,!}\,\frac{\Delta^2 u_{-1}}{\frac{1}{2}(\Delta u_0 + \Delta u_{-1})} \ldots\ldots\ldots\ldots\text{(v)}.$$

Very often good results are readily obtained by applying Lagrange's interpolation formula which for three values of u reduces to

$$u_0(\theta) = (1-\theta^2)\,u_0 - \tfrac{1}{2}\theta(1-\theta)\,u_{-1} + \tfrac{1}{2}\theta(1+\theta)\,u_1 \ldots\ldots\ldots\ldots\ldots\text{(vi)}.$$

Or, we may use the mean of two such formulae and take

$$u_0(\theta) = (1-\theta)(1-\tfrac{1}{4}\theta)\,u_0 + \tfrac{1}{4}\theta(5-\theta)\,u_1 - \tfrac{1}{4}\theta(1-\theta)(u_{-1}+u_2) \quad\ldots\ \text{(vii)}.$$

The resulting quadratics are respectively:

$$\theta^2\left(\tfrac{1}{2}(u_1+u_{-1}) - u_0\right) + \theta\,\tfrac{1}{2}(u_1 - u_{-1}) + u_0 - u_0(\theta) = 0 \ \ldots\ldots\ldots\text{(vi)}^{\text{bis}},$$

and $\theta^2\,\tfrac{1}{4}(u_0 - u_1 + u_{-1} + u_2) + \theta\,\tfrac{1}{4}(5u_1 - 5u_0 - u_{-1} - u_2) + u_0 - u_0(\theta) = 0 \ \ldots\text{(vii)}^{\text{bis}}.$

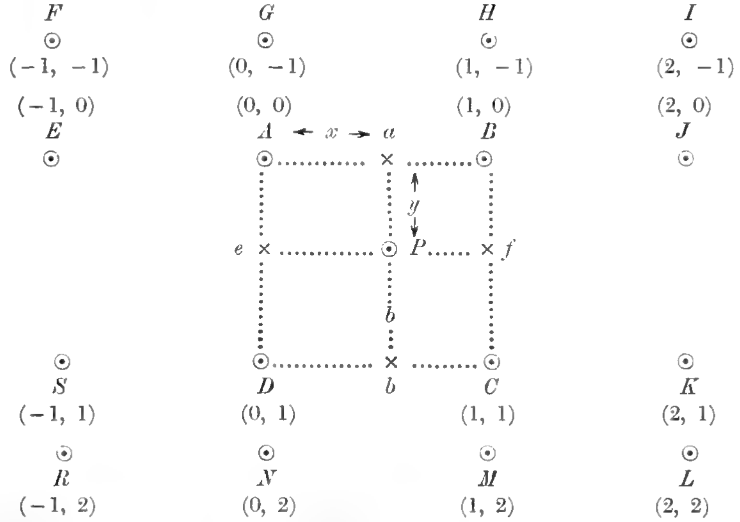
(2) There are some tables in this book which are of double entry, e.g. those for the Tetrachoric Functions and for the $G(r, \nu)$ Integrals. The simplest solid interpolation formula, using second differences, is:

$$u_{x,y} = u_{0,0} + x\Delta u_{0,0} + y\Delta' u_{0,0}$$
$$+ \tfrac{1}{2}\{x(x-1)\,\Delta^2 u_{0,0} + 2xy\,\Delta\Delta' u_{0,0} + y(y-1)\,\Delta'^2 u_{0,0}\} \ldots\ldots\text{(viii)},$$

where Δ denotes a difference with regard to x, and Δ' with regard to y. But if we consider $u_{x,y}$ to be the ordinate of a surface, and the figure, p. xv, to represent the xy plane of such a surface, then it is clear that, if P be the point x, y, and A, B, C, D, &c. the adjacent points at which the ordinates are known from the table of double entry, only the points A, B, C, D, J, and N are used by the above formula; and of these points, not equal weight is given to the fundamental points A, B, C, D, for C only appears in a second difference. If another point of the fundamental square other than A be taken as origin, we get a divergent, occasionally a widely divergent result. If we use only four points—A, B, C, D—to determine the value of the function at P, then we might take the ordinate at P of the plane which (by the method of least squares) most nearly passes through the four points of the surface vertically above A, B, C, D. We have then

$$u_{x,y} = \tfrac{1}{4}(u_{0,0} + u_{1,0} + u_{0,1} + u_{1,1}) + \tfrac{1}{2}(u_{1,0} - u_{0,0} + u_{1,1} - u_{0,1})(x - \cdot 5)$$
$$+ \tfrac{1}{2}(u_{0,1} - u_{0,0} + u_{1,1} - u_{1,0})(y - \cdot 5) \quad \ldots\ldots\text{(ix)},$$

but by trial it has been found that this formula gives occasionally worse results than that for first differences, using only three points. To find by the methods of simple interpolation (with first or first and second differences) the points a and b, and then interpolate P between them, generally gives a fairly good result; but this result usually differs somewhat from that obtained by first simply interpolating e and f and then interpolating between e and f[*]. Various other methods for interpolation in n-dimensioned space will be found discussed by Palin Elderton in *Biometrika*[†]. The ideal method can hardly yet be said to be known, and it may well vary from table to table and from one part of the same table to another. One or other of the above methods will, however, suffice in practice for most statistical purposes.



I consider now the individual tables.

TABLE I (p. 1)

Table of Deviates of the Normal Curve for each Permille of Frequency. (Calculated by Sheppard and published by Galton in *Biometrika*, Vol. v. p. 405.)

If N be the total number in a population, $z\delta x$ the frequency between x and $x+\delta x$, σ the standard-deviation, then the frequency curve of the population assuming its distribution to be Gaussian or normal will be:

$$z = \frac{N}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ix)}$$

---

[*] *B. A. Report*, Dover 1899. Tables of $G(r, \nu)$-Integrals, Report of the Committee (Drawn up by K. Pearson).

[†] Vol. VI. p. 94.

the origin being the mean. Table I. gives the value of x/σ for each thousandth of the area of this curve,—each 'permille'—reckoned from left to right.

In entering the table we enter from the left-hand column and top row if the permille be less than 500. For example, if the frequency below a particular value were 387 per thousand, the corresponding deviate would be $-0{\cdot}2871$, the number placed at the intersection of the ·38 row from left and ·007 column from top. The negative sign is always to be given when reading permilles below 500, because the deviate will be in defect of the mean, supposing increasing variates to be plotted as usual from left to right.

On the other hand if the permille be greater than 500 we enter the table from the right-hand column and bottom row. For example, if the permille be 748, the deviate is $+0{\cdot}6682$, the number placed at the intersection of the ·74 row from right and ·008 column from bottom of the table. The plus sign must be given, as the deviation is in excess of the mean, if the convention as to plotting variables has been observed.

Illustration: The following observations were made on the nature of the degree taken by 1011 Cambridge undergraduates measured at the Anthropological Society's Laboratory:

Poll	487	Second Class	182
Third Class	189	First Class	153

Find the deviates of these on a normal or Gaussian scale.

The sums from the lowest to each class top are 487, 676, 858 and 1011 respectively. If we term with Francis Galton the one man in a thousand of surpassing intelligence or special ability a "genius," we have on multiplying by ·0009891197 the reciprocal of 1011, the series for entering Table I. Thus we find:

		·4817	·6686	·8487	and ·9990
Hence:	(·481)	·0476	(·668) ·4344	(·848) 1·0279	(·999) 3·0902
	(·482)	·0451	(·669) ·4372	(·849) 1·0322	—
	Δ	·0025	Δ ·0028	Δ ·0043	—
	Δ × ·7	·00175	Δ × ·6 ·00168	Δ × ·7 ·00301	—
Deviates:		− ·0458	+ ·4361	+ 1·0309	+ 3·0902

Supposing with Pearson* that 100 units of intelligence ("mentaces") separate the lowest man of the First Class from the highest man of the Poll, we have $+1{\cdot}0309-(-{\cdot}0458)=100/\sigma$, where σ is the standard deviation of intelligence. Thus $\sigma=100/1{\cdot}0767=92{\cdot}88$ mentaces. Hence we conclude that the range of Third Class men is from $-4{\cdot}25$ (i.e. $92{\cdot}88\times(-{\cdot}0458)$) below to $+40{\cdot}50$

---

* *Biometrika*, Vol. v. p. 109.

(i.e. 92·88 × (+ ·4361)) above the average undergraduate. The range of Second Class men is from + 40·50 to + 95·75 mentaces above the average undergraduate, and the range of First Class men all those with more than 95·75 mentaces above the average. The "genius" corresponds to an excess of no less than 287·02 mentaces. If we suppose that one individual in 1000 is completely feeble-minded or practically wanting in all intelligence, we should credit roughly the average man with 300 mentaces, and we should then have our range of intelligence on a Gaussian scale:

Poll: below 296 mentaces;
Third Class: above 296 and below 340 mentaces;
Second Class: above 340 and below 396 mentaces;
First Class: above 396 mentaces;
"Genius": above 587 mentaces.

In rough numbers: Poll, below 300; Third Class, 300 to 350; Second Class, 350 to 400; First Class, over 400; "Genius," over 600.

Of course there is much that is hypothetical here, but the numbers give us some appreciation of the distribution of ability, and they serve to illustrate the construction of a Gaussian or normal scale. When more than three or four significant figures are needed Tables II and III must be used.

TABLES II AND III (pp. 2—10)

Tables of the Probability Integral: Area and Ordinate of the Normal Curve in terms of the Abscissa; and Abscissa and Ordinate in Terms of Difference of Areas. (Calculated by Dr W. F. Sheppard, and published in *Biometrika*, Vol. II. pp. 174—190.)

"Sheppard's Tables" were the first to express the Gaussian* or normal probability integral in terms of the standard deviation; they are so familiar to statisticians that it would almost seem a work of supererogation to explain their 'use,' which is further too manifold for full description. We can only give a few sample illustrations.

It is most important when using these tables to pay attention to the signs of the differences recorded at the tops of the columns.

Illustration (i). The mean length of cubit in 1063 adult English males is recorded as 18″·31 ± ·019 and of their 1063 adult sons as 18″·52 ± ·021. Determine the odds against these two measurements being really identical, i.e. random samples from the same population. We assume that the deviation of means and their differences follow the normal law. The difference is 0″·21 and the probable error of this difference $= \sqrt{(\cdot 019)^2 + (\cdot 021)^2} = 0''\cdot 0283$. Since the probable error

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* The term is usual, but inaccurate. Laplace had reached the probability integral and suggested its tabulation several years before Gauss.

$= {\cdot}67449 \times$ standard deviation, we have the standard deviation of the difference $= 0''{\cdot}04196$. Hence the deviation in terms of the standard deviation

$$= 0{\cdot}21/(0{\cdot}04196) = 5{\cdot}0048.$$

Table II, p. 8, gives the area $\frac{1}{2}(1+\alpha)$ of the normal curve up to the abscissa x/σ. Noting the remark at the foot of the table, we have

$$\begin{array}{lll} x/\sigma = 5{\cdot}00, & \frac{1}{2}(1+\alpha) = {\cdot}999{,}999{,}7133, & \\ x/\sigma = 5{\cdot}01, & \frac{1}{2}(1+\alpha) = {\cdot}999{,}999{,}7278, & \\ & \qquad\Delta = \qquad\qquad\quad 145, & \\ & \qquad\Delta \times 48 \qquad\qquad\; 70, & \\ x/\sigma = 5{\cdot}0048, & \frac{1}{2}(1+\alpha) = {\cdot}999{,}999{,}7203. & \end{array}$$

Hence $$\tfrac{1}{2}(1-\alpha) = {\cdot}000{,}000{,}2797.$$

Accordingly if we suppose the deviation as likely to be in defect as in excess, the probability that we shall reach the observed deviation, or exceed it, is $2 \times \frac{1}{2}(1-\alpha)$, and that we shall not is $\frac{1}{2}(1+\alpha) - \frac{1}{2}(1-\alpha)$, or the odds against the result on a pure random sampling chance are ·999,999,4406 to ·000,000,5594, or 1,787,629 to 1, i.e. overwhelming odds. Thus we may reasonably argue that sons in the professional classes in 1900 were substantially differentiated from their fathers by a longer forearm of about $\frac{1}{5}''$.

Illustration (ii). Find the value in mentaces of the mean intelligence of Poll-men, First, Second and Third Class men as given by the numbers in the *Illustration* to Table I.

The equation to the normal or Gaussian curve being

$$z = \frac{N}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(x/\sigma)^2},$$

we easily find that if there be 'tabled' ordinates z_1 and z_2* at the abscissae x_1 and x_2, which cut off an area n_{12}, then the mean $\bar{x}_{12}$ of this area is given by

$$\bar{x}_{12} = \sigma \,.\, (z_1 - z_2)/(n_{12}/N) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{x}).$$

It will be sufficient to take the values of the abscissae already found, i.e.

$$\begin{array}{ll} x_1/\sigma = -\ \ {\cdot}0458, & x_2/\sigma = +\ \ {\cdot}4361, \\ x_3/\sigma = +\,1{\cdot}0309, & x_4/\sigma = +\,3{\cdot}0902. \end{array}$$

We require the z's for these. For example:

$$\begin{array}{rll} x = {\cdot}04, & z = {\cdot}398{,}6233 & \\ {\cdot}05, & z = {\cdot}398{,}4439 & \\ \theta = {\cdot}58, & \Delta_1 & -1793 \\ & \Delta_2 & -397. \end{array}$$

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* The symbol z here used is that of the Tables, i.e. $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x/\sigma)^2}$.

Therefore by formula (i) p. xiii:

$$z_1 = \cdot 398{,}6233 - \cdot 58\,[1793] + \frac{\cdot 58 \times \cdot 42}{2}\,[397]$$

$$= \left.\begin{aligned} &\cdot 398{,}6233 \\ &\quad -1040 \\ &\quad + \quad 48 \end{aligned}\right\} = \cdot 398{,}5241.$$

Or, we might proceed as follows: for the Poll-men $\frac{1}{2}(1-\alpha) = \cdot 4817$, hence $\alpha = \cdot 0366$. But from Table III, p. 9, which gives z for α:

$$\begin{aligned} \alpha &= \cdot 03, & z &= \cdot 398{,}6603 \\ \alpha &= \cdot 04, & z &= \cdot 398{,}4408 \\ \theta &= \cdot 66, & \Delta_1 &= \quad -2194 \\ & & \Delta_2 &= \quad -627. \end{aligned}$$

Hence by formula (i):

$$z_1 = \cdot 398{,}6603 - \cdot 66\,[2194] + \frac{\cdot 66 \times \cdot 34}{2}\,[627]$$

$$= \left.\begin{aligned} &\cdot 398{,}6603 \\ &\quad -1448 \\ &\quad +70 \end{aligned}\right\} = \cdot 398{,}5225.$$

We conclude therefore that z would be correct to five figures with second differences, and that for four figures, first differences from either Table II or Table III will suffice.

If we use formula (ii) p. xiii—Everitt's formula—we find from Table II:

$$z_1 = \cdot 398{,}6233 - \cdot 58\,[1793] + \frac{\cdot 58 \times \cdot 6636}{6}\,[397] + \frac{\cdot 42 \times \cdot 8236}{6}\,[398]$$

$$= \left.\begin{aligned} &\cdot 398{,}6233 \\ &\quad -1040 \\ &\quad + \quad 13 \\ &\quad + \quad 5 \end{aligned}\right\} = \cdot 398{,}5211,$$

and from Table III:

$$z_1 = \cdot 398{,}6603 - \cdot 66\,[2194] + \frac{\cdot 66 \times \cdot 5644}{6}\,[627] + \frac{\cdot 34 \times \cdot 8844}{6}\,[627]$$

$$= \left.\begin{aligned} &\cdot 398{,}6603 \\ &\quad -1448 \\ &\quad + \quad 37 \\ &\quad + \quad 31 \end{aligned}\right\} = \cdot 398{,}5223.$$

Working with formula (iii), Table II gives us $z_1 = \cdot 398{,}5242$ and Table III $z_1 = \cdot 398{,}5225$ with second differences. We shall not therefore without higher differences get from any of our formulae closer than $\cdot 398{,}522$ with a possible error of 1 or 2 in the last place. This is, of course, amply sufficient for statistical purposes, where four figures as a rule would be sufficient.

Using formula (i) p. xiii we obtain:

$$z_1 = {\cdot}39852, \quad z_3 = {\cdot}23450,$$
$$z_2 = {\cdot}36275, \quad z_4 = {\cdot}00337.$$

Whence:

$$\bar{x}_{\infty 1} = + \frac{0 - {\cdot}39852}{{\cdot}4817}\sigma = - {\cdot}8273\sigma = - 76{\cdot}84 \text{ mentaces},$$

$$\bar{x}_{12} = + \frac{{\cdot}39852 - {\cdot}36275}{{\cdot}1869}\sigma = + {\cdot}1925\sigma = + 17{\cdot}88 \text{ mentaces},$$

$$\bar{x}_{23} = + \frac{{\cdot}36275 - {\cdot}23450}{{\cdot}1801}\sigma = + {\cdot}7121\sigma = + 66{\cdot}14 \text{ mentaces},$$

$$\bar{x}_{34} = + \frac{{\cdot}23450 - {\cdot}00337}{{\cdot}1503}\sigma = + 1{\cdot}5378\sigma = + 142{\cdot}83 \text{ mentaces},$$

$$\bar{x}_{4\infty} = + \frac{{\cdot}00337 - 0}{{\cdot}0010}\sigma = + 3{\cdot}3700\sigma = + 313{\cdot}01 \text{ mentaces}.$$

Assuming as before the average man to have 300 mentaces of intelligence we find:

Average Poll-man has 223 mentaces.
Average Third Class man has 318 mentaces.
Average Second Class man has 366 mentaces.
Average First Class man has 443 mentaces.
Average man of "genius" has 613 mentaces.

Thus the average First Class Honours man is twice as able as the average Poll-man, and the average "genius" has not quite twice the ability of the average Third Class Honours man.

Illustration (iii). It is required to determine normal curve frequencies corresponding to the following frequencies of the cephalic index in Bavarian skulls.

Here the mean and standard deviation found by moments in the usual way are

$$m = 83{\cdot}069, \quad \sigma = 3{\cdot}432.$$

The deviations from the mean were next expressed in terms of the standard deviation, i.e. these deviations are

$$-13{\cdot}569, \quad -12{\cdot}569, \quad \ldots -0{\cdot}569, \quad +{\cdot}431, \quad +1{\cdot}431, \quad +2{\cdot}431, \quad \ldots +14{\cdot}431,$$

and they are multiplied on a calculator by the reciprocal of the standard deviation, whence the column x/σ is found. Table II gives us $\frac{1}{2}(1+\alpha)$ knowing x/σ; this has been calculated by first differences only. We shall consider as an illustration to Table XII, whether the normal distribution thus reached is to be considered a good fit to the observations.

Index	Observed	x/σ	$\frac{1}{2}(1+\alpha)$	Calculated Frequency
69·5—70·5	1	−3·9539	·99996	Under 70·5 ·1
70·5—71·5	1	−3·6625	·99988	·2
71·5—72·5	—	−3·3711	·99963	·6
72·5—73·5	2·5	−3·0797	·99896	1·5
73·5—74·5	1·5	−2·7883	·99735	3·3
74·5—75·5	3·5	−2·4969	·99374	6·7
75·5—76·5	12·5	−2·2055	·98629	12·7
76·5—77·5	17	−1·9141	·97219	22·1
77·5—78·5	37	−1·6228	·94768	35·3
78·5—79·5	55	−1·3314	·90846	51·9
79·5—80·5	71·5	−1·0400	·85082	70·1
80·5—81·5	82	− ·7486	·77294	87·0
81·5—82·5	116	− ·4572	·67623	99·4
82·5—83·5	98	− ·1658	·56584	104·2
83·5—84·5	107	·1256	·54997	100·5
84·5—85·5	82	·4170	·66165	89·1
85·5—86·5	74	·7084	·76064	72·6
86·5—87·5	58	·9998	·84129	54·3
87·5—88·5	34·5	1·2912	·90167	37·4
88·5—89·5	19	1·5825	·94324	23·7
89·5—90·5	10	1·8739	·96953	13·8
90·5—91·5	8	2·1653	·98482	7·4
91·5—92·5	3	2·4567	·99299	3·6
92·5—93·5	1·5	2·7481	·99700	1·6
93·5—94·5	2	3·0395	·99882	·7
94·5—95·5	1·5	3·3309	·99957	·3
95·5—96·5	—	3·6223	·99985	Over 95·5 ·1
96·5—97·5	—	3·9137	·99995	—
97·5—98·5	1	4·2050	·99999	—
Totals ...	900	—	—	900·2

Table IV (p. 11)

Extension of the Table of the Probability Integral $F=\frac{1}{2}(1-\alpha)$. (Calculated by Julia Bell, M.A., *Drapers' Research Memoirs*, Biometric Series, VIII, p. 27.)

It has been found needful occasionally to determine probabilities for deviations exceeding considerably the limit $x/\sigma=6$ of Sheppard's Table II.

Illustration. If $x/\sigma=34{\cdot}31$, determine to two significant figures the probability of a deviation occurring as large or larger than this.

The table gives us:

33	238·39135		
		14·56180	
34	252·95315		·43393
		14·99573	
35	267·94888		·43394
		15·42967	
36	283·37855		

Hence using formula (ii) p. xiii:

$$(-\log F) = 252{\cdot}95315 + {\cdot}31\,[14{\cdot}99573] - \frac{{\cdot}31 \times {\cdot}9039}{6} \times {\cdot}43393 - \frac{{\cdot}69 \times {\cdot}5239}{6} \times {\cdot}43394$$

$$= \left.\begin{array}{r} 252{\cdot}95315 \\ +\ 4{\cdot}64868 \\ -\ {\cdot}04641 \end{array}\right\} = 257{\cdot}55542.$$

Hence
$$\log F = -257{\cdot}55542 = \overline{258}{\cdot}44458,$$
$$F = 2{\cdot}7834/10^{258},$$
which measures the improbability required.

TABLE V (pp. 12—18) AND TABLE VI (p. 18)

Probable Errors of Means, Standard Deviations and Coefficients of Variation. (Table V calculated by Winifred Gibson, B.Sc.; Table VI by Dr Raymond Pearl and T. Blakeman, M.A. *Biometrika*, Vol. IV. pp. 385—393.)

If m be a mean, σ a standard deviation and $V = 100\sigma/m$ a coefficient of variation, for a population of n, we have

Probable Error of Mean

$$= {\cdot}6744898\sigma/\sqrt{n} = \chi_1\sigma \quad\text{(xi)},$$

Probable Error of Standard Deviation

$$= {\cdot}6744898\sigma/\sqrt{2n} = \chi_2\sigma \quad\text{(xii)},$$

Probable Error of the Coefficient of Variation

$$= {\cdot}6744898V \times \left\{1 + 2\left(\frac{V}{100}\right)^2\right\}^{\frac{1}{2}} \Big/ \sqrt{2n} \quad\text{(xiii)},$$

$$= {\cdot}6744898/\sqrt{2n} \times \psi$$

$$= \chi_2 \times \psi \quad\text{(xiv)}.$$

Table V gives χ_1 and χ_2 for each value of n up to 1000, Table VI gives ψ for each value of V proceeding by units from 0 to 50.

When the frequency n is greater than 1000, the tables may still be used by taking out a square factor, which can be divided out at sight.

Illustration (i). $n = 2834 = 4 \times 708{\cdot}5$.

$$n = 708,\ \chi_1 = {\cdot}02535; \quad n = 709,\ \chi_1 = {\cdot}02533.$$

$$\therefore\ n = 708{\cdot}5,\ \chi_1 = {\cdot}02534, \text{ and } \therefore \text{ for } n = 2834,$$

we have
$$\chi_1 = {\cdot}01267.$$

Illustration (ii). In the case of the 900 Bavarian crania of the *Illustration* (iii) to Table II the values

$$m = 83{\cdot}069, \quad \sigma = 3{\cdot}432,$$

and therefore $V = 4{\cdot}1315$ were found. It is required to find the probable errors of these values.

For 900, $\chi_1 = {\cdot}02248$ and $\chi_2 = {\cdot}01590$, hence the probable errors of m and σ are

$$\text{p.e. of } m = \chi_1 \sigma = {\cdot}077,$$

$$\text{p.e. of } \sigma = \chi_2 \sigma = {\cdot}055.$$

Next for $V = 4{\cdot}1315$,

$$\begin{aligned}\psi_1 &= 4{\cdot}00639 + {\cdot}1315\,[1{\cdot}00609] - \tfrac{1}{2}({\cdot}1315)({\cdot}8685) \times [299] \\ &= 4{\cdot}13852.\end{aligned}$$

$$\begin{aligned}\text{p.e. of } V &= \chi_2 \times \psi = {\cdot}01590 \times 4{\cdot}13852 \\ &= {\cdot}0658.\end{aligned}$$

Hence our results should be recorded as

$$\begin{aligned} m &= 83{\cdot}069 \pm {\cdot}077, \\ \sigma &= 3{\cdot}432 \pm {\cdot}055, \\ V &= 4{\cdot}1315 \pm {\cdot}0658. \end{aligned}$$

TABLE VII (p. 19)

Abac for Probable Errors of r. (Calculated by Dr David Heron, drawn by H. Gertrude Jones, *Biometrika*, Vol. VII. p. 411.)

The probable error of a coefficient of correlation

$$= \frac{{\cdot}67449}{\sqrt{n}} (1 - r^2).$$

To ascertain the value of this function approximately, turn the page horizontal, enter with the proper frequency on the scale at base, follow the corresponding vertical until the sloping line with the given correlation is reached, then move along the horizontal to the left until the scale of probable error is reached, which will give the required approximate probable error of the correlation in a population of the given size.

Illustration. $r = {\cdot}671$ and $n = 415$.

Probable error of $r = {\cdot}018$. The actual value is ${\cdot}0182$.

TABLE VIII (pp. 20—21)

Values of $1 - r^2$. (Calculated by H. E. Soper, M.A.)

Illustration. As in the last example let

$$r = {\cdot}671 \text{ and } n = 415.$$

Then probable error of $r = \chi_1 \times (1 - r^2)$

$$\begin{aligned} &= {\cdot}549{,}759 \text{ (from Table VIII)} \\ &\times {\cdot}03311 \text{ (from Table V)} \\ &= {\cdot}0182. \end{aligned}$$

The value of $1 - r^2$ for r to four instead of to three figures can be obtained by interpolation.

TABLE IX (pp. 22—23)

Values of the Incomplete Normal Moment Functions. (Calculated by Dr Alice Lee, *Biometrika*, Vol. VI. p. 59.)

The nth incomplete normal moment function is defined to be

$$\mu_n(x) = \frac{1}{\sqrt{2\pi}} \int_0^x x^n e^{-\frac{1}{2}x^2}\, dx \quad \text{.......................(xv)}.$$

We take

$$\left. \begin{aligned} m_n(x) &= \mu_n(x)/\{(n-1)(n-3)(n-5)\ldots 1\} \text{ if } n \text{ be even} \\ &= \mu_n(x)/\{(n-1)(n-3)(n-5)\ldots 2\} \text{ if } n \text{ be odd} \end{aligned} \right\} \quad \text{......(xvi)},$$

and $m_n(x)$ is the function tabled.

In multiple correlation (supposed normal), the frequency surface is

$$\left. \begin{aligned} z &= \frac{N}{(2\pi)^{\frac{1}{2}n}\sigma_1\sigma_2\ldots\sigma_n\sqrt{R}} e^{-\frac{1}{2}\chi^2} \\ \text{where} \qquad \chi^2 &= \frac{1}{R}\{S(R_{pp}x_p^2/\sigma_p^2) + 2S'(R_{pq}x_px_q/\sigma_p\sigma_q)\} \end{aligned} \right\} \text{...........(xvii)},$$

and

$$R = \begin{vmatrix} 1, & r_{12}, & r_{13} \ldots r_{1n} \\ r_{21}, & 1, & r_{23} \ldots r_{2n} \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ r_{n1}, & r_{n2}, & r_{n3} \ldots 1 \end{vmatrix} \quad \text{............. ....(xviii)},$$

while R_{pp} and R_{pq} are the usual minors.

χ^2 = constant is the "ellipsoid" of equal frequency in n-dimensional space.

The total frequency, i.e. the volume of the surface, inside any ellipsoid χ is

$$I_\chi = \int_0^\chi z\,dV$$

and

$$\left. \begin{aligned} I_\chi/N &= \frac{\sqrt{2\pi}\,\mu_{n-1}(\chi)}{2.4.6\ldots(n-2)} \text{ if } n \text{ be even} \\ &= \frac{2\mu_{n-1}(\chi)}{1.3.5\ldots(n-2)} \text{ if } n \text{ be odd} \end{aligned} \right\} \quad \text{..............(xix)}.$$

Thus a knowledge of the incomplete normal moment functions enables us to predict for multiple variables whether an outlying observation consisting of a system of n variate values is or is not reasonably probable.

If $I_{\chi_0}/N = \frac{1}{2}$, we obtain the 'ellipsoidal' contour χ_0 within which half the frequency lies. This χ_0 is the "generalised probable error" of Pearson and Lee.

Values of the "generalised probable error" coefficients are given in Table X for $n=1$ to 11, and by means of a smooth curve the results may probably be extended to $n=15$. The values found for this extension are:

	$n=12$	$n=13$	$n=14$	$n=15$
χ_0	3·367	3·513	3·654	3·791

Illustration (i). Let us consider long bone data for Frenchmen. $1=F=$ femur, $2=H=$ humerus, $3=T=$ tibia, $4=R=$ radius*, then by formula (xviii) p. xxiv:

$$R=\begin{vmatrix} 1, & \cdot 8421, & \cdot 8058, & \cdot 7439 \\ \cdot 8421, & 1, & \cdot 8601, & \cdot 8451 \\ \cdot 8058, & \cdot 8601, & 1, & \cdot 7804 \\ \cdot 7439, & \cdot 8451, & \cdot 7804, & 1 \end{vmatrix}$$

Further in cms:

$$\begin{aligned} m_1 &= 45{\cdot}23, & \sigma_1 &= 2{\cdot}372, \\ m_2 &= 33{\cdot}01, & \sigma_2 &= 1{\cdot}538, \\ m_3 &= 36{\cdot}81, & \sigma_3 &= 1{\cdot}799, \\ m_4 &= 24{\cdot}39, & \sigma_4 &= 1{\cdot}170. \end{aligned}$$

What is the chance that the following individual may be considered French?

$$F'=36{\cdot}97,\quad H'=26{\cdot}82,\quad T'=30{\cdot}56,\quad R'=20{\cdot}68.$$

The deviations in terms of their standard deviations are:

$$x_1=(F'-m_1)/\sigma_1=-3{\cdot}482,\quad x_2=(H'-m_2)/\sigma_2=-4{\cdot}059,$$
$$x_3=(T'-m_3)/\sigma_3=-3{\cdot}474,\quad x_4=(R'-m_4)/\sigma_4=-3{\cdot}171.$$

Further:

$$\frac{R_{11}}{R}=3{\cdot}7810,\quad \frac{R_{22}}{R}=6{\cdot}5496,\quad \frac{R_{33}}{R}=4{\cdot}3406,\quad \frac{R_{44}}{R}=3{\cdot}6508,$$
$$\frac{R_{12}}{R}=2{\cdot}0231,\quad \frac{R_{13}}{R}=1{\cdot}1404,\quad \frac{R_{14}}{R}=0{\cdot}2130,\quad \frac{R_{23}}{R}=2{\cdot}1946,$$
$$\frac{R_{24}}{R}=2{\cdot}3175,\quad \frac{R_{34}}{R}=0{\cdot}6842.$$

Whence $\chi^2=16{\cdot}741{,}035$ and $\chi=4{\cdot}0916$,

n is even, hence: $$I_\chi/N=\frac{\sqrt{2\pi}\,.\,\mu_3\,(4{\cdot}0916)}{2}=\sqrt{2\pi}\,.\,m_3,$$

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* For particulars of these length measurements the reader must consult *R. S. Proc.* Vol. 61, pp. 343 *et seq.* and *Phil. Trans.* Vol. 192, A, p. 180.

and from the Table, p. 22, we have by formula (i), p. xiii:

$$m_3(4{\cdot}0916) = {\cdot}397{,}7378 + {\cdot}916[3650] - \tfrac{1}{2}({\cdot}916)({\cdot}084)[1043]$$
$$= {\cdot}398{,}0682.$$

Hence $$I_\chi/N = \sqrt{2\pi} \times {\cdot}398{,}0682 = {\cdot}9978.$$

Thus the odds are 9978 to 22, say 454 to 1 against a deviation-complex as great as or greater than this occurring in a French male skeleton, i.e. the bones very improbably were those of a Frenchman. Actually they were those of a male of the Aino race.

Illustration (ii). The following are the ordinates of a frequency distribution for the speed of American trotting horses*. It is assumed that they form a truncated normal curve, and we require to determine (i) the mean of the whole population, (ii) its standard deviation, and (iii) what fraction the 'tail' is of the whole population.

The values of frequency in an arbitrary scale are:

Seconds	Frequency	Seconds	Frequency
29—28	92·8	*20—19*	45·8
28—27	100·4	*19—18*	38·4
27—26	95·0	*18—17*	27·8
26—25	71·2	*17—16*	19·8
25—24	67·6	*16—15*	10·7
24—23	61·3	*15—14*	15·8
23—22	61·4	*14—13*	7·9
22—21	44·8	*13—12*	5·0
21—20	44·5	*12—11*	2·1
		11—10	5·6

Taking the working origin at 20—19 seconds, we find

$$\nu_1' = -3{\cdot}9214, \quad \nu_2' = 32{\cdot}545{,}666$$

for raw moment coefficients. Hence, if d be the distance from 29 seconds, i.e. the stump of the tail from the mean, and Σ the standard deviation of the tail about its mean:

$$d = 9{\cdot}5 - 3{\cdot}9214 = 5{\cdot}5786 \text{ secs.},$$
$$\Sigma^2 = \nu_2' - \nu_1'^2 = 17{\cdot}168{,}288,$$

and accordingly $$\Sigma^2/d^2 = {\cdot}5517.$$

If this value be compared with those for ψ_1 in Table XI, p. 25, it will be seen that we have got slightly more than the half of a normal curve, i.e. not a true tail. We cannot therefore use Table XI, but must fall back on Table IX.

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* Galton, *R. S. Proc.* Vol. 62, p. 310. See for another method of fitting, Pearson, *Biometrika*, Vol. II. p. 3.

Let x be the distance from stump to centre of curve, n equal the area of truncated portion, and N be whole population. Then

$$n/N = \int_0^\infty + \int_0^{x/\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2} dx' = \tfrac{1}{2} + m_0(x/\sigma) \text{..............(xx)};$$

$$\begin{aligned} n\bar{x} &= N\sigma \left\{ \int_0^\infty + \int_{-x/\sigma}^0 \frac{x'}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2} dx' \right\}, \\ &= N\sigma \left\{ \frac{1}{\sqrt{2\pi}} - \int_0^{x/\sigma} \frac{x'}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2} dx' \right\}, \\ &= N\sigma \left\{ \frac{1}{\sqrt{2\pi}} - m_1(x/\sigma) \right\} \text{ ....(xxi)}; \end{aligned}$$

$$\begin{aligned} n\mu_2' &= N\sigma^2 \left\{ \int_0^\infty + \int_{-x/\sigma}^0 \frac{x'^2}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2} dx' \right\} \\ &= N\sigma^2 \left\{ \tfrac{1}{2} + \int_0^{x/\sigma} \frac{x'^2}{\sqrt{2\pi}} e^{-\frac{1}{2}x'^2} dx' \right\} \\ &= N\sigma^2 \left\{ \tfrac{1}{2} + m_2(x/\sigma) \right\} \text{ ....(xxii)}. \end{aligned}$$

Now $\qquad d = \bar{x} + x$, and $\Sigma^2 = \mu_2' - \bar{x}^2$.

Hence
$$\frac{d}{\sigma} = \frac{\dfrac{1}{\sqrt{2\pi}} - m_1(x/\sigma) + \dfrac{x}{\sigma}\left\{\tfrac{1}{2} + m_0(x/\sigma)\right\}}{\tfrac{1}{2} + m_0(x/\sigma)} \text{ ....(xxiii)},$$

$$\frac{\Sigma^2}{\sigma^2} = \frac{\left\{\tfrac{1}{2} + m_2(x/\sigma)\right\}\left\{\tfrac{1}{2} + m_0(x/\sigma)\right\} - \left\{\dfrac{1}{\sqrt{2\pi}} - m_1(x/\sigma)\right\}^2}{\left\{\tfrac{1}{2} + m_0(x/\sigma)\right\}^2} \text{ ......(xxiv)},$$

and
$$\frac{\Sigma^2}{d^2} = \frac{\left(\tfrac{1}{2} + m_2\right)\left(\tfrac{1}{2} + m_0\right) - \left(\dfrac{1}{\sqrt{2\pi}} - m_1\right)^2}{\left\{\dfrac{1}{\sqrt{2\pi}} - m_1 + x'\left(\tfrac{1}{2} + m_0\right)\right\}^2} \text{ ....(xxv)},$$

say, for brevity.

Here m_1 and m_2 are given by Table IX and $\frac{1}{2} + m_0$ is the $\frac{1}{2} + \frac{1}{2}\alpha$ of Table II.

Formula (xxv) has not yet been tabled for different values of x', as it occurs much more rarely than the corresponding function for a true tail.

If we take three values $x' = 0$, 0·1 and 0·2, we have, from Tables II and IX,

$x' = 0$, $\quad \frac{1}{2} + m_0 = \cdot 500{,}0000$, $\quad \frac{1}{2} + m_2 = \cdot 500{,}0000$, $\quad \frac{1}{\sqrt{2\pi}} - m_1 = \cdot 398{,}9423$,
$x' = \cdot 1$, $\quad$ „ $= \cdot 539{,}8278$, $\quad$ „ $= \cdot 500{,}1325$, $\quad$ „ $= \cdot 396{,}9526$,
$x' = \cdot 2$, $\quad$ „ $= \cdot 579{,}2597$, $\quad$ „ $= \cdot 501{,}0512$, $\quad$ „ $= \cdot 391{,}0427$.

Whence from formula (xxv) for the three values of x

$$\Sigma^2/d^2 = \cdot 5708, \quad \cdot 5528 \text{ and } \cdot 5345.$$

But our value of Σ^2/d^2 is ·5517. Thus we find by interpolation

$$x' = \cdot 1060.$$

It remains to determine m_0, m_1 and m_2 for this value of x', or simpler $\frac{1}{2} + m_0$, $\frac{1}{2} + m_2$ and $\frac{1}{\sqrt{2\pi}} - m_1$ from the above values for $x' = \cdot 1$ and $x' = \cdot 2$. We find

$$\tfrac{1}{2} + m_0 = \cdot 542{,}194, \quad \tfrac{1}{2} + m_2 = \cdot 500{,}184, \quad \frac{1}{\sqrt{2\pi}} - m_1 = \cdot 396{,}598.$$

Whence
$$d/\sigma = \frac{\cdot 396{,}598}{\cdot 542{,}194} + \cdot 1060 = \cdot 8375.$$

Thus
$$\sigma = 5 \cdot 5786/(\cdot 8375) = 6 \cdot 6610 \text{ secs.},$$
$$x = x' \times \sigma = \cdot 7061 \text{ secs.},$$
$$n/N = \cdot 5422.$$

This gives a mean of 28·29 secs. with a variability measured by 6·66 secs. Those actually registered as trotters are 54 °/₀ of the population.

TABLE X (p. 24)

See under Table IX, p. xxv.

TABLE XI (p. 25)

Constants of Normal Curve from Moments of Tail about Stump. (Pearson and Lee, *Biometrika*, Vol. VI. pp. 65 and 68.)

This Table may be of service in cases of the following kind:

(*a*) In some cases a record is actually truncated as in the case of the American Trotters dealt with on p. xxvi. Or, again, we may take a record of stature obtained by measuring all men who exceed 69″, or a record of mental capacity found by measuring all persons with low intelligence in a community.

(*b*) When we recognise heterogeneity, e.g. when we have a mixture of male and female bones, or two strains of trypanosomes, we can occasionally get a rough approximate analysis by supposing the tails to represent homogeneous material and then fitting them with normal curves. From the tails we get two components, and if their compound agrees fairly well with the observed total we have performed an analysis far more rapidly than by using the nonic equation*. Difficulties arise owing to deviations from Gaussian frequency being not infrequent; different dichotomic lines may give different results, and owing to paucity of material in the 'tails' and corresponding irregularity there will be large probable errors.

Cases under (*a*) and (*b*) will be treated by exactly the same process, but our Table supposes that the distribution considered is less than half the normal distribution. The rules for determining the mean, standard deviation and total frequency of the untruncated population are given on p. 25 under Table XI itself.

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* Pearson, *Phil. Trans.* Vol. 185, A, p. 84.

Illustration (i). The following distribution represents the 'tail' of a group of 301 mentally defective children measured by G. Jaederholm using Binnet-Simon test methods*:

Mental Defect in Years.

	−3·45—−3·95	−3·95—−4·45	−4·45—−4·95	−4·95—−5·45	−5·45—−5·95	−5·95—−6·45	−6·45—−6·95	−6·95—−7·45	Total
Frequency ...	34	18	13	3	4	4	1	1	78

We find, with origin at $-3{\cdot}45$,

$$d = 1{\cdot}8077 \text{ in } \tfrac{1}{2} \text{ years and } \Sigma^2 = 2{\cdot}7258.$$

Hence $$\psi_1 = \Sigma^2/d^2 = {\cdot}834.$$

The Table (p. 25) gives us $h' = 2{\cdot}186$ and $\psi_2 = 2{\cdot}833$. Hence the mean is at distance from $-3{\cdot}45$ on left $= 2{\cdot}186 \times \sigma$, and for the standard deviation

$$\sigma = \psi_2 \times d = 2{\cdot}833 \times 1{\cdot}8077 = 5{\cdot}1212 \ \tfrac{1}{2} \text{ years} = 2{\cdot}5606 \text{ years},$$

whence mean is at distance $h = 5{\cdot}597$ years from $-3{\cdot}45$.

Now $h' = 2{\cdot}186$ corresponds by Table II to $\frac{1}{2}(1+\alpha) = {\cdot}98559$.

$$\therefore\ n/N = {\cdot}01441, \text{ or } N = 78/({\cdot}01441) = 5413.$$

Thus the distribution is the tail of a population of 5413 individuals with a mean at 2·15 years of mental *excess*, and a standard deviation of 2·56 years.

The example is merely illustrative and of no importance in itself.

Illustration (ii). Assuming that the correlation data follow the normal law, determine indirectly the 'plural' partial correlation† of habits of mother and health of baby for the limited universe of unclean homes.

The correlations between the various characters found by tetrachoric tables are

Habits of mother and health of baby: $r_{12} = {\cdot}3060$,
Habits of mother and cleanliness of home: $r_{13} = {\cdot}7958$,
Health of baby and cleanliness of home: $r_{23} = {\cdot}2578$.

There are 947 out of 2931 homes which are not clean. Data from Bradford.

Here $$n/N = 947/2931 = {\cdot}3231 = \tfrac{1}{2}(1-\alpha);$$

hence $\alpha = {\cdot}3538$, and by Table III

$$x' = x/\sigma = {\cdot}459{,}049.$$

---

* *Mendelism and the Problem of Mental Defect.* II. *On the Continuity of Mental Defect.* Dulau & Co., 37, Soho Square, W.

† *Biometrika*, Vol. IX. p. 289.

Now on the assumption of a normal distribution:

$$n\mu'_s = \int_x^\infty x^s \frac{N}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}(x^2/\sigma^2)}\,dx$$

$$= N\sigma^s \int_{x'}^\infty x'^s e^{-\frac{1}{2}x'^2}\,dx'$$

$$= N\sigma^s \{\mu_s(\infty) - \mu_s(x')\}.$$

Here Table IX (p. 22) shows that if s be odd, $m_s(\infty) = \cdot 398{,}9423$, i.e. $1/\sqrt{2\pi}$, and (p. 23) if s be even, $m_s(\infty) = \cdot 500{,}0000$.

Hence in obtaining the moment coefficients of the tail, about the mean of the whole population, $m_s(x')$ should be subtracted from ·398,9423 or from ·500,0000 before the results are multiplied by $(s-1)(s-3)\ldots 2$ or $(s-1)(s-3)\ldots 1$, when s is odd or even respectively. It is convenient to term $\mu_s(\infty) - \mu_s(x')$ the complementary incomplete moment function of order s*.

For $s = 1$ and $s = 2$, we have

$$n\mu_1' = \sigma N \{m_1(\infty) - m_1(x')\},$$

$$n\mu_2' = \sigma^2 N \{m_2(\infty) - m_2(x')\},$$

for in this case the multiplying factors to proceed from $m_s(x')$ to $\mu_s(x')$ are both unity.

Now $x' = x/\sigma$ can be found when n is known from Tables II or III. Hence we have for the distance of centroid of tail from its stump, and for the square of its standard-deviation about its centroid:

$$\left.\begin{aligned} d &= \mu_1' - h'\sigma = \sigma\left[\frac{N}{n}\{m_1(\infty) - m_1(x')\} - h'\right] \\ \Sigma^2 &= \sigma^2\frac{N}{n}\{m_2(\infty) - m_2(x')\} - \mu_1'^2 \end{aligned}\right\} \ldots\ldots\ldots\ldots\text{(xxvi)}.$$

Of course $m_1(\infty) - m_1(x')$ is the z of Sheppard's Tables II and III.

Returning to our numerical example, we have from Table IX (p. 22):

$$m_1(\cdot 45905) = \cdot 030{,}6721 + \cdot 5905\,[162049] - \tfrac{1}{2}(\cdot 5905)(\cdot 4095)\,[26358]$$

$$= \cdot 039{,}9222,$$

$$m_1(\infty) - m_1(\cdot 45905) = \cdot 359{,}02.$$

Found directly from Sheppard's Tables, it equals ·35905.

Similarly from Table IX (p. 23):

$$m_2(\cdot 45905) = \cdot 008{,}1136 + \cdot 5905\,[73{,}162] - \tfrac{1}{2}(\cdot 5905)(\cdot 4095)\,[30661]$$

$$= \cdot 012{,}0630,$$

and $$m_2(\infty) - m_2(\cdot 45905) = \cdot 487{,}9370.$$

---

* It is the function used by Dr Alice Lee and myself, *Biometrika*, Vol. VI. p. 65 to form Table XI, p. 25, but by an oversight not adequately distinguished in symbol from $\mu_s(x')$ of p. 60 of the same memoir.

We thus reach

$$d = \sigma\,({\cdot}35902/{\cdot}3231 - {\cdot}45905) = {\cdot}65212\sigma,$$

and

$$\Sigma^2 = \sigma^2\,\{{\cdot}48794/{\cdot}3231 - (1{\cdot}11117)^2\} = {\cdot}274{,}786\sigma^2.$$

Or the distance of centroid from stump, and the standard deviation of the tail are respectively

$$d = {\cdot}65212\sigma \quad\text{and}\quad \Sigma = {\cdot}52426\sigma.$$

We now find

$$\lambda = \sqrt{1 - \Sigma^2/\sigma^2} = {\cdot}85155.$$

The formula for the 'plural' correlation coefficient is*

$${}_3\mathfrak{r}_{12} = \frac{r_{12} - r_{13}'\,r_{23}'}{\sqrt{1 - r_{13}'^2}\,\sqrt{1 - r_{23}'^2}} \quad \text{........................(xxvii),}$$

where

$$r_{13}' = \lambda r_{13}, \quad r_{23}' = \lambda r_{23},$$
$$= {\cdot}2195, \qquad = {\cdot}6777.$$

Thus

$${}_3\mathfrak{r}_{12} = {\cdot}2191.$$

Actually taking out the universe of not clean homes, the correlation of habits of mother and health of child is ·1615. The difference is considerable, but ${}_3\mathfrak{r}_{12}$ is deduced from the entire population of 2931 homes, while ·1615 depends on only a third of this number. The 'singular' partial correlation is ${}_3r_{12} = {\cdot}1723$, i.e. the average relation between habits of mother and health of child for each individual grade of cleanliness of home.

TABLE XII (p. 26)

Tables for testing Goodness of Fit. (W. Palin Elderton, *Biometrika,* Vol. I. pp. 155—163.)

The theory of testing frequency distributions for goodness of fit was first given by Pearson† and may be summed up as follows:

If a frequency distribution or table contains n' 'cells' and the contents of these cells be m_1', m_2', m_3' ... m_n' in number, while m_1, m_2, m_3 ... m_n be the numbers that would occur in these cells on any theory; then calculate

$$\chi^2 = S\left\{\frac{(m_r' - m_r)^2}{m_r}\right\}$$
$$= \text{sum}\left(\frac{\text{square of difference of theoretical and observed frequencies}}{\text{theoretical frequency}}\right) \text{..(xxviii),}$$

and the probability that random sampling would lead to as large or larger deviation between theory and observation is

$$\left.\begin{aligned} P &= \sqrt{\frac{2}{\pi}}\int_\chi^\infty e^{-\frac{1}{2}\chi^2}\,d\chi + \sqrt{\frac{2}{\pi}}\,e^{-\frac{1}{2}\chi^2}\left(\frac{\chi}{1} + \frac{\chi^3}{1\,.\,3} + \frac{\chi^5}{1\,.\,3\,.\,5} + \dots + \frac{\chi^{n'-3}}{1\,.\,3\,.\,5\,\dots\,(n'-3)}\right) \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } n' \text{ be even} \\ &= e^{-\frac{1}{2}\chi^2}\left(1 + \frac{\chi^2}{2} + \frac{\chi^4}{2\,.\,4} + \frac{\chi^6}{2\,.\,4\,.\,6} + \dots + \frac{\chi^{n'-3}}{2\,.\,4\,.\,6\,\dots\,(n'-3)}\right) \qquad \text{if } n' \text{ be odd} \end{aligned}\right\}$$
$$\text{...........(xxix).}$$

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* Pearson, *Phil. Trans.* Vol. 200, A, p. 25.

† *Phil. Mag.* Vol. L. pp. 157—175, 1900.

Short provisional tables of P were given in the article referred to and were replaced in the following year by the present standard tables of Palin Elderton.

In using the test for goodness of fit, due regard should be paid to the conditions under which it is deduced. It is assumed that the frequencies form a normal system of variates. This is legitimate only when in the binomial $(p+q)^n$, q is not very small as compared with p. If q be not very small as compared with p, even for n finite, the binomial approaches closely to the normal curve. Accordingly in using the test it is desirable to club together small frequencies at the tails of curves or margins of surfaces. The difficulty becomes very obvious when theory can go by fractions, but observations only by units.

The theory can be extended to cover much ground in all sorts of sampling*.

Illustration. The following data for observed frequencies of cephalic index in Bavarian crania and for corresponding frequencies of a fitted Gaussian curve have already been considered on p. xx. Test the goodness of fit.

	Observed (m')	Gaussian (m)	$m'-m$	$\frac{(m'-m)^2}{m}$
Under *75·5*	9·5	12·4	− 2·9	·68
75·5—76·5	12·5	12·7	− 0·2	·00
76·5—77·5	17	22·1	− 5·1	1·18
77·5—78·5	37	35·3	+ 1·7	·08
78·5—79·5	55	51·9	+ 3·1	·19
79·5—80·5	71·5	70·1	+ 1·4	·03
80·5—81·5	82	87·0	− 5·0	·29
81·5—82·5	116	99·4	+16·6	2·77
82·5—83·5	98	104·2	− 6·2	·37
83·5—84·5	107	100·5	+ 6·5	·42
84·5—85·5	82	89·1	− 7·1	·57
85·5—86·5	74	72·6	+ 1·4	·03
86·5—87·5	58	54·3	+ 3·7	·25
87·5—88·5	34·5	37·4	− 2·9	·22
88·5—89·5	19	23·7	− 4·7	·93
89·5—90·5	10	13·8	− 3·8	1·05
90·5—91·5	8	7·4	+ 0·6	·05
Over *91·5*	9	6·3	+ 2·7	1·16
Totals ...	900	900·2	18 Groups	$\chi^2=10{\cdot}27$

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[*] A word of caution must be given about a recent extension by Slutsky (see *Journal of Royal Statistical Society*, Vol. LXXVII. pp. 78—84) who has applied it to test the goodness of fit of regression curves. In such cases the means and standard deviations of each array should, I think, be deduced from the theoretical surface, and the method would then agree with that illustrated on pp. xxiv—xxvi, i.e. on the probability of a given complex of variates differing from the run of values of a given population significantly. Slutsky after assuming that the observed frequencies and standard deviations of the arrays may replace the theoretical values, deduces his P from Elderton's Tables instead of from the incomplete normal moment tables. He finds for the fit of a straight regression line, used to predict the probable price of rye at Samara from the price a month previously, $\chi^2=22{\cdot}2$, giving $P={\cdot}02$, a bad fit. Had he, however, used the theoretical standard-deviation of an array, i.e. $\sigma\sqrt{1-r^2}$, instead of the very irregular observed standard deviations of individual arrays, he would have found $\chi^2=8{\cdot}84$, leading to $P={\cdot}64$ an excellent fit, which would probably have been still further improved by the use of theoretical total frequencies for the arrays, based, say, on a Gaussian distribution.

Taking from the column for $n' = 18$ (p. 27) the values for $\chi^2 = 10$ and 11, we interpolate $P = {\cdot}891$ for $\chi^2 = 10{\cdot}27$ by first differences, and conclude that in 89 out of 100 trials we should get in random sampling a fit as bad or worse than that observed, if the real distribution were Gaussian. Accordingly we say that a Gaussian curve describes excellently the distribution of Bavarian cephalic indices.

TABLES XIII—XVI (pp. 29—30)

Auxiliary Tables provided by W. Palin Elderton (*Biometrika*, Vol. I. pp. 162—163), *useful for calculating values of P for χ^2 outside the range of the existing table.*

For such cases we must turn back to the fundamental formulae (xxix) of p. xxxi, and the numerical values of considerable portions of these formulae will be found evaluated in these auxiliary tables.

Illustration. Find P for $n' = 11$ and $\chi^2 = 78$, $\frac{1}{2}\chi^2 = 39$, hence by formula we have

$$P = e^{-39}\left(1 + 39 + \tfrac{1}{2}(39)^2 + \frac{(39)^3}{3!} + \frac{(39)^4}{4!}\right)$$
$$= e^{-39}(40 + 760{\cdot}5 + 9886{\cdot}5 + 96393{\cdot}375)$$
$$= e^{-39} \times 107080{\cdot}375,$$

where the powers of 39 are taken out of Table XXVII (p. 38).

Hence using Table XIII,

$$\log P = \overline{17}{\cdot}0625{,}1520 + 5{\cdot}0297{,}0988 = \overline{12}{\cdot}0922{,}2508,$$

which gives us $$P = 1{\cdot}23659/10^{12}.$$

As a rule we can select n' to be odd, but, if it is necessarily even, there is more trouble, not in the determination of the series, but in the evaluation of the integral

$$I = \sqrt{\frac{2}{\pi}} \int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2} d\chi.$$

A table of the values of $F = \frac{1}{2}I$ for $\chi = 5$ to 500 has been given as Table IV (p. 11). This gives $\chi^2 = 25$ to 250000 but the intervals are large.

If greater accuracy be required then Schlömilch's formula*

$$I = \sqrt{\frac{2}{\pi}} \int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2} d\chi = \sqrt{\frac{2}{\pi}}\,\chi e^{-\frac{1}{2}\chi^2} \left\{\frac{1}{\chi^2} - \frac{1}{\chi^2(\chi^2+2)} + \frac{1}{\chi^2(\chi^2+2)(\chi^2+4)} - \frac{5}{\chi^2(\chi^2+2)(\chi^2+4)(\chi^2+6)} + \frac{9}{\chi^2(\chi^2+2)(\chi^2+4)(\chi^2+6)(\chi^2+8)} - \frac{129}{\chi^2(\chi^2+2)(\chi^2+4)(\chi^2+6)(\chi^2+8)(\chi^2+10)} + \dots\right\} \quad \text{(xxx)}$$

must be used.

Here $\sqrt{\frac{2}{\pi}}\,\chi e^{-\frac{1}{2}\chi^2}$ will be found in Table XIII, and the series converges fairly rapidly.

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* *Compendium der höheren Analysis*, Bd. II. S. 270, Braunschweig, 1879.

Table XVII (p. 31)

Values of (− log P) corresponding to given values of χ^2 in a fourfold table. (K. Pearson: On a Novel Method of regarding the Association of two Variates classed solely in Alternate Categories. *Drapers' Company Research Memoirs, Biometric Series,* VIII. Dulau & Co.)

If individuals be classed by the characters into A and not-A, B and not-B, we form a tetrachoric table of the form

	A	Not-A	Totals
B ...	a	b	$a+b$
Not-B	c	d	$c+d$
Totals	$a+c$	$b+d$	N

For such a table:

$$\chi^2 = \frac{N(ab-cd)^2}{(a+b)(c+d)(b+d)(a+c)} \qquad \text{(xxxi)},$$

gives a measure of the probability of independence, and, if the two attributes are highly associated, χ^2 will be large and P the probability of independence very small and largely outside Palin Elderton's Table XII. Table XVII provides for such cases.

Illustrations. The following tables are given by Mr G. U. Yule in his *Theory of Statistics**. His conclusions with regard to them are:

1. *Datura*: "No Association."
2. Eye Colour in Father and Son: "Shows the tendency to resemblance."
3. Houses in course of erection, Urban and Rural: "Distinct Positive Association."
4. Imbecility and Deaf-Mutism: "High Degree of Association."
5. Developmental Defects and Dullness: "Very high indeed."

It is required to measure the degree of probability that the variates in these five cases are independent.

(1) *Datura.*

Colour of Flower.

		Violet	White	Totals
Fruit.	Prickly ...	47	21	68
	Smooth ...	12	3	15
	Totals ...	59	24	83

(2) *Eye-Colour in Father and Son.*

Father.

		Light	Not Light	Totals
Son.	Light ...	471	148	619
	Not Light ...	151	230	381
	Totals ...	622	378	1000

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* Pp. 37, 34, 62, 33, 34 and 45 respectively.

(3) *Houses in course of erection in Urban and Rural Districts.*

	Built	Building	Totals
Urban ...	4960	50	5010
Rural ...	1749	12	1761
Totals ...	6709	62	6771

(4) *Imbecility and Deaf-Mutism.*

	Imbecile	Non-Imbecile	Totals ...
Deaf Mute ...	451	14,795	15,246
Non-Deaf Mute ...	48,431	32,464,323	32,512,754
Totals ...	48,882	32,479,118	32,528,000

(5) *Developmental Defects and Dullness.*

	With Defects	Without	Totals
Dull ...	888	1186	2074
Not-Dull ...	1420	22,793	24,213
Totals ...	2308	23,979	26,287

ϕ^2 the mean square contingency $= \chi^2/N$ and ϕ is the product-moment coefficient on the assumption that the 'presence of the character' is to be considered as a concrete unit*. The coefficient of mean square contingency $C_2 = \sqrt{\phi^2/(1+\phi^2)}$.

The following table gives the values of χ^2, ϕ^2, ϕ and C_2, and the values of P deduced.

	χ^2	ϕ^2	ϕ	C_2	P
(1) Datura	·7080	·085,301	·2921	·2803	·8713
(2) Eye-Colour	133·3265	·133,327	·3651	·3430	$1\cdot035/10^{28}$
(3) Houses	1·4393	·000,2125	·0146	·0146	·6948
(4) Imbecility and Deaf-Mutism	8014·62	·000,2464	·0157	·0157	$3\cdot179/10^{1739}$
(5) Defects and Dullness ...	3256·797	·123,894	·3519	·3320	$2\cdot846/10^{706}$

For example, from Table XVII by Formula (i) we have for $\chi^2 = 133\cdot3265$:

$$(-\log P) = 20\cdot809 + \frac{33\cdot3265}{50}[10\cdot770] - \tfrac{1}{2}\left(\frac{33\cdot3265}{50}\right)\left(\frac{16\cdot6735}{50}\right)[\cdot026]$$
$$= 27\cdot985,$$

and therefore $\log P = \overline{28}\cdot015$,

which leads to $P = 1\cdot035/10^{28}$.

Or again, for $\chi^2 = 8014\cdot62$:

$$-\log P = 1735\cdot324 + \frac{14\cdot62}{500}[108\cdot561] - \tfrac{1}{2}\left(\frac{14\cdot62}{500}\right)\left(\frac{485\cdot38}{500}\right)[\cdot000]$$
$$= 1738\cdot498,$$

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* Pearson and Heron, *Biometrika*, Vol. IX. p. 167.

and therefore $$\log P = \overline{1739}{\cdot}502,$$
and $$P = 3{\cdot}179/10^{1739}.$$

In the first and third cases a different treatment must be used. For $\chi^2 = 1{\cdot}4393$ we use Table XII.

We have for $n' = 4$:

$$P = {\cdot}801253 + {\cdot}4393\,[-228846] - \tfrac{1}{2}({\cdot}4393)\,({\cdot}5607)\,[+48064]$$
$$= {\cdot}6948.$$

Had we worked from Table XVII by Formula (i), we should have had
$$P = {\cdot}6950.$$

For $\chi^2 = {\cdot}7080$, we can use Table XII, remembering that for $\chi^2 = 0$, $P = 1$. We have

$$P = 1{\cdot}000{,}000 + {\cdot}708\,[-198{,}747] - \tfrac{1}{2}({\cdot}708)\,({\cdot}292)\,[-30{,}099]$$
$$= {\cdot}8624.$$

Had we worked from Table XVII by Formula (i), we should have had $P = {\cdot}865$, close enough for practical purposes.

The true value of P worked from

$$P = 2\left\{\frac{1}{\sqrt{2\pi}}\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\,d\chi + \frac{1}{\sqrt{2\pi}}\,e^{-\frac{1}{2}\chi^2}\,\chi\right\}$$

by using Table II is $P = {\cdot}8713$. See p. xxxviii.

Examining the values of P we see that having regard to the errors of random sampling we can only say that there is no relation between rural and urban districts and houses building or built; there is clearly no 'distinct association,' for in 69 out of 100 cases in sampling from independent material we should get more highly associated results. There is likewise no association on the given material in the *Datura* characters. The other three cases have clearly very marked association, quite independent of any influence of random sampling. If we regard these three tables the order of ascending association judged by either ϕ or C_2 is (4), (5), (2), as against Mr Yule's (2), (4), (5). If we disregard the non-significance and take merely intensity of association, without regard to random sampling, the order is (3), (4), (1), (5), (2), as against Mr Yule's order (1), (2), (3), (4), (5).

The best method of inquiry at present for relative association in the case of four-fold tables is, I hold, first to investigate P and throw out as not associated those cases like the 'Houses, built and building' above. Then to use either "tetrachoric r_t" or C_2 according as we are justified in considering the variates as continuous or not. r_P (see p. xxxvii) may be used as control.

TABLES XVIII—XX (pp. 31—32)

Tables for determining the Equiprobable Tetrachoric Correlation r_P. (Pearson and Bell: On a Novel Method of regarding the Association of two Variates classed solely in Alternate Categories. *Drapers' Company Research Memoirs, Biometric Series,* VIII. Dulau & Co.)

We have seen under the discussion of the previous Table how to find a measure of the improbability of two variates being independent, when they are classed in alternate categories. The difficulty in such cases is to appreciate the relative importance of very large inverse powers of 10. The object of the present tables is to enable us to deduce a tetrachoric correlation, r_t, of which the improbability is the same as that of the given system supposing it to arise, when the two variates have the same marginal frequencies but are really independent. In order to do this we have to determine ${}_0\sigma_r$ for the given marginal frequencies, i.e. the standard deviation of r_t on the assumption that r is really zero. This may be easily found from Abac Diagram XXI or from Table XXIV (see below). Table XVIII then gives us the value of $(-\log P)$ for each value of r_t and ${}_0\sigma_r$. If we now turn to our original table and calculate its χ^2, this as we have seen will correspond to a given $(-\log P)$. We now make the $(-\log P)$ from our χ^2 correspond to the $(-\log P)$ from our r_t and ${}_0\sigma_r$, this gives us a value of r_t which has the same degree of improbability as our observed table. In other words, instead of trying to appreciate the meaning of inverse high powers of 10, we say that a table of the same marginal frequency would be as improbable if it had a tetrachoric correlation r_t arising from random sampling of independent variates. Thus we read our improbability on a scale of tetrachoric correlation. We use our correlation merely as a scale to measure probability on.

As $\log \chi^2$ provides a more satisfactory basis for interpolation, and as many readers use logarithm tables and not calculators, $\log \chi^2$ will be the form in which χ^2 will be often presented. Table XX provides the value of r_t corresponding to given ${}_0\sigma_r$ and given $\log \chi^2$.

We will assume for the present that ${}_0\sigma_r$ can be readily found from the marginal totals: see p. xli below.

Illustration. Obtain the values of r_P for the five tables given above on pp. xxxiv—v.

The values of $\log \chi^2$ and ${}_0\sigma_r$ are as follows:

	$\log \chi^2$	${}_0\sigma_r$
(1) Datura	$\bar{1}$·8500	·1941
(2) Eye-Colour, Father and Son ...	2·1249	·0514
(3) Houses in Course of Erection ...	·1582	·0634
(4) Imbecility and Deaf-Mutism ...	3·9039	·0175
(5) Developmental Defects and Dullness	3·5128	·0201

Of the values here recorded for $\log \chi^2$ and ${}_0\sigma_r$, those for the first and third cases lie beyond the limits of our Table XX. But if the point ${}_0\sigma_r = {}$·0634 and $\log \chi^2 = {}$·1582 be noted on the Abac XXII, p. 34, it will be seen that, having due regard to the spacings of the correlation curves, the value of the equiprobable correlation is under ·03, say ·027. In other words no significant association can be asserted.

In the case of ${}_0\sigma_r = {\cdot}1941$ we are thrown back on the original formulae*. In the first place we must find P for the given value of χ^2, i.e. ·7080 (see p. xxxv). But for $n' = 4$ from formula (xxix),

$$\begin{aligned} P &= 2\left\{\frac{1}{\sqrt{2\pi}}\int_\chi^\infty e^{-\frac{1}{2}\chi^2}\,d\chi + \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\chi^2}\chi\right\} \\ &= 2\,\{{\cdot}200{,}0578 + {\cdot}280{,}0088 \times {\cdot}84142\} \\ &= {\cdot}871{,}3256. \end{aligned}$$

To obtain r we have to use the formula below, where ${}_0\sigma_r = {\cdot}1941$, and $m = \frac{1}{2}\left(\frac{1}{{}_0\sigma_r{}^2} - 3\right)$, the μ_0, μ_4, μ_8 being the normal moment functions of Table IX.

$$\begin{aligned} P = \frac{2}{{}_0\sigma_r\sqrt{2m}}\Big[&\{\mu_0(\sqrt{2m}) - \mu_0(\sqrt{2m}\,r)\} - \frac{1}{8m}\{\mu_4(\sqrt{2m}) - \mu_4(\sqrt{2m}\,r)\} \\ &- \frac{1}{24m^2}\{\mu_6(\sqrt{2m}) - \mu_6(\sqrt{2m}\,r)\} + \frac{1}{128m^2}\{\mu_8(\sqrt{2m}) - \mu_8(\sqrt{2m}\,r)\}\Big] \end{aligned} \quad \text{..............(xxxii)}.$$

Substituting the values of ${}_0\sigma_r = {\cdot}1941$ and $\sqrt{2m} = 4{\cdot}852{,}107$, we have for

$$r = {\cdot}03, \quad P = {\cdot}90550,$$
$$r = {\cdot}04, \quad P = {\cdot}86501.$$

Whence for $P = {\cdot}87133$, we have $r = {\cdot}038$.

We now turn to the three cases which fall inside Table XX.

(2) *Eye-colour, Father and Son.*

$$\log\chi^2 = 2{\cdot}1249 \qquad {}_0\sigma_r = {\cdot}0514,$$

$$\begin{array}{lll} {}_0\sigma_r = {\cdot}05 & r = 0{\cdot}5 & \log\chi^2 = 2{\cdot}0942 \\ & r = 0{\cdot}6 & \log\chi^2 = 2{\cdot}2748 \\ {}_0\sigma_r = {\cdot}06 & r = 0{\cdot}6 & \log\chi^2 = 2{\cdot}1239 \\ & r = 0{\cdot}7 & \log\chi^2 = 2{\cdot}2935. \end{array}$$

Linear differences will suffice

$${}_0\sigma_r = {\cdot}05 \qquad r = 0{\cdot}5 + \frac{{\cdot}0307}{{\cdot}1806}[{\cdot}1] = 0{\cdot}517,$$

$${}_0\sigma_r = {\cdot}06 \qquad r = 0{\cdot}6 + \frac{{\cdot}0010}{{\cdot}1696}[{\cdot}1] = 0{\cdot}601.$$

Hence ${}_0\sigma_r = {\cdot}0514$ gives

$$\begin{aligned} r &= {\cdot}517 + \frac{14}{100} \times {\cdot}084 \\ &= {\cdot}517 + {\cdot}012 = {\cdot}529. \end{aligned}$$

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* *Drapers' Company Research Memoirs.* Biometric Series VII. "A Novel Method," etc.: see pp. 12, 13.

Interpolating for ${}_0\sigma_r$ first,

$$r = \cdot 5 \qquad {}_0\sigma_r = \cdot 0514 \qquad \log \chi^2 = 2\cdot 0737,$$

$$r = \cdot 6 \qquad {}_0\sigma_r = \cdot 0514 \qquad \log \chi^2 = 2\cdot 2537.$$

Hence for $\log \chi^2 = 2\cdot 1249$:

$$r_P = \cdot 5 + \frac{\cdot 0512}{\cdot 1800} [\cdot 1] = \cdot 528.$$

We conclude that the equiprobable correlation is ·53.

(4) *Imbecility and Deaf-mutism.*

$$\log \chi^2 = 3\cdot 9039 \qquad {}_0\sigma_r = \cdot 0175,$$

$$r = 0\cdot 95,\ {}_0\sigma_r = \cdot 01,\ \log \chi^2 = 4\cdot 3673;\quad {}_0\sigma_r = \cdot 02,\ \log \chi^2 = 3\cdot 7660.$$

Hence: $r = 0\cdot 95,\ {}_0\sigma_r = \cdot 0175,\ \log \chi^2 = 3\cdot 9163.$

Again:

$$r = 0\cdot 90,\ {}_0\sigma_r = \cdot 01,\ \log \chi^2 = 4\cdot 2207;\quad {}_0\sigma_r = \cdot 02,\ \log \chi^2 = 3\cdot 6197.$$

Hence: $r = 0\cdot 90,\ {}_0\sigma_r = \cdot 0175,\ \log \chi^2 = 3\cdot 7699.$

Interpolating $\log \chi^2 = 3\cdot 9039$ between $3\cdot 9163$ and $3\cdot 7699$, we find

$$r_P = 0\cdot 946.$$

(5) *Developmental Defects and Dullness.*

$$\log \chi^2 = 3\cdot 5128, \quad {}_0\sigma_r = \cdot 0201.$$

$$r = 0\cdot 8,\ {}_0\sigma_r = \cdot 02,\ \log \chi^2 = 3\cdot 4097;\quad {}_0\sigma_r = \cdot 03,\ \log \chi^2 = 3\cdot 0598.$$

Hence: ${}_0\sigma_r = \cdot 0201,\quad \log \chi^2 = 3\cdot 4062.$

$$r = 0\cdot 9,\ {}_0\sigma_r = \cdot 02,\ \log \chi^2 = 3\cdot 6197;\ {}_0\sigma_r = \cdot 03,\ \log \chi^2 = 3\cdot 2690.$$

Hence: $\log \chi^2 = 3\cdot 6162$, for ${}_0\sigma_r = \cdot 0201$.

Thus, by interpolating $\log \chi^2 = 3\cdot 5128$ between $3\cdot 4062$ and $3\cdot 6162$, we find

$$r_P = \cdot 851.$$

We have accordingly the following results:

	C_2	P	r_P	r_t	Q
(1) Datura	·2803	·8713	·038	$-\cdot 188 \pm \cdot 140$	$-\cdot 282$
(2) Eye-Colour	·3430	$1\cdot 035/10^{28}$	·529	$\cdot 550 \pm \cdot 027$	·581
(3) Houses	·0146	·6948	·027	$-\cdot 081 \pm \cdot 043$	$-\cdot 190$
(4) Imbecility and Deaf-Mutism	·0157	$3\cdot 179/10^{1739}$	·946	$\cdot 330 \pm \cdot 012$	·907
(5) Defects and Dullness ...	·3320	$2\cdot 846/10^{706}$	·851	$\cdot 652 \pm \cdot 009$	·846

It will be seen that equiprobable r_P confirms generally the results from P, i.e. the tables for 'Datura' and 'Houses' give no sensible association. r_t also confirms this view and shows that 'Houses' is even lower in the scale than 'Datura.' The order of r_P is the same as that of Yule's coefficient of association Q, but neither r_P, r_t, C_2, P or Q support the conclusions stated to flow from the percentages on

p. xxxiv. Both r_P and Q give very high results for (4) and (5), and this is in accordance with the view elsewhere expressed that for extreme dichotomies Q is not to be trusted. It may further be doubted, whether for such dichotomies the theory of the distribution of deviations on which r_P is based can in its turn be accepted. On the whole r_t seems to me the most satisfactory coefficient of association, to be controlled by results for r_P in the cases where neither the dichotomies are extreme, nor the numbers so large or so small as to fall outside the moderate range of Tables XVIII—XX or Abacs XXI and XXII.

Abacs XXI and XXII (pp. 33—34).

See after Tables XXIII and XXIV.

Tables XXIII and XXIV

Tables for determining approximately the probable error of a tetrachoric correlation. (Pearson, *Biometrika*, Vol. IX. pp. 22—27. Tables calculated by Julia Bell, M.A.)

Given a tetrachoric table

a	b	$a+b$
c	d	$c+d$
$a+c$	$b+d$	N

so arranged that $a+c > b+d$ and $a+b > c+d$,

then if $\frac{1}{2}(1+\alpha_1) = (a+b)/N, \quad \frac{1}{2}(1+\alpha_2) = (a+c)/N,$

and r_t be the correlation, we have approximately:

$$\text{Probable error of } r_t = \chi_1 \cdot \chi_{r_t} \cdot \chi_{\alpha_1} \cdot \chi_{\alpha_2},$$

where $\chi_1 = {\cdot}67449/\sqrt{N},$

and is tabled in Table V, p. 12,

$$\chi_{\alpha_1} = \frac{\sqrt{\frac{1}{2}(1+\alpha_1)\frac{1}{2}(1-\alpha_1)}}{H}, \quad \chi_{\alpha_2} = \frac{\sqrt{\frac{1}{2}(1+\alpha_2)\frac{1}{2}(1-\alpha_2)}}{K} \quad \ldots(\text{xxxiii}),$$

H and K being found from the z column of Table II, p. 2, and

$$\chi_{r_t} = \sqrt{1-r_t^2}\sqrt{1-\left(\frac{\sin^{-1} r_t}{90^\circ}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(\text{xxxiv}),$$

$\sin^{-1} r_t$ being read in degrees. χ_{α_1} and χ_{α_2} are tabled in Table XXIV and χ_r in Table XXIII (p. 35).

This value of the probable error is only approximate and may diverge considerably from the true value* for extreme dichotomies. In such cases the full formula must be used.

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* *Phil. Trans.* Vol. 195, p. 14. χ_0 in formula (l) should of course not be included under the radical.

When r_t is zero in the population and not in the sample, the standard deviation ${}_0\sigma_r$ of $r=0$ is given accurately by $\dfrac{1}{\sqrt{N}}\chi_{a_1}\chi_{a_2}$.

Illustration (i). Tetrachoric r_t for the Table

22,793	1,420	24,213
1,186	888	2,074
23,979	2,308	26,287

is ·652. Find approximately its probable error.

From Table XXIII:

$$r = {\cdot}65,\ \chi_r = {\cdot}6785\,;\quad r = {\cdot}66,\ \chi_r = {\cdot}6675.$$

$$\therefore\ \chi_r = {\cdot}6785 - {\cdot}0110 \times {\cdot}2 = {\cdot}6763.$$

Now $\tfrac{1}{2}(1+\alpha_1) = {\cdot}9211,\quad \tfrac{1}{2}(1+\alpha_2) = {\cdot}9122.$

Hence from Table XXIII,

$$\chi_{a_1} = 1{\cdot}8249 + {\cdot}11\,[754] = 1{\cdot}8332,$$

$$\chi_{a_2} = 1{\cdot}7623 + {\cdot}22\,[626] = 1{\cdot}7761,$$

$$\chi_{a_1}\chi_{a_2} = 3{\cdot}2559.$$

χ_1 cannot be found from Table V in this case as N is beyond its range. But it equals

$${\cdot}67449/\sqrt{26287} = {\cdot}67449/162{\cdot}13 = {\cdot}00416.$$

Thus finally $\quad$ p.e. of $r_t = {\cdot}00416 \times {\cdot}6763 \times 3{\cdot}2559$

$$= {\cdot}009.$$

Illustration (ii). Find the value of ${}_0\sigma_r$ for the table:

471	148	619
151	230	381
622	378	1000

Here $\quad \tfrac{1}{2}(1+\alpha_1) = {\cdot}619$ and $\tfrac{1}{2}(1+\alpha_2) = {\cdot}622.$

$$\chi_{a_1} = 1{\cdot}2712 + {\cdot}9\,[36] = 1{\cdot}2744,$$

$$\chi_{a_2} = 1{\cdot}2748 + {\cdot}2\,[39] = 1{\cdot}2756.$$

$${}_0\sigma_r = \chi_{a_1}\chi_{a_2}/\sqrt{1000} = {\cdot}0514.$$

In a similar manner the values for all the ${}_0\sigma_r$'s in the table on p. xxxvii were found.

ABAC XXI (p. 33)

For determination of the standard-deviation of the correlation coefficients obtained by random sampling from a four-fold table in which the correlation is zero. (*Drapers' Company Research Memoirs*, Biometric Series, VIII. G. H. Soper's Abac.)

Method of use: Enter with the total frequency of the sample on the left-hand scale, and with the first value of $\frac{1}{2}(1+\alpha)$ on the bottom scale. The horizontal through the former and the vertical through the latter meet at a point. At this point pass up the diagonal to the left-hand scale again. Where you meet that scale pass along the horizontal until you meet the vertical through the second value of $\frac{1}{2}(1+\alpha)$. Then from this point pass along the diagonal again to the left-hand scale, whence traverse the horizontal to the right-hand scale and there the required value of ${}_0\sigma_r$ may be read off.

Illustration (i). Find the value of ${}_0\sigma_r$ for the case just given of

$$N = 1000, \quad \tfrac{1}{2}(1+\alpha_1) = \cdot619, \quad \tfrac{1}{2}(1+\alpha_2) = \cdot622.$$

The vertical through ·619 meets the 1000 horizontal in a point whose diagonal reaches the left-hand scale almost exactly in 620. Whence passing horizontally we reach the vertical through ·622 in a point about midway between two diagonal lines. Passing up midway between these two diagonals, we reach almost exactly the 380 line on the left-hand scale. Passing across to the right-hand scale along this line, we see that we are slightly above the middle of the division between ·050 and ·052, say ·0512. The actual value of ${}_0\sigma_r$ is ·0514.

Illustration (ii). Let

$$N = \cdot6771, \quad \tfrac{1}{2}(1+\alpha_1) = \cdot7399; \quad \tfrac{1}{2}(1+\alpha_2) = \cdot9908.$$

A similar process gives first 450 on left-hand scale and then about 248, whence crossing to right-hand scale we find ${}_0\sigma_r = \cdot0635$ instead of ·0634 actual.

ABAC XXII (p. 34)

Abac to determine from log χ^2 and ${}_0\sigma_r$ the value of the equiprobable correlation r_P, for a fourfold table. (*Drapers' Company Research Memoirs*, Biometric Series, VIII. G. H. Soper's Abac.)

The rule is very simple: Enter the Abac with the proper value of ${}_0\sigma_r$ on the scale at the foot and rise on the vertical till the horizontal through the proper value of log χ^2 on the left-hand scale is reached. Then follow the curve through the meet of these two lines to the right-hand scale, where the requisite correlation will be found inscribed.

Illustration. Take the Table for Eye Colour in Father and Son given on p. xxxiv. Here, as just shewn, ${}_0\sigma_r = \cdot0514$ and (p. xxxvii) log $\chi^2 = 2\cdot1249$. If we enter with the vertical through ·0514 on the scale at the bottom, and the horizontal through 2·1249 on the left-hand scale, the curve through their point of intersection reaches the right-hand scale just below the ·53 mark, say ·529. This agrees with the correlation found above (p. xxxviii) by interpolation from Table XX.

TABLE XXV (p. 36)

Value of the probability that the mean of a small sample of n, drawn at random from a population following the normal law, will not exceed (in the algebraic sense) the mean of that population by more than z times the standard deviation of the sample. ("Student": *Biometrika*, Vol. VI. p. 19.)

When n is greater than 10, it will be sufficient as a rule to use the approximate result

$$P = \frac{\sqrt{n-3}}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(n-3)x^2}{2}} dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (\text{xxxv})$$

as a measure of the probability. This may be found from Table II.

Illustration (i). Experiments of A. R. Cushney and A. R. Peebles on the difference in effect of Dextro-hyoscyamine hydrobromide and Laevo-hyoscyamine hydrobromide*.

Patient	Additional Hours of Sleep (Laevo – Dextro)
1	+1·2
2	+2·4
3	+1·3
4	+1·3
5	0
6	+1·0
7	+1·8
8	+0·8
9	+4·6
10	+1·4
Mean	+1·58
Standard Deviation	1·17

$$z = \frac{+1{\cdot}58}{1{\cdot}17} = +1{\cdot}35.$$

Table XXV shows that for $z = 1{\cdot}35$:

$$P = {\cdot}99854,$$

or the odds are 666 to 1 that leavo- is a better soporific than dextro-hyoscyamine hydrobromide.

Illustration (ii). Difference in weight of crops of potatoes grown by Dr Voelcker with (i) sulphate of potash and (ii) kainite as artificial manure.

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* *Journal of Physiology*, 1904.

Gain by sulphate of potash.

1904	(*a*)	10 cwt. 3 qr. 20 lbs.
„	(*b*)	1 ton 10 cwt. 1 qr. 26 lbs.
1905	(*a*)	6 cwt. 0 qr. 3 lbs.
„	(*b*)	13 cwt. 2 qr. 8 lbs.

Average gain = 15·25 cwt., and the standard deviation = 9 cwt., $z = 15{\cdot}25/9 = 1{\cdot}694$.

Here $n = 4$, and Table XXV gives us

$$P = {\cdot}9653 + 0{\cdot}94 \times [46] = {\cdot}9696,$$

or the odds are about 32 to 1 that the sulphate of potash is a better dressing than kainite for potatoes.

Illustration (iii). Test whether it is of advantage to kiln-dry barley seed before sowing. The following table gives price of head corn in shillings per quarter for 11 sowings, the first seven in 1899 and the last four in 1900.

	Not Kiln-dried	Kiln-dried	Δ
1899	26·5	26·5	0
	28	26·5	1·5
	29·5	28·5	1
	30	29	1
	27·5	27	0·5
	26	26	0
	29	26	3
1900	29·5	28·5	1
	28·5	28	0·5
	30	29	1
	28·5	28	0·5
Mean		—	·91
Standard Deviation			·79

The Gaussian curve gives

$$P = \sqrt{\frac{8}{2\pi}} \int_{-\infty}^{x} e^{-\frac{8}{2}x^2}\, dx.$$

Here $$x = {\cdot}91/{\cdot}79 = 1{\cdot}1519,$$

and if $$x' = x/\sqrt{1/8} = 1{\cdot}1519 \times 2{\cdot}8284 = 3{\cdot}258,$$

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{3\cdot258} e^{-\frac{1}{2}x'^2}\, dx',$$

which evaluated by Table II, p. 6, gives

$$P = {\cdot}99944,$$

or the odds are 2845 to 1 in favour of not kiln-drying seed barley.

If we had actually worked with the non-approximate formula, we should have found

$$P = \cdot 9976,$$

or odds of 416 to 1, considerably less than the approximate formula provide, but not enough difference to vitiate any conclusion likely to be drawn in practice*.

TABLE XXVI (p. 37)

Table for use in plotting Type III Curves, i.e.

$$y = y_0 e^{-p\frac{x}{a}} \left(1 + \frac{x}{a}\right)^p \quad \text{..........................(xxxvi)}$$

(W. P. Elderton, *Biometrika*, Vol. II. p. 270.)

Rule: Taking p for the curve, multiply the values in the Table by p in succession on the machine with p on as multiplier. Then *subtract* the results from the logarithm of y_0, and we have the logarithms of the ordinates of the curve at the abscissae found by multiplying X in the first column of the Table by a of the curve. The curve can then be plotted. Its origin will be the mode. It is usually quite unnecessary to use the whole series of ordinates, either alternate ordinates will suffice, or we cut off one or both tails at a considerable distance from their tabulated values.

Illustration. The frequency curve of barometric heights at Dunrobin Castle is given by the curve

$$y = 39 \cdot 140 e^{-22 \cdot 9323 \frac{x}{17 \cdot 7661}} \left(1 + \frac{x}{17 \cdot 7661}\right)^{22 \cdot 9323}.$$

The range $X = - \cdot 65$ to $+ \cdot 90$ is easily seen to be sufficient. Column (i) of the accompanying table gives aX for these values, the second gives

$$22 \cdot 9323 \times (\log_{10}(1+X) - X \log_{10} e);$$

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* The three illustrations above are drawn from "Student's" original paper. He gives (*l. c.* p. 19) the values for P as drawn from the Gaussian for $n=10$ to compare with those obtained from the full formula. They are,—corrected for slips:

z	Full Formula	Gaussian	z	Full Formula	Gaussian
·1	·61462	·60411	1·1	·99539	·99819
·2	·71846	·70159	1·2	·99713	·99925
·3	·80423	·78641	1·3	·99819	·99971
·4	·86970	·85520	1·4	·99885	·99989
·5	·91609	·90691	1·5	·99926	·99996
·6	·94732	·94375	1·6	·99951	·99999
·7	·96747	·96799	1·7	·99968	—
·8	·98007	·98285	1·8	·99978	—
·9	·98780	·99137	1·9	·99985	—
1·0	·99252	·99592	2·0	·99990	—

Clearly even for $n=10$, the Gaussian ascends too rapidly in P, and this must be borne in mind in deducing conclusions for $z=1$ and upwards when $n=11$ to 20, say.

actually these values are *negative* and must be *subtracted* from log y_0, i.e. 1·592,621; the resulting values are given in the third column. In column (iv) are given the antilogarithms of the numbers in column (iii), and these must be plotted to the values in column (i) to obtain the graph of the curve which is a good fit.

(i)	(ii)	(iii)	(iv)
$x=aX$	$p[\log_{10}(1+X)-X\log e]$*	$\log y$	y
− 9·77	2·474,991	− ·882,370	·13
− 8·88	1·923,630	− ·331,009	·47
− 7·99	1·472,368	·120,253	1·32
− 7·11	1·103,755	·488,866	3·08
− 6·22	·804,557	·788,064	6·14
− 5·33	·564,456	1·028,165	10·67
− 4·44	·375,287	1·217,334	16·49
− 3·55	·230,493	1·362,128	23·02
− 2·67	·124,683	1·467,938	29·37
− 1·78	·053,386	1·539,235	34·61
− 0·89	·012,888	1·579,733	38·00
0·00	·000,000	1·592,621	39·14
0·89	·012,039	1·580,582	38·07
1·78	·046,713	1·545,908	35·15
2·67	·101,957	1·490,664	30·95
3·55	·176,074	1·416,547	26·09
4·44	·267,482	1·325,139	21·14
5·33	·374,828	1·217,793	16·51
6·22	·496,920	1·095,701	12·47
7·11	·632,702	·959,919	9·12
7·99	·781,189	·811,432	6·48
8·88	·941,509	·651,112	4·48
9·77	1·112,905	·479,716	3·02
10·66	1·294,689	·297,932	1·99
11·55	1·486,196	·106,425	1·28
12·44	1·686,831	− ·094,210	·80
13·32	1·896,111	− ·303,490	·50
14·21	2·113,510	− ·520,889	·30
15·10	2·338,613	− ·745,992	·18
15·99	2·570,963	− ·978,342	·11

Once the reader is used to the process it will be found to work readily, and the same multipliers are kept on the mechanical calculator throughout.

TABLES XXVII AND XXVIII (pp. 38—41)

Tables of the Powers and Sums of the Powers of the natural numbers from 1 *to* 100. (W. Palin Elderton, *Biometrika*, Vol. II. p. 474.)

These tables can be used in a great variety of ways, for example in finding the roots of equations, or in fitting parabolae of various orders to curves.

Illustration (i). Find the positive root of the equation:

$$\phi(r) = {\cdot}002{,}726r^7 + {\cdot}057{,}149r^6 + {\cdot}017{,}192r^5 + {\cdot}083{,}578r^4 + {\cdot}088{,}331r^3 + {\cdot}134{,}717r^2 + r - {\cdot}560{,}386 = 0.$$

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* Actually these values are negative, and are therefore *subtracted* from log y_0 to give (iii).

The positive root is less than ·56, but the term in r^2 shows that it must be less than ·52. Take ·52 and ·50 as trials. From Table XXVII we have

1st	·520,000	and	·500,000,
2nd	·270,400	„	·250,000,
3rd	·140,608	„	·125,000,
4th	·071,162	„	·062,500,
5th	·038,020	„	·031,250,
6th	·019,771	„	·015,625,
7th	·010,281	„	·007,813.

Multiply out by the coefficients of $\phi(r)$, retaining the products always on the arithmometer. We find

$$\phi(\cdot 52) = + \cdot 016{,}384,$$
$$\phi(\cdot 50) = - \cdot 008{,}990.$$

Interpolating $$r = \cdot 52 - \tfrac{16384}{25374} \times 2 = \cdot 5071,$$

which is correct to last figure.

Illustration (ii). Fit a cubic parabola to the data below, giving the average age of husband to each age of wife in Italy (see *Biometrika*, Vol. II. p. 20). We will suppose each observation to be of equal weight,—this is of course not the fact, but it will illustrate the general method of fitting parabolic curves. In the paper just cited illustrations are given up to parabolae of the sixth order. The object here is to show the use of Table XXVII.

Age of Bride	Probable Age of Groom	Age of Bride	Probable Age of Groom	Age of Bride	Probable Age of Groom
15·5	25·0	25·5	27·0	35·5	36·0
16·5	25·2	26·5	27·5	36·5	37·0
17·5	25·4	27·5	28·0	37·5	38·5
18·5	25·5	28·5	29·0	38·5	39·5
19·5	25·5	29·5	30·0	39·5	41·5
20·5	25·5	30·5	32·0	40·5	41·5
21·5	25·75	31·5	33·0	41·5	42·5
22·5	26·0	32·5	33·5	42·5	43·5
23·5	26·0	33·5	34·0	43·5	43·5
24·5	26·8	34·5	34·5	44·5	43·5
—	—	—	—	45·5	43·5

The ages of groom have been taken as approximate means. Now we can take our axis of x, the age of bride through 30·5, and the age of groom to be measured from 32·0. x will accordingly range from -15 to $+15$, and the age $32 + y$ of groom will range from $y = -7$ to $y = 11{\cdot}5$. We can now re-arrange the above table in a form suitable for working on the following table. Then the squares, cubes, and if necessary, higher powers of x are taken from Table XXVII, p. 38, and are given as Columns (iii) and (iv) below. The entries in Column (i) are then multiplied by those in (ii), (iii) and (iv) by *continuous* process on the machine, and it is not needful to enter separate products, the sums being reached which are placed at the foot. Next from Table XXVIII we read off

$$S(x)=0,\quad S(x^2)=2(S(15^2)),\quad S(x^3)=0,\quad S(x^4)=2(S(15^4)),$$
$$S(x^5)=0,\quad S(x^6)=2(S(15^6)).$$

These give us:

$$S(x^2)=2480,\quad S(x^4)=356{,}624,\quad S(x^6)=6096{,}5840.$$

We have now all the numerical data for a solution. Let the required cubic be

$$y=c_0+c_1x+c_2x^2+c_3x^3.$$

Then we must make $u=S(y-c_0-c_1x-c_2x^2-c_3x^3)^2$ a minimum. The resulting equations are

$$\begin{aligned}
S(y) &= c_0S(1)+c_1S(x)+c_2S(x^2)+c_3S(x^3),\\
S(xy) &= c_0S(x)+c_1S(x^2)+c_2S(x^3)+c_3S(x^4),\\
S(x^2y) &= c_0S(x^2)+c_1S(x^3)+c_2S(x^4)+c_3S(x^5),\\
S(x^3y) &= c_0S(x^3)+c_1S(x^4)+c_2S(x^5)+c_3S(x^6).
\end{aligned}$$

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
y	x	x^2	x^3	xy	x^2y	x^3y
-7·0	-15	225	-3375	—	—	—
-6·8	-14	196	-2744	—	—	—
-6·6	-13	169	-2197	—	—	—
-6·5	-12	144	-1728	—	—	—
-6·5	-11	121	-1331	—	—	—
-6·5	-10	100	-1000	—	—	—
-6·25	-9	81	-729	—	—	—
-6·0	-8	64	-512	—	—	—
-6·0	-7	49	-343	—	—	—
-5·2	-6	36	-216	—	—	—
-5·0	-5	25	-125	—	—	—
-4·5	-4	16	-64	—	—	—
-4·0	-3	9	-27	—	—	—
-3·0	-2	4	-8	—	—	—
-2·0	-1	1	-1	—	—	—
0	0	0	0	—	—	—
1·0	1	1	1	—	—	—
1·5	2	4	8	—	—	—
2·0	3	9	27	—	—	—
2·5	4	16	64	—	—	—
4·0	5	25	125	—	—	—
4·5	6	36	216	—	—	—
6·5	7	49	343	—	—	—
7·5	8	64	512	—	—	—
9·5	9	81	729	—	—	—
9·5	10	100	1000	—	—	—
10·5	11	121	1331	—	—	—
11·5	12	144	1728	—	—	—
11·5	13	169	2197	—	—	—
11·5	14	196	2744	—	—	—
11·5	15	225	3375	—	—	—
$S(x)=23·65$	—	—	—	$S(xy)=1833·45$	$S(x^2y)=4560·35$	$S(x^3y)=248{,}807·85$

Write $b_0 = c_0, \quad b_1 = 10c_0, \quad b_2 = 100c_2, \quad b_3 = 1000c_3.$

Then our equations are

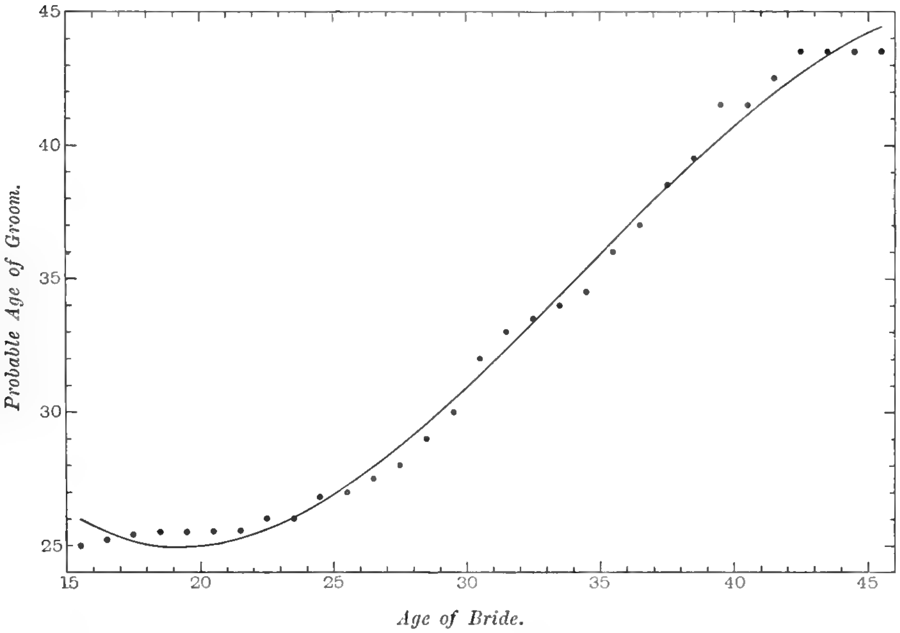
$$\begin{aligned}
\cdot 23650 &= b_0 \times \cdot 31000 && && + b_2 \times \cdot 24800, && \\
1\cdot 83345 &= && b_1 \times \cdot 24800 && && + b_3 \times \cdot 35662, \\
\cdot 45603 &= b_0 \times \cdot 24800 && && + b_2 \times \cdot 35662, && \\
2\cdot 48808 &= && b_1 \times \cdot 35662 && && + b_3 \times \cdot 60966;
\end{aligned}$$

giving

$$\begin{aligned}
b_0 &= -\cdot 58626, & \therefore\ c_0 &= -\cdot 58626, \\
b_2 &= 1\cdot 686453, & c_2 &= \cdot 016{,}8645, \\
b_1 &= 9\cdot 59613, & c_1 &= \cdot 959{,}613, \\
b_3 &= -1\cdot 532{,}144, & c_3 &= -\cdot 001{,}532{,}144,
\end{aligned}$$

and the required cubic is

$$y = -\cdot 58626 + \cdot 959{,}613x + \cdot 016{,}8645x^2 - \cdot 001{,}532{,}144x^3.$$



The graph of the cubic and the observations are given in the accompanying diagram. If X and Y be the actual ages of bride and groom, then

$$Y = 61\cdot 30457 - 4\cdot 344{,}941X + \cdot 157{,}0553X^2 - \cdot 001{,}53214X^3.$$

For higher parabolic curves fitted to the same data, see *Biometrika*, Vol. II. pp. 21—22.

TABLE XXIX (pp. 42—51)

Tables of the Tetrachoric Functions. (P. F. Everitt, *Biometrika*, Vol. VII. pp. 437—451.)

The purpose of these tables is to expedite the calculation of tetrachoric r_t, the correlation coefficient from a four-fold table, when we suppose the variates to be Gaussian in the law of their frequency.

Let the table be

a	b	$a+b$
c	d	$c+d$
$a+c$	$b+d$	N

where a is the quadrant in which the mean falls, then $b+d$ and $c+d$ are clearly each less than $\frac{1}{2}N$. Let

$$\tau_0 = (b+d)/N = \tfrac{1}{2}(1-\alpha_1), \quad \tau_0' = (c+d)/N = \tfrac{1}{2}(1-\alpha_2),$$

then
$$d/N = \tau_0\tau_0' + \tau_1\tau_1' r + \tau_2\tau_2' r^2 + \ldots + \tau_n\tau_n' r^n + \ldots \ldots\ldots\ldots(\text{xxxvii})$$

is the equation to determine r the tetrachoric correlation, and Table XXIX gives the values for given τ_0, i.e. $\frac{1}{2}(1-\alpha)$ of the following six tetrachoric functions $\tau_1, \tau_2 \ldots \tau_6$, and further of h, the ratio of the abscissa of the dichotomic line to the standard deviation of the corresponding variate.

It is occasionally needful to go beyond the first six tetrachoric functions. In this case the following finite difference formula is available:

$$\tau_n = hp_n\tau_{n-1} - q_n\tau_{n-2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{xxxviii}),$$

where
$$p_n = 1/\sqrt{n}, \quad q_n = (n-2)/\sqrt{n(n-1)} \ldots\ldots\ldots\ldots\ldots\ldots(\text{xxxix}).$$

The following table gives the values of p_n and q_n from $n = 7$ to 24.

n	p_n	q_n	n	p_n	q_n
7	·37796	·77152	16	·25000	·90370
8	·35355	·80178	17	·24254	·90951
9	·33333	·82496	18	·23570	·91466
10	·31623	·84327	19	·22942	·91925
11	·30151	·85812	20	·22361	·92338
12	·28868	·87039	21	·21822	·92711
13	·27735	·88070	22	·21320	·93048
14	·26726	·88950	23	·20851	·93356
15	·25820	·89709	24	·20412	·93638

Illustration (i). Find the correlation between dullness and developmental defects as indicated in the following table for 26,287 children.

	Without Defects	With Defects	Totals
Not Dull ...	22,793	1,420	24,213
Dull ...	1,186	888	2,074
Totals ...	23,979	2,308	26,287

Here

$$\tau_0 = \tfrac{2074}{26287} = \cdot 078{,}898,$$
$$\tau_0' = \tfrac{2308}{26287} = \cdot 087{,}800.$$

Whence by interpolation from Table, p. 43:

$$\begin{array}{llll} \tau_1 = & \cdot 14712, & \tau_1' = & \cdot 15945, \\ \tau_2 = & \cdot 14694, & \tau_2' = & \cdot 15268, \\ \tau_3 = & \cdot 05977, & \tau_3' = & \cdot 05431, \\ \tau_4 = - & \cdot 04262, & \tau_4' = - & \cdot 05137, \\ \tau_5 = - & \cdot 06702, & \tau_5' = - & \cdot 06755, \\ \tau_6 = - & \cdot 00752, & \tau_6' = & \cdot 00017, \\ h = & 1\cdot 41253, & k = & 1\cdot 35442. \end{array}$$

Proceeding to apply the difference formula (xxxviii) for four further functions we have

$$\begin{array}{llll} \tau_7 = & \cdot 04770, & \tau_7' = & \cdot 05221, \\ \tau_8 = & \cdot 02985, & \tau_8' = & \cdot 02486, \\ \tau_9 = - & \cdot 02530, & \tau_9' = - & \cdot 03185, \\ \tau_{10} = - & \cdot 03647, & \tau_{10}' = - & \cdot 03460. \end{array}$$

Hence the equation for r is

$$\begin{aligned} \cdot 026{,}854 = {} & \cdot 023{,}458r + \cdot 022{,}435r^2 + \cdot 003{,}246r^3 \\ & + \cdot 002{,}189r^4 + \cdot 004{,}527r^5 - \cdot 000{,}001r^6 \\ & + \cdot 002{,}490r^7 + \cdot 000{,}742r^8 + \cdot 000{,}806r^9 \\ & + \cdot 001{,}262r^{10}. \end{aligned}$$

Whence we find $r = \cdot 652 \pm \cdot 009$.

Illustration (ii). Find the tetrachoric correlation for the four-fold table given for *Houses in course of Erection* on p. xxxv. Here

$$\tfrac{1}{2}(1 - \alpha_1) = \tau_0 = \tfrac{1761}{6771} = \cdot 260{,}080\,; \quad \tfrac{1}{2}(1 - \alpha_2) = \tau_0' = \tfrac{62}{6771} = \cdot 009{,}157.$$

By simple linear interpolation,

$$\begin{array}{ll} \tau_1 = \cdot 32442, & \tau_1' = \cdot 02468, \\ \tau_2 = \cdot 14753, & \tau_2' = \cdot 04116, \\ \tau_3 = - \cdot 07766, & \tau_3' = \cdot 04599, \\ \tau_4 = - \cdot 11015, & \tau_4' = \cdot 03048. \end{array}$$

Hence the equation for r:

$$-\cdot 000{,}6093 = \cdot 008{,}007r + \cdot 006{,}072r^2 - \cdot 003{,}572r^3 - \cdot 003{,}357r^4.$$

Whence $r = -\cdot 081 \pm \cdot 043$.

Or, the association is not definitely significant.

Illustration (iii). Find the tetrachoric r for the Table of Bradford Parents:

Mother's Habits.

		Good	Bad	Totals
Father's Habits.	Good ...	994	67	1061
	Bad ...	159	476	635
	Totals ...	1153	543	1696

Here a brief experience will show the reader that to proceed by tetrachoric functions will require a very large amount of labour.

We have

$$\tfrac{1}{2}(1-\alpha_1) = \tau_0 = 543/1696 = \cdot 32017\,;\quad \tfrac{1}{2}(1-\alpha_2) = \tau_0' = 635/1696 = \cdot 37441,$$
$$d/N = 476/1696 = \cdot 28066.$$

We have accordingly the following series of tetrachoric functions—the first 6 from the table, the remaining 18 from the difference formula.

d/N	$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
·28066	·32017	·35769	·11817	− ·11415	− ·09489	·05674	·08012
—	·37441	·37901	·08581	− ·13887	− ·07178	·08288	·06325
τ_7	τ_8	τ_9	τ_{10}	τ_{11}	τ_{12}	τ_{13}	τ_{14}
− ·02963	− ·06913	·01368	·06032	− ·00324	− ·05294	− ·00401	·04659
− ·05629	− ·05709	·04034	·05223	− ·02957	− ·04819	·02176	·04558
τ_{15}	τ_{16}	τ_{17}	τ_{18}	τ_{19}	τ_{20}	τ_{21}	τ_{22}
·00922	− ·04103	− ·01304	·03609	·01586	− ·03167	− ·01793	·02768
− ·01575	− ·04245	·01103	·03966	− ·00723	− ·03714	·00411	·03484
τ_{23}	τ_{24}	h	—	—	—	—	—
·01944	− ·02407	·46732	—	—	—	—	—
− ·00151	− ·03272	·32020	—	—	—	—	—

Considering only the equation as far as Everitt's Tables extend, we have

$$\phi(r) = -\cdot 16079 + \cdot 13557r + \cdot 01014r^2 + \cdot 01585r^3 + \cdot 00681r^4 + \cdot 00470r^5 + \cdot 00507r^6 = 0.$$

This leads to $r = \cdot 9365$, but the series indicates that the terms are far from converging rapidly.

The first 12 tetrachoric functions were then used, the last six being found by the table of p_n and q_n above, and the value of r was found to be ·9152.

Then 18 functions were used and gave $r = \cdot 9114$.

Lastly 24 tetrachoric functions were used, and the equation below obtained, which led to $r = \cdot 9105$.

$$\begin{aligned}\phi(r) = -\cdot 16079 &+ \cdot 13557r + \cdot 01014r^2 + \cdot 01585r^3 + \cdot 00681r^4 + \cdot 00470r^5 \\ &+ \cdot 00507r^6 + \cdot 00167r^7 + \cdot 00395r^8 + \cdot 00055r^9 + \cdot 00315r^{10} \\ &+ 00010r^{11} + \cdot 00255r^{12} - \cdot 00009r^{13} + \cdot 00212r^{14} - \cdot 00015r^{15} \\ &+ 00174r^{16} - \cdot 00014r^{17} + \cdot 00143r^{18} - \cdot 00011r^{19} + \cdot 00118r^{20} \\ &- \cdot 00057r^{21} + \cdot 00096r^{22} - \cdot 00003r^{23} + \cdot 00079r^{24}.\end{aligned}$$

It will be seen that even with this very large amount of labour we cannot be sure of having reached a final result*. To obviate this the following table was constructed by Everitt, and there is no doubt that the extension of this table to the whole range of correlation would much simplify the discovery of tetrachoric r_t. At present the calculation of high values of r_t, for negative correlations is in hand.

TABLE XXX (pp. 52—27)

Supplementary Tables for determining High Correlations from Tetrachoric Groupings. (P. F. Everitt, *Biometrika*, Vol. VIII. pp. 385—395.)

Using the notation of p. l,

$$\frac{d}{N} = \frac{1}{2\pi\sqrt{1-r^2}} \int_h^\infty \int_k^\infty e^{-\frac{1}{2}\frac{1}{1-r^2}(x^2+y^2-2rxy)}\, dx\, dy \ldots\ldots\ldots\ldots\text{(xl)}$$

in the case of a tetrachoric table, or

$$\left.\begin{aligned}\frac{d}{N} &= \frac{1}{\sqrt{2\pi}} \int_k^\infty e^{-\frac{1}{2}y^2}\, Y dy \\ \text{where } Y &= \frac{1}{\sqrt{2\pi}} \int_t^\infty e^{-\frac{1}{2}X dX},\ \text{if } t = \frac{h - yr}{\sqrt{1-r^2}}\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\text{(xli)}.$$

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* Mr H. E. Soper working out this example draws my attention to the fact that convergence is closely given by a form : $r_n = r_\infty(1 + a\,.\,c^n)$, where n is the number of terms used and a and c are constants.

Hence $$(r_n - r_\infty)(r_{n+2m} - r_\infty) = (r_{n+m} - r_\infty)^2,$$

or $$r_\infty = \frac{r_n r_{n+2m} - r^2_{n+m}}{r_n + r_{n+2m} - 2r_{n+m}}.$$

In our case take $n = 6$, $m = 6$, and we find

$$r_\infty = \frac{r_6 r_{18} - r_{12}^2}{r_6 + r_{18} - 2r_{12}} = \cdot 9106.$$

The value r_{24} is ·9105. In this case $a = \cdot 1567$ and $c = \cdot 7574$, but we cannot assert that these would be constants for all tables. If we use r_{12}, r_{18} and r_{24}, we find $r_\infty = \cdot 9102$.

Hence r, h being known, Y is a tabled integral for each value of Y. Accordingly by aid of Table II we know $\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}$, and using a quadrature formula, d/N can be found for each value of h, k and r.

Table XXX gives, for $r = {\cdot}80$, $\cdot 85$, $\cdot 90$, $\cdot 95$ and $1{\cdot}00$, and values of h and k proceeding by $\cdot 1$, the values of d/N. For given values of h, k and d/N, we can then find r by interpolation from these tables. The process is far shorter than that required by Table XXIX when we have to proceed to many terms. Unfortunately opportunity has not yet arisen for fully completing similar tables for r negative and over $\cdot 80$.

Illustration. Determine the correlation in habits between Mother and Father in Bradford. The data are

Mother.

		Habits Good	Habits Bad	Totals
Father.	Habits Good ...	994	67	1061
	Habits Bad ...	159	476	635
	Totals	1153	543	1696

Here $(b+d)/N = {\cdot}32017$, $(c+d)/N = {\cdot}37441$, and therefore $h = {\cdot}46722$, $k = {\cdot}32020$ from Table II. Also $d/N = 476/1696 = {\cdot}28066$.

Inspection of Table XXX shows that r will be likely to lie between $\cdot 90$ and $\cdot 95$.

We extract from the Table for d/N:

$r = {\cdot}90$	$h = {\cdot}4$	$h = {\cdot}5$
$k = {\cdot}3$	·2943	·2728
$k = {\cdot}4$	·2784	·2602

$r = {\cdot}95$	$h = {\cdot}4$	$h = {\cdot}5$
$k = {\cdot}3$	·3135	·2898
$k = {\cdot}4$	·2980	·2787

Hence:

$r = {\cdot}90$	$h = {\cdot}4$	$h = {\cdot}5$
$k = {\cdot}32020$	·2911	·2703

$r = {\cdot}95$	$h = {\cdot}4$	$h = {\cdot}5$
$k = {\cdot}32020$	·3104	·2876

Thus:

$r = {\cdot}90$	$h = {\cdot}46722$
$k = {\cdot}32020$	·2771

$r = {\cdot}95$	$h = {\cdot}46722$
$k = {\cdot}32020$	·2951

We have now the desired h and k and have to interpolate $d/N = \cdot 28066$ between ·2771 and ·2951. There results $r = \cdot 9099$.

This is in excellent agreement with the value ·9105 deduced from 24 terms, or from the final value ·9102, which can be deduced from the 12, 18 and 24 term values on the logarithmic rate of decrease hypothesis: see footnote p. liii.

TABLE XXXI (pp. 58—61)

The Γ-Function. (J. H. Duffell: *Biometrika*, Vol. VII. pp. 43—47.)

It is well known that $\Gamma(x+1) = x\Gamma(x)$, and this property enables us to raise or lower the argument of the Γ-function at will. As a rule in most statistical investigations we require $\Gamma(x+1)/x^x e^{-x}$. The following formula due to Pearson will then be found to give $\Gamma(x+1)/x^x e^{-x}$ with great exactness:

$$\log\left(\frac{\Gamma(x+1)}{x^x e^{-x}}\right) = \cdot 0399{,}0899 + \tfrac{1}{2}\log x + \cdot 080{,}929 \sin\frac{25^\circ\cdot 623}{x} \qquad \ldots\text{(xlii)}.$$

For values of $x+1$ less than 6 and often for values less than 10, we find $\log\Gamma(x+1)$ or $\log\Gamma(p)$ from Table XXXI by reduction to p between 1 and 2.

The reader's attention must be especially drawn as to the rules, given on the Table itself, as to (i) characteristic, (ii) change of third figure of mantissa at a bar, and (iii) the sign of the differences on the facing pages of the tables. The difference tabled under 1·144, say, is the drop from 1·144 to 1·145.

Illustration (i). Find $\Gamma(\cdot 2346)$.

By the reduction formula $\Gamma(\cdot 2346) = \Gamma(1\cdot 2346)/\cdot 2346$.

Hence $\log\Gamma(\cdot 2346) = \log\Gamma(1\cdot 2346) - \bar{1}\cdot 370{,}3280$.

$$\log\Gamma(1\cdot 234) = \bar{1}\cdot 958{,}9685 \qquad \Delta = -\,1069,$$

$$\log\Gamma(1\cdot 235) = \bar{1}\cdot 958{,}8616 \qquad \cdot 6\Delta = -\,[641\cdot 4].$$

$$\therefore\ \log\Gamma(1\cdot 2346) = \bar{1}\cdot 958{,}9685 - [641]$$

$$= \bar{1}\cdot 958{,}9044.$$

$$\log\Gamma(\cdot 2346) = \begin{array}{r} \bar{1}\cdot 958{,}9044 \\ -\,\bar{1}\cdot 370{,}3280 \\ \hline \cdot 588{,}5764 \end{array}$$

Or $\Gamma(\cdot 2346) = 3\cdot 87772$.

Illustration (ii). Find $\Gamma(8\cdot7614)$.

$$\Gamma(8\cdot7614) = 7\cdot7614 \times 6\cdot7614 \times 5\cdot7614 \times 4\cdot7614 \times 3\cdot7614 \times 2\cdot7614 \times 1\cdot7614\,\Gamma(1\cdot7614).$$

$$\begin{array}{rl} \log\Gamma(8\cdot7614) = & \cdot889{,}9401 + \log\Gamma(1\cdot7614) \\ & \cdot830{,}0366 \\ & \cdot760{,}5280 \\ & \cdot677{,}7347 \\ & \cdot575{,}3495 \\ & \cdot441{,}1293 \\ & \cdot245{,}8580 \\ \hline & = 4\cdot420{,}5762 + \log\Gamma(1\cdot7614). \end{array}$$

$$\log\Gamma(1\cdot7614) = \bar{1}\cdot964{,}5473 + \cdot4\,[1113]$$
$$= \bar{1}\cdot964{,}5918.$$
$$\therefore\ \log\Gamma(8\cdot7614) = 4\cdot385{,}1680.$$

Hence $$\Gamma(8\cdot7614) = 24275\cdot49.$$

TABLE XXXII (pp. 62—63)

TABLE XXXIII, A and B (p. 64).

Subtense from Arc and Chord in the case of the Common Catenary. (Julia Bell and H. E. Soper: see *Biometrika*, Vol. VIII. pp. 316, 338, and Vol. IX. pp. 401—2.)

If c be the parameter of the common catenary, then we know that

$$y = c\cosh u \quad \text{.................................(xliii)},$$

where $u = x/c$ is its equation.

If the chord be $2x$, then

$$\left.\begin{array}{l} \text{subtense/chord} = (y-c)/(2x) \\ \qquad = \dfrac{(\sinh \frac{1}{2}u)^2}{u} \end{array}\right\} \quad \text{.....................(xliv)},$$

$$\text{arc/chord} = \frac{\sinh u}{u} \quad \text{...............................(xlv)},$$

$$\frac{\text{arc} - \text{chord}}{\text{chord}} = \frac{\sinh u - u}{u} = \frac{\beta}{100} \text{.......................(xlvi)},$$

$$\frac{\text{subtense}}{\text{chord}} = \frac{(\sinh \frac{1}{2}u)^2}{u} = \frac{\alpha}{100} \quad \text{.....................(xlvii)}.$$

Corresponding values of α and β are given in the Tables XXXII and XXXIII.

Illustration (i). A cable of 132·5 is suspended over the gap between two towers of the same height, 115 feet apart. What will be the droop of the cable?

$$\beta = 100\frac{(132\cdot5 - 115)}{115} = 11\cdot52.$$

Table XXXIII A, gives us $\alpha = 21\cdot62 = 100$ subtense/chord.

$$\therefore \text{ subtense} = \cdot2162 \times 115$$
$$= 24\cdot86.$$

Thus the droop is 24·86 ft.

Illustration (ii). A catenary arch is to have a rise of 50 ft., centre line measurement, and a span of 200. What is the length of the centre line?

$$\alpha = 100 \times 50/200 = 25\cdot0,$$

but $\alpha = 25$ by Table XXXII gives $\beta = 15\cdot1$.

$$100 \text{ (arc} - \text{chord)/chord} = 15\cdot1.$$
$$\therefore \text{ arc} = 230\cdot2 \text{ ft.}^*$$

Illustration (iii). For some races the shape of the nasal bridge is very approximately a catenary. Thus if the nasal chord from dacryon to dacryon be measured and also the tape measure from dacryon to dacryon, we obtain the mesodacryal index β. The tables enable us to pass to the mesodacryal index α, and thus ascertain the nasal subtense, which is slightly harder of direct measurement than the arcual or tape measure.

In the skull of a male gorilla the mesodacryal chord was 22·6 mm., and the mesodacryal arc 30 mm. Determine the mesodacryal subtense

$$\beta = 100\frac{30 - 22\cdot6}{22\cdot6} = \frac{100 \times 7\cdot4}{22\cdot6} = 32\cdot74.$$

Hence, from Table XXXII:

$$\alpha = 38\cdot84 = 100 \text{ subtense}/22\cdot6.$$
$$\therefore \text{ subtense} = 22\cdot6 \times \cdot3884 = 8\cdot8 \text{ mm.}$$

The actual value of the mesodacryal subtense measured on the skull was 8·7 mm.

ABAC XXXIV (p. 65)

Diagram to find the Correlation Coefficient r from Mean Contingency on the Hypothesis of a Normal Frequency Distribution. (Pearson: *Drapers' Company Research Memoirs,* No. 1, "On the Theory of Contingency.")

If n_{pq} be the frequency in the cell of the pth column and qth row of a correlation or contingency table, and m_p be the total frequency in the pth column, n_q the

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[*] Should there be any use for this table for constructional purposes, which there ought to be when the value of the catenary arch is more fully recognised, I will in a later edition of this work give the value of u corresponding to each β, so that the parameter c can be at once read off and the form of the arch readily plotted. It might also be desirable to give the values of α and β to two decimal places. We have these data in our MS. copies.

total frequency in the qth row, and N the whole population, then if the two variates are independent, the frequency to be expected in the p, qth cell will be

$$N \times \frac{n_q}{N} \times \frac{m_p}{N} = \frac{n_q m_p}{N},$$

and the observed excess over this, i.e. $n_{pq} - \frac{n_q m_p}{N}$, is termed the 'contingency' in this cell. The total contingency must be of course zero, i.e. the sum of all the cell contingencies. If, however, we take only the positive excess contingencies and divide them by N, i.e. $\psi = \frac{1}{N} \Sigma_+ \left(n_{pq} - \frac{n_q m_p}{N}\right)$, we obtain the so-called 'mean contingency.' On the assumption of normal frequency distribution it is possible to deduce the actual correlation from ψ, provided that the cells are sufficiently small for summation to replace integration. As in practice our cells are hardly likely to exceed 8 × 8, and may be smaller and unequal in area, we shall generally find a value below that of the true correlation, even if the system be accurately normal. A corrective factor corresponding to the class-index correlation has not yet been theoretically deduced. But experience seems to show that to add *half* the correction due to class-index correlations gives good results. That is to say, that, if r_ψ be the correlation found from the Abac, p. 65, and r_{xC_x} and r_{xC_y} be the class-index correlations for x and y, we should take for the true correlation:

$$\begin{aligned} r &= r_\psi + \tfrac{1}{2}\left[\frac{r_\psi}{r_{xC_x} r_{yC_y}} - r_\psi\right] \\ &= \tfrac{1}{2}\left[r_\psi + \frac{r_\psi}{r_{xC_x} r_{yC_y}}\right] \quad \text{..........................(xlviii).} \end{aligned}$$

It is clear that this is the same thing as taking the mean of the crude mean contingency correlation and its value as corrected for the class-index correlations. The following illustrations may indicate the method of procedure.

Illustration (i). Find the correlation from the table on p. lix by mean contingency. The first number in each cell is the frequency reduced to 1000, the second number is that to be expected on the basis of independent probability, and the third is the mean contingency of the cell.

The sum of the positive contingencies is 94·136, hence the mean contingency is ·094. Entering the diagram with ·094 on the base scale, we pass up the vertical to the curve, and then along the horizontal to the left hand scale and find $r_\psi = {\cdot}285$.

The class-index correlation for the vertical marginal frequency is $r_{yC_y} = {\cdot}9645$, and that for the horizontal marginal frequency is ·9624*. Hence

$$r_\psi / (r_{xC_x} r_{yC_y}) = {\cdot}307,$$

and $$r = \tfrac{1}{2}({\cdot}307 + {\cdot}285) = {\cdot}296.$$

The table is actually a true Gaussian distribution with correlation equal to ·300.

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[*] *Biometrika*, Vol. IX. p. 218.

First Variate *A*.

Second Variate *B*.

	1	*2*	*3*	*4*	*5+6*	*7*	*8*	Totals
1	**4·04** (1·224) *2·816*	**17·16** (10·948) *6·212*	**7·55** (8·976) *−1·426*	**3·30** (6·120) *−2·820*	**0·91** (2·346) *−1·436*	**0·92** (3·434) *−2·514*	**0·12** (0·952) *−0·832*	34
2	**17·41** (10·836) *6·574*	**123·59** (96·922) *26·668*	**79·76** (79·464) *0·296*	**44·64** (54·180) *−9·540*	**14·61** (20·769) *−6·159*	**17·67** (30·401) *−12·731*	**3·32** (8·428) *−5·108*	301
3	**8·86** (10·224) *−1·364*	**93·00** (91·448) *1·552*	**78·31** (74·976) *3·334*	**52·04** (51·120) *0·920*	**19·20** (19·596) *−0·396*	**26·40** (28·684) *−2·284*	**6·19** (7·952) *−1·762*	284
4	**2·83** (4·932) *−2·102*	**37·73** (44·114) *−6·384*	**37·24** (36·168) *1·072*	**27·51** (24·660) *2·850*	**10·95** (9·453) *1·497*	**16·31** (13·837) *2·473*	**4·43** (3·836) *0·594*	137
5+6	**1·62** (3·780) *−2·160*	**25·21** (33·810) *−8·600*	**27·75** (27·720) *0·030*	**22·09** (18·900) *3·190*	**9·26** (7·245) *2·015*	**14·64** (10·605) *4·035*	**4·43** (2·940) *1·490*	105
7	**1·02** (3·528) *−2·508*	**19·50** (31·556) *−12·056*	**24·47** (25·872) *−1·402*	**21·39** (17·640) *3·750*	**9·58** (6·762) *2·818*	**16·36** (9·898) *6·462*	**5·68** (2·744) *2·936*	98
8	**0·22** (1·476) *−1·256*	**5·81** (13·202) *−7·392*	**8·92** (10·824) *−1·904*	**9·03** (7·380) *1·650*	**4·49** (2·829) *1·661*	**8·70** (4·141) *4·559*	**3·83** (1·148) *2·682*	41
Totals	36	322	264	180	69	101	28	1000

Illustration (ii). Find r_ψ by mean contingency for the table on p. lx:

The sum of the positive contingencies is 169·846, or we have mean contingency $\psi = \cdot 170$, whence the diagram leads us to $r_\psi = \cdot 480$. The marginal frequencies are the same as in *Illustration* (i). Thus we have

$$r_\psi/(r_{x\,C_x} r_{y\,C_y}) = \cdot 517,$$

$$r = \tfrac{1}{2}(\cdot 517 + \cdot 480) = \cdot 499.$$

The table gives actually a true Gaussian distribution with correlation ·500. It will be seen from *Illustrations* (i) and (ii), that if the distribution be Gaussian, even if the marginal frequencies are in fairly irregular groupings, r_ψ will be reasonably close to the true contingency, and corrected as suggested above will give excellent results.

First Variate *A*.

Second Variate *B*.	*1*	*2*	*3*	*4*	*5+6*	*7*	*8*	Totals
1	**7·38** (1·224) *6·156*	**19·85** (10·948) *8·902*	**4·94** (8·976) *−4·036*	**1·38** (6·120) *−4·740*	**0·26** (2·346) *−2·086*	**0·18** (3·434) *−3·254*	**0·01** (0·952) *−0·951*	34
2	**20·58** (10·836) *9·744*	**145·47** (96·922) *48·548*	**78·94** (79·464) *−0·524*	**35·98** (54·180) *−18·200*	**9·72** (20·769) *−11·049*	**9·27** (30·401) *−21·131*	**1·04** (8·428) −7·388	301
3	**6·01** (10·224) *−4·214*	**93·63** (91·182) *2·182*	**85·41** (74·976) *10·434*	**54·34** (51·120) *3·220*	**18·59** (19·596) *−1·006*	**22·33** (28·684) *−6·354*	**3·69** (7·952) *−4·262*	284
4	**1·26** (4·932) *−3·672*	**31·81** (44·114) *−12·304*	**39·49** (36·168) *3·322*	**31·03** (24·660) *6·370*	**12·29** (9·453) *2·837*	**17·36** (13·837) *3·523*	**3·76** (3·836) *−·076*	137
5+6	**0·53** (3·780) *−3·250*	**18·11** (33·810) *−15·700*	**27·79** (27·720) *0·070*	**25·14** (18·90) *6·240*	**11·09** (7·245) *3·845*	**17·62** (10·605) *7·015*	**4·72** (2·940) *1·780*	105
7	**0·22** (3·528) *−3·308*	**11·02** (21·556) *−20·536*	**21·59** (25·872) *−4·282*	**23·66** (17·640) *6·020*	**11·86** (6·762) *5·098*	**21·89** (9·898) *11·992*	**7·76** (2·744) *5·016*	98
8	**0·02** (1·476) *−1·456*	**2·11** (12·202) *−11·092*	**5·84** (10·824) *−4·984*	**8·47** (7·380) *1·090*	**5·19** (2·829) *2·361*	**12·35** (4·141) *8·209*	**7·02** (1·148) *5·872*	41
Totals	36	322	264	180	69	101	28	1000

TABLES XXXV—XLVI (pp. 66—87)

Criteria for Frequency Types and Probable Errors of Frequency Constants. (A. J. Rhind: *Biometrika*, Vol. VII. pp. 127—147 and pp. 386—397.)

It is desirable to consider all these tables under one heading, namely the general investigation of frequency type and of the probable errors of frequency constants.

The main lines of Pearson's theory of frequency are involved in the following statements:

If the differential equation to the uni-modal frequency distribution be

$$\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{f(x)} \quad\text{.................................(xlix)},$$

we may suppose $f(x)$ expanded in a series of powers of x, and so

$$\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x+c_2x^2+\ldots+c_nx^n+\ldots} \quad\text{.....................(l)}.$$

then a, c_0, c_1, c_2, ... c_n ... can be uniquely determined from the 'moment coefficients' of the frequency distribution. These constants are functions of certain other constants β_1, β_2-3, β_3, β_4-15, ... which vanish for the Gaussian curve, and are small for any distribution not widely divergent from the Gaussian. Further c_0, c_1, c_2 ... c_n ... converge, if, as usual, these constants are less than unity, the factors of convergence being of the order $\sqrt{\beta\text{-constant}}$. As a matter of fact c_n involves the $(n+2)$th moment coefficient, and thus we obtain values of the c-constants subject to very large errors, if we retain terms beyond c_2. If we stop at c_2 then our differential equation is of the form

$$\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x+c_2x^2} \quad\text{..............................(li)},$$

and we need only $\beta_1=\mu_3^2/\mu_2^3$ and $\beta_2=\mu_4/\mu_2^2$, where μ_2, μ_3, μ_4 are the second, third and fourth moment coefficients about the mean.

If we take the form $\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0}$, we reach the Gaussian, in which each contributory cause-group is independent, and if the number of groups be not very large, each cause-group is of equal valency and contributes with equal frequency results in excess and defect of its mean contribution. If we take $\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x}$, then each contributory cause-group is still of equal valency and independent, but does not give contributions in excess and defect of equal frequency.

Finally if we take $\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x+c_2x^2}$, then contributory cause-groups are not of equal valency, they are not independent, but their results correlated, and further contributions in excess and defect are not equally probable. The use of this form $\frac{1}{y}\frac{dy}{dx}=\frac{x-a}{c_0+c_1x+c_2x^2}$ was adopted to allow of this wide generalisation of the Gaussian hypothesis.

If we adopt it, every β-constant is expressible by means of the formulae:

$$\beta_n\,(\text{even})=(n+1)\{\tfrac{1}{2}\beta_{n-1}+(1+\tfrac{1}{2}\alpha)\,\beta_{n-2}\}/(1-\tfrac{1}{2}(n-1)\,\alpha) \quad\text{...........(lii)},$$

$$\beta_n\,(\text{odd})\;=(n+1)\{\tfrac{1}{2}\beta_1\beta_{n-1}+(1+\tfrac{1}{2}\alpha)\,\beta_{n-2}\}/(1-\tfrac{1}{2}(n-1)\,\alpha) \quad\text{......(liii)},$$

where

$$\alpha=(2\beta_2-3\beta_1-6)/(\beta_2+3) \quad\text{..........................(liv)},$$

in terms of lower β-constants.

Table XLII, (*a*)—(*d*) gives the values of β_3, β_4, β_5 and β_6 in terms of β_1 and β_2. Hence as soon as β_1 and β_2 are calculated we can find the numerical values of

$$\beta_3 = \mu_3\mu_5/\mu_2^4, \quad \beta_4 = \mu_6/\mu_2^3, \quad \beta_5 = \mu_7\mu_3/\mu_2^5, \quad \beta_6 = \mu_8/\mu_2^4 \quad \ldots\ldots\ldots(\text{lv}),$$

theoretically. Although these values will not be those which would be absolutely deduced from the data themselves, they will, considering the large probable errors of μ_5, μ_6, μ_7 and μ_8 be reasonable approximations to them. The values of the probable errors of β_1 and β_2 are determinable by formulae involving $\beta_1, \beta_2 \ldots \beta_8$.

From these formulae, Tables XXXVII and XXXVIII, giving the values of $\sqrt{N}\Sigma_{\beta_1}$ and $\sqrt{N}\Sigma_{\beta_2}$ have been constructed. Hence multiplying by χ_1 from Table V, we obtain

$$\frac{\cdot 67449}{\sqrt{N}}\Sigma_{\beta_1} \quad \text{and} \quad \frac{\cdot 67449}{\sqrt{N}}\Sigma_{\beta_2}$$

the probable errors of β_1 and β_2.

If we add to the standard deviations of β_1 and β_2, the correlation between deviations in β_1 and β_2, namely $R_{\beta_1\beta_2}$, which correlation is given in Table XXXIX, we can find the probable errors of any functions of β_1 and β_2. Two such important functions are the distance d from mean to mode and the skewness *sk* of the distribution. The probable errors of d and *sk* can be found from Tables XL and XLI respectively, the former by multiplying the tabulated value $\sqrt{N}\Sigma_d/\sigma$ by $\sigma \times \chi_1$ (from Table V), and the latter by multiplying the tabulated value $\sqrt{N}\Sigma_{sk}$ by χ_1 (from Table V).

Thus far we have only been concerned with the constants which describe certain physical characters of the frequency distribution without regard to the type of curve suited to the distribution. We now turn to the latter subject.

It is known that the type of frequency depends upon a certain criterion κ_2. Hence near the critical values of κ_2 more than one type of curve may describe the frequency within the limit of the probable error of κ_2. Table XLIII gives the probable error of κ_2, if the entries in that table be multiplied by the χ_1 of Table V.

The following are the series of Type curves which arise according to the value of the criteria

$$\kappa_1 = 2\beta_2 - 3\beta_1 - 6 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{lvi}),$$

$$\kappa_2 = \frac{\beta_1(\beta_2+3)^2}{4(4\beta_2-3\beta_1)(2\beta_2-3\beta_1-6)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(\text{lvii}).$$

β_2 is by necessity $> \frac{3}{4}\beta_1$. Hence for our curves all possible values of β_1, β_2 lie in the positive quadrant between the lines $\beta_2 = \frac{3}{4}\beta_1$ and $\beta_2 = \frac{15}{8}\beta_1 + \frac{9}{2}$, the latter being if we go to β_8 the limit of failure of Type IV, for its μ_6 becomes infinite. Beyond the latter line distributions are heterotypic.

Criterion	*Type*	*Equation to Curve*
$\kappa_2 = 0 \quad \beta_1 = 0, \quad \beta_2 > 3$	VII	$y = y_0 \dfrac{1}{\left(1 + \dfrac{x^2}{a^2}\right)^m}$ ....(lviii).
$\kappa_2 = 0 \quad \beta_1 = 0, \quad \beta_2 = 3$	Normal	$y = y_0 e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$ ....(lix).
$\kappa_2 = 0 \quad \beta_1 = 0, \quad \beta_2 < 3$	II_a	$y = y_0 \left(1 - \dfrac{x^2}{a^2}\right)^m$ ....(lx).
$\kappa_2 = 0 \quad \beta_1 = 0, \quad \beta_2 < 1{\cdot}8$	II_b	$y = y_0 \dfrac{1}{\left(1 - \dfrac{x^2}{a^2}\right)^m}$ ....(lxi).
$\kappa_2 > 0 < 1$	IV	$y = y_0 \dfrac{e^{-\nu \tan^{-1} \frac{x}{a}}}{\left(1 + \dfrac{x^2}{a^2}\right)^m}$ ....(lxii).
$\kappa_2 = 1$*	V	$y = y_0 e^{-\gamma/x} x^{-p}$ ....(lxiii).
$\kappa_2 > 1 < \infty$	VI	$y = y_0 (x - a)^{m_1}/x^{m_2}$ ....(lxiv).
$\kappa_2 = \infty$, i.e. $2\beta_2 - 3\beta_1 - 6 = 0$	III	$y = y_0 e^{-p\frac{x}{a}} \left(1 + \dfrac{x}{a}\right)^p$ ....(lxv).
$\kappa_2 < 0$, i.e. negative. Below $f = 0$	I_{I}	$y = y_0 \left(1 + \dfrac{x}{a_1}\right)^{m_1} \left(1 - \dfrac{x}{a_2}\right)^{m_2}$ ......(lxvi).
$\kappa_2 < 0$. Inside $f = 0$	J_{I}	$y = y_0 \left(1 + \dfrac{x}{a_1}\right)^{m_1} \Big/ \left(1 - \dfrac{x}{a_2}\right)^{m_2}$ ...(lxvii).
$\kappa_2 < 0$. Above $f = 0$	U_{I}	$y = y_0 \dfrac{1}{\left(1 + \dfrac{x}{a_1}\right)^{m_1} \left(1 - \dfrac{x}{a_2}\right)^{m_2}}$ ...(lxviii).

For $\kappa_2 < 0, f = 0$ represents the biquadratic

$$\beta_1 (8\beta_2 - 9\beta_1 - 12)(4\beta_2 - 3\beta_1) = (10\beta_2 - 12\beta_1 - 18)^2 (\beta_2 + 3)^2 \quad \text{...(lxix).}$$

Type I is thus divided into three subclasses, limited range curves, J-shaped curves and U-shaped curves.

Diagram XXXV enables the reader at once to find the type appropriate to his distribution, and Diagram XXXVI gives the same figure on a much larger scale to indicate the changes that occur with large values of β_1 and β_2.

Knowing the values of β_1 and β_2 the computer can fix his point on the Diagram XXXV, but he may come so near a critical point or line, that one curve may appear as reasonable as another. It is clear, for example, that in the neighbourhood of the Gaussian point G, he might possibly use II, I_{I}, III, VI, V,

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* The branch of the cubic $\kappa_2 = 1$ with which we are concerned passes through the Gaussian point, at which $p = \infty$, and along this branch p is always > 5.

IV or VIII, and as all these types at that point transform into each other, the forms actually deduced will be almost identical, however different their equations. But there will be other occasions when doubt as to the use of the simpler of two curves may arise; for example if $\beta_1 = {\cdot}8$, $\beta_2 = 4{\cdot}15$, are we justified in using Type III as simpler than Type I?

Now we have to remember that the variates β_1, β_2 form a frequency surface, of which the equation is

$$z = \frac{M}{2\pi\Sigma_{\beta_1}\Sigma_{\beta_2}\sqrt{1 - R^2_{\beta_1\beta_2}}} e^{-\frac{1}{2}\frac{1}{(1 - R^2{}_{\beta_1\beta_2})}\left(\frac{\beta_1{}^2}{\Sigma_{\beta_1}{}^2} + \frac{\beta_2{}^2}{\Sigma_{\beta_2}{}^2} - \frac{2R_{\beta_1\beta_2}\beta_1\beta_2}{\Sigma_{\beta_1}\Sigma_{\beta_2}}\right)} \text{......(lxx)},$$

and that the contours of this surface projected onto the β_1, β_2 plane of Diagram XXXV form a series of similar and similarly placed ellipses. Within any one of these ellipses a certain amount of the volume of the β_1, β_2-frequency lies, and therefore if this system of contours were properly placed round the β_1, β_2 point on Diagram XXXV we could tell at once the probability that the given point, owing to random sampling, should fall outside a given elliptic contour.

The ellipse which has for principal semi-axes $1{\cdot}177\Sigma_1$ and $1{\cdot}177\Sigma_2$, where Σ_1 and Σ_2 are the principal axes of the ellipse:

$$1 = \frac{1}{1 - R^2_{\beta_1\beta_2}}\left(\frac{\beta_1{}^2}{\Sigma_{\beta_1}{}^2} + \frac{\beta_2{}^2}{\Sigma_{\beta_2}{}^2} - \frac{2R_{\beta_1\beta_2}\beta_1\beta_2}{\Sigma_{\beta_1}\Sigma_{\beta_2}}\right) \text{ ....(lxxi)},$$

covers an area on which stands just one half the frequency, i.e. it is the ellipse determined by the generalised probable error of two variates (see Table X, p. 24).

The semi-minor axis $1{\cdot}177\Sigma_1$ and the semi-major axis $1{\cdot}177\Sigma_2$ of this "Probability Ellipse" multiplied by $\sqrt{N}$ are given in Tables XLIV and XLV respectively, and Table XLVI gives the angle in degrees between the major axis of this ellipse and the axis of β_2. It is thus possible to construct from Tables XLIV—XLVI the "probability ellipse" round a given point β_1, β_2, and to test the area within which half the frequency lies. If the probability required be not $\frac{1}{2}$, but much less, then we note that the probability, that a point will lie outside the ellipse with semi-axes $\lambda\Sigma_1$ and $\lambda\Sigma_2$ is $P = e^{-\frac{1}{2}\lambda^2}$.

Let $$\lambda\Sigma_2 = 1{\cdot}177\sqrt{N}\Sigma_2 \times \frac{{\cdot}67449}{\sqrt{N/q}} \text{ ....(lxxii)},$$

or $$\lambda^2 = q \times {\cdot}630{,}672,$$

and $$P = e^{-q \times {\cdot}315{,}336}.$$

Hence $$\log P = -q \times {\cdot}136{,}949.$$

Accordingly

$q = 10$: $P = {\cdot}0427$,

$q = 12$: $P = {\cdot}0227$,

$q = 15$: $P = {\cdot}0088$,

$q = 20$: $P = {\cdot}0018$.

Hence we select the grade of working probability we require, roughly 1 in 23, 1 in 44, 1 in 114 or 1 in 555, and this determines q. Divide N the total frequency by q and look up in Table V, χ_1 for N/q, multiply this by the $1{\cdot}177\sqrt{N}\Sigma_2$ of Table XLV, p. 84, and we obtain the semi-major axis of the required ellipse. Multiply the same χ_1 by $1{\cdot}177\sqrt{N}\Sigma_1$ of Table XLIV and we have the semi-minor axis. We can then construct round the point β_1, β_2 this ellipse and ascertain if it cuts critical boundaries on Diagram XXXV, p. 66, the orientation being given by Table XLVI, p. 86. Less accurately, but for practical purposes effectively, we may work on Diagram XLVII, p. 88. We proceed just as before, to select our q and so determine our $\lambda\Sigma_2$ and $\lambda\Sigma_1$. Then we take the ratio of Σ_1/Σ_2. We now pick out of the ellipses on p. 88 the set having the nearest Σ_1/Σ_2 value and out of this set the ellipse with the nearest $\lambda\Sigma_2$ value of its semi-major axis. This ellipse or if necessary an interpolated one is transferred to tracing paper and placed with its centre at the given point (β_1, β_2), and its major axis touching the dotted curve. If this ellipse does not cut a critical line, we can be certain that to the given degree of probability the curve is of the type into the area of which its β_1, β_2 point falls.

It would be impossible in an Introduction to these tables to give the whole theory of frequency curves*. But one or two formulae may be usefully placed here for reference.

$$\text{Distance } d \text{ from mode to mean} = \frac{\sigma\sqrt{\beta_1}\,(\beta_2+3)}{2\,(5\beta_2-6\beta_1-9)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(lxxiii)},$$

$$\text{Skewness } sk = \frac{\sqrt{\beta_1}\,(\beta_2+3)}{2\,(5\beta_2-6\beta_1-9)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(lxxiv)},$$

$$N\Sigma_{\beta_1}{}^2 = \beta_1\,(4\beta_4 - 24\beta_2 + 36 + 9\beta_1\beta_2 - 12\beta_3 + 35\beta_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(lxxv)},$$

$$N\Sigma_{\beta_2}{}^2 = \beta_6 - 4\beta_2\beta_4 + 4\beta_2{}^3 - \beta_2{}^2 + 16\beta_2\beta_1 - 8\beta_3 + 16\beta_1) \quad \ldots\ldots\ldots\ldots\text{(lxxv } bis\text{)},$$

$$\Sigma_{\beta_1}\Sigma_{\beta_2}R_{\beta_1\beta_2} = 2\beta_5 - 3\beta_4\beta_1 - 4\beta_3\beta_2 + 6\beta_2{}^2\beta_1 + 3\beta_1\beta_2 - 6\beta_3 + 12\beta_1{}^2 + 24\beta_1 \quad \text{(lxxvi)}.$$

It is from the above formulae that the Tables now under discussion have been calculated.

Illustration. The following percentages of black measured with a colour top are stated to occur with the recorded frequencies in the skin colour of white and negro crosses†.

Discuss the type of frequency curve suited to the data and determine the chief physical constants of the distribution and their probable errors.

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* The general theory is given in "Skew Variation in Homogeneous Material," *Phil. Trans.* Vol. 186 (1895), A, pp. 343—414: Supplement, Vol. 197 (1901), A, pp. 443—459; "On the Mathematical Theory of Errors of Judgment," *Phil. Trans.* Vol. 198 (1902), pp. 274—279; "Das Fehlergesetz und seine Verallgemeinerungen durch Fechner und Pearson," A Rejoinder, *Biometrika*, Vol. IV. pp. 169—212. "Skew Frequency Curves," A Rejoinder to Professor Kapteyn, *Ibid.* Vol. V. pp. 168—171, and "On the curves which are most suitable for describing the frequency of Random Samples of a Population," *Ibid.* Vol. V. pp. 172—175.

† Extracted from C. B. Davenport, *Heredity of Skin Color in Negro-White Crosses*, Carnegie Institution of Washington, 1913.

The working origin was taken at 20, the centre of the group 18—22. The centre of the first group at 1·47 % is $\frac{1}{5}(20 - 1{\cdot}47) = 3{\cdot}706$ on the negative side of the working origin and may be taken to contribute -56, $+206$, -763, $+2830$ to the first, second, third and fourth moments respectively. The working unit being 5 %, the raw moment coefficients are:

Percentage	Frequency	Percentage	Frequency
*0— 2**	15	*43—47*	45
3— 7	120	*48—52*	24
8—12	139	*53—57*	14
13—17	157	*58—62*	6
18—22	158	*63—67*	3
23—27	139	*68—72*	3
28—32	117	*73—77*	2
33—37	92	*78—82*	2
38—42	50	*83—87*	—

$$\nu_1' = {\cdot}567{,}2191, \qquad \nu_2' = 7{\cdot}703{,}4990,$$
$$\nu_3' = 28{\cdot}982{,}5042, \qquad \nu_4' = 253{\cdot}268{,}8730.$$

Whence transferring to mean and correcting, we have

$$\text{Mean} = 22{\cdot}8361, \qquad \sigma = 2{\cdot}70156,$$
$$\mu_2 = 7{\cdot}298{,}428, \qquad \mu_3 = 16{\cdot}238{,}780, \qquad \mu_4 = 198{\cdot}909{,}921.$$

These lead to

$$\beta_1 = {\cdot}678{,}295, \qquad \beta_2 = 3{\cdot}734{,}202,$$
$$\kappa_1 = 2\beta_2 - 3\beta_1 - 6 = -{\cdot}566{,}483, \qquad \kappa_2 = -1{\cdot}052{,}180,$$
$$sk = {\cdot}495{,}087, \qquad \text{Distance from Mean to Mode} = d = 1{\cdot}337{,}508.$$

These values, except the mean, are all in working units. Therefore in percentages of black:

$$\sigma = 13{\cdot}5078 \text{ and } d = 6{\cdot}6875.$$

We can now find the probable errors of these constants. We first want χ_1 from Table V, but 1086 is outside the limit of n. We therefore take χ_2 for 543 and have $\chi_1 = {\cdot}02047$, and we find $\chi_2 = {\cdot}014{,}47$. We can repeat our constants with their probable errors

$$\text{Mean} = 22{\cdot}8361 \pm {\cdot}2765, \qquad \sigma = 13{\cdot}5078 \pm {\cdot}1955.$$

Then from Table XXXVII,

$$\beta_2 = 3{\cdot}7: \quad \sqrt{N}\Sigma_{\beta_1} = 4{\cdot}70 + \tfrac{283}{500}[2] = 4{\cdot}71,$$
$$\beta_2 = 3{\cdot}8: \quad \sqrt{N}\Sigma_{\beta_1} = 5{\cdot}05 + \tfrac{283}{500}[2] = 5{\cdot}06.$$

Hence for $\beta_2 = 3{\cdot}7342: \quad \sqrt{N}\Sigma_{\beta_1} = 4{\cdot}71 + \tfrac{342}{1000}[35],$

$$\sqrt{N}\Sigma_{\beta_1} = 4{\cdot}83.$$

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* 4 at 0 and 11 at 2, giving a mean at 1·47 %.

Similarly from Table XXXVIII:

$$\beta_2 = 3{\cdot}7: \quad \sqrt{N}\Sigma_{\beta_2} = 12{\cdot}02 - \tfrac{283}{500}[66] = 11{\cdot}65.$$

$$\beta_2 = 3{\cdot}8: \quad \sqrt{N}\Sigma_{\beta_2} = 13{\cdot}60 - \tfrac{283}{500}[72] = 13{\cdot}19.$$

Hence for $\beta_2 = 3{\cdot}7342: \quad \sqrt{N}\Sigma_{\beta_2} = 11{\cdot}65 + \tfrac{342}{1000}[1{\cdot}54],$

$$\sqrt{N}\Sigma_{\beta_2} = 12{\cdot}18.$$

Thus we find, multiplying by χ_1:

$$\beta_1 = {\cdot}6783 \pm {\cdot}0989,$$

$$\beta_2 = 3{\cdot}7342 \pm {\cdot}2493.$$

It is clear that the β_1 and β_2 are significantly different from the Gaussian $\beta_1 = 0$ and $\beta_2 = 3$.

We next turn to the skewness, using Table XLI:

$$\beta_2 = 3{\cdot}7: \quad \sqrt{N}\Sigma_{sk} = 1{\cdot}98 + \tfrac{283}{500}[21] = 2{\cdot}10,$$

$$\beta_2 = 3{\cdot}8: \quad \sqrt{N}\Sigma_{sk} = 1{\cdot}88 + \tfrac{283}{500}[16] = 1{\cdot}97.$$

Hence for $\beta_2 = 3{\cdot}7342: \quad \sqrt{N}\Sigma_{sk} = 2{\cdot}10 - \tfrac{342}{1000}[13]$

$$= 2{\cdot}06.$$

Thus the skewness $= {\cdot}4951 \pm {\cdot}0422$, or the distribution is significantly skew.

Passing to Table XL for the probable error of d, we have

$$\beta_2 = 3{\cdot}7: \quad \sqrt{N}\Sigma_d/\sigma = 2{\cdot}14 + \tfrac{283}{500}[20] = 2{\cdot}25,$$

$$\beta_2 = 3{\cdot}8: \quad \sqrt{N}\Sigma_d/\sigma = 2{\cdot}03 + \tfrac{283}{500}[17] = 2{\cdot}13.$$

Hence for $\beta_2 = 3{\cdot}7342: \quad \sqrt{N}\Sigma_d/\sigma = 2{\cdot}25 - \tfrac{342}{1000}[12]$

$$= 2{\cdot}21.$$

Thus Probable Error of $d = \chi_1 \times \sigma \times 2{\cdot}21 = {\cdot}6111$,

and $d = 6{\cdot}6875 \pm {\cdot}6111$.

The probable error of κ_1 is to be found from the relation:

$$(\sqrt{N}\Sigma_{\kappa_1})^2 = 4(\sqrt{N}\Sigma_{\beta_2})^2 + 9(\sqrt{N}\Sigma_{\beta_1})^2 - 12(\sqrt{N}\Sigma_{\beta_2})(\sqrt{N}\Sigma_{\beta_1}) \times R_{\beta_1\beta_2}. \quad \text{(lxxvii)}$$

Thus we require $R_{\beta_1\beta}$. Table XXXIX, p. 72, will provide this:

$$\beta_2 = 3{\cdot}7: \ R_{\beta_1\beta_2} = {\cdot}892 + \tfrac{283}{500}[5] = {\cdot}895,$$

$$\beta_2 = 3{\cdot}8: \ R_{\beta_1\beta_2} = {\cdot}893 + \tfrac{283}{500}[5] = {\cdot}896.$$

Hence for $\beta_2 = 3{\cdot}7342$ we may take $R_{\beta_1\beta_2} = {\cdot}895$. Accordingly

$$(\sqrt{N}\Sigma_{\kappa_1})^2 = 593{\cdot}4096 + 209{\cdot}9601 - 631{\cdot}8278$$

$$= 171{\cdot}5419.$$

Or, $\sqrt{N}\Sigma_{\kappa_1} = 13{\cdot}0974$.

Hence p.e. of $\kappa_1 = \chi_1 \times \sqrt{N}\Sigma_{\kappa_1} = {\cdot}2681$,

or, $\kappa_1 = -{\cdot}566,483 \pm {\cdot}2681$.

It would look therefore as if κ_1 were significantly negative, but it is just possible that κ_1 might be zero. Such a big probable error for κ_1 suggests our being in the neighbourhood of a critical limit. This is verified on examining Table XLIII to find the value of $\sqrt{N}\Sigma_{\kappa_2}$. We see that it is over 80, and we thus conclude that the probable error of κ_2 may lie between 1 and 2. Thus we cannot be definitely certain of the sign or magnitude of κ_2, when we are even relatively in the neighbourhood of $\kappa_1 = 0$.

If we turn to Diagram XXXV (p. 66) we see that the point $\beta_1 = \cdot 678$ and $\beta_2 = 3\cdot 734$ is not very close but is approaching the line along which Type III is applicable, and this is the source of the disturbance noted.

We accordingly try to measure the probability that Type III would be as satisfactory as Type I within the area of which our β_1, β_2 values actually lie. We must take $1\cdot 177\sqrt{N}\Sigma_1$ from Table XLIV and $1\cdot 177\sqrt{N}\Sigma_2$ from Table XLV. We have

$\beta_1 = \cdot 6789$, $\beta_2 = 3\cdot 7$ then $1\cdot 177\sqrt{N}\Sigma_1 = 2\cdot 3$
„ „ $\beta_2 = 3\cdot 8$ „ „ $= 2\cdot 5$.

Hence for $\beta_2 = 3\cdot 7342$, $1\cdot 177\sqrt{N}\Sigma_1 = 2\cdot 3 + \frac{342}{1000}[2] = 2\cdot 37$.
Again: $\beta_2 = 3\cdot 7$ $1\cdot 177\sqrt{N}\Sigma_2 = 16 - \frac{283}{500}[1] = 15\cdot 43$,
$\beta_2 = 3\cdot 8$ $1\cdot 177\sqrt{N}\Sigma_2 = 18 - \frac{283}{500}[1] = 17\cdot 43$,
$\beta_2 = 3\cdot 7342$ $1\cdot 177\sqrt{N}\Sigma_2 = 15\cdot 43 + \frac{342}{1000}[2] = 16\cdot 11$.

Thus: $\Sigma_1/\Sigma_2 = 2\cdot 37/16\cdot 11 = \cdot 147 = \cdot 15$, say. Or, if we turn to Diagram XLVII (p. 88), our system of ellipses is half-way between the 3rd ($\Sigma_1/\Sigma_2 = \cdot 14$) and the 4th ($\Sigma_1/\Sigma_2 = \cdot 16$). Now if such a system of ellipses be traced off and centred at the point $\beta_1 = \cdot 678$, $\beta_2 = 3\cdot 734$ on Diagram XLVII to the right and then the major-axis be brought into parallelism with the dotted lines, we find that the biggest of these ellipses $\lambda\Sigma_2 = \cdot 5$ fails to reach the critical line III. But the semi-major axis of the probability-ellipse is $1\cdot 177\Sigma_2 = 16\cdot 11/\sqrt{N} = \cdot 493$. Hence we must conclude that it is more probable that the curve is of Type I than of Type III. This is readily determined and is usually sufficient guide. Actually the value of $\lambda\Sigma_2$ must be about $\cdot 6$ before we get an ellipse to approximately touch the Type III line. But $\Sigma_2 = \cdot 493/1\cdot 177 = \cdot 419$, and accordingly $\lambda = \cdot 6/\cdot 419 = 1\cdot 432$, which gives $P = e^{-\frac{1}{2}\lambda^2} = \cdot 36$ nearly, or the odds are 16 to 9 that the point would not lie outside this contour. But if it did lie outside this contour, the chance of its being on or over the Type III line corresponds to only a very small section of the total frequency outside this contour. If we invert the problem and put the system of ellipses on the nearest point of the Type III line we find that the odds are very much in favour of the point $\beta_1 = \cdot 678$, $\beta_2 = 3\cdot 734$ lying outside such a system. On the whole it is reasonable to conclude that Type I is properly used although we should probably not get bad results from a Type III curve. In some respects a suitable fit would be obtained by using Type I, and fixing its start at zero*, but the vagueness of what is meant by 'percentage of black' as a factor, when the entire pigmentation of the skin probably arises from a single melanin pigment, only varying in concentration in the pigment granules and in the density of granules themselves. We have therefore contented ourselves by fitting a Type I curve, as further illustration of the use of the tables in the present work. The theory of fitting is given in the paper cited below†. Following the usual notation we find:

$$\begin{aligned} r &= 6(\beta_2 - \beta_1 - 1)/(3\beta_1 - 2\beta_2 + 6) = 21\cdot7755, \\ e &= r^2/\{4 + \tfrac{1}{4}\beta_1 (r+2)^2/(r+1)\} = 57\cdot764{,}468, \\ b^2 &= \mu_2 r^2 (r+1)/\epsilon = (36\cdot9391)^2. \end{aligned}$$

Hence:

$$\begin{aligned} m_1 &= 2\cdot0917, \quad m_2 = 17\cdot6838, \\ a_1 &= 3\cdot9071, \quad a_2 = 33\cdot0320, \end{aligned}$$

and:

$$y = y_0 \left(1 + \frac{x}{3\cdot9071}\right)^{2\cdot0917} \left(1 - \frac{x}{33\cdot0320}\right)^{17\cdot6838}.$$

To find y_0 since m_2 is large, we use the approximation to the formula:

$$y_0 = \frac{N}{b} \frac{(m_1 + m_2 + 1)}{\left\{\dfrac{\Gamma(m_1+1)}{e^{-m_1} m_1^{m_1}}\right\}} \times \frac{\left\{\dfrac{\Gamma(m_1 + m_2 + 1)}{e^{-(m_1+m_2)} (m_1+m_2)^{m_1+m_2}}\right\}}{\left\{\dfrac{\Gamma(m_2+1)}{e^{-m_2} m_2^{m_2}}\right\}} \quad \text{......(lxxviii)},$$

namely,

$$y_0 = \frac{N}{b} \frac{(m_1 + m_2 + 1)}{\Gamma(m_1+1)/(e^{-m_1} m_1^{m_1})} \sqrt{\frac{m_1+m_2}{m_2}}\, e^{\frac{1}{12}\left(\frac{1}{m_1+m_2} - \frac{1}{m_2}\right)} \quad \text{...(lxxix)}$$

the evaluation of the two Γ-functions for $m_1 + m_2 + 1$ and $m_2 + 1$ following easily by Stirling's Theorem. If we write $Z = \Gamma(3\cdot0917)/\{e^{-2\cdot0917}(2\cdot0917)^{2\cdot0917}\}$ we have

$$\begin{aligned} \log Z = {} & \log 2\cdot0917 \quad - 2\cdot0917 \log 2\cdot0917, \\ & + \log 1\cdot0917, \\ & + \log \Gamma(1\cdot0917), \\ & + 2\cdot0917 \log e. \end{aligned}$$

From Table XXXI (p. 58) we find $\log \Gamma(1\cdot0917) = \bar{1}\cdot979{,}8897$ and $\log e$ is given by Table LV (p. 143). Hence we determine, $\log Z = \cdot576{,}5176$. Evaluating the rest of the expression for $\log y_0$ we have:

$$\begin{aligned} \log y_0 &= 2\cdot233{,}3936, \\ y_0 &= 171\cdot157. \end{aligned}$$

Thus our curve is

$$y = 171\cdot157 \left(1 + \frac{x}{3\cdot9071}\right)^{2\cdot0917} \left(1 - \frac{x}{33\cdot0320}\right)^{17\cdot6838}$$

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* For method, see *Phil. Trans.* Vol. 186, A, pp. 370, 371.

† *Phil. Trans.* Vol. 186, A, pp. 367—370. See also Palin Elderton: *Frequency Curves and Correlation*, Layton Brothers.

with origin at the mode $=16\cdot1486$ in actual percentages, $=3\cdot2297$ in working units. To calculate y we take the origin at 0 and have

$$\log y=\overline{26}\cdot134{,}8705+2\cdot0917\log(x+\cdot6774)+17\cdot6838\log(36\cdot2617-x),$$

where x may be put 0, 1, 2, 3, 4, 5... working units corresponding to 0, 5, 10, 15, 20, 25... actual percentages. The curve is shewn in the accompanying diagram, and considering the nature of the data is a reasonable graduation.

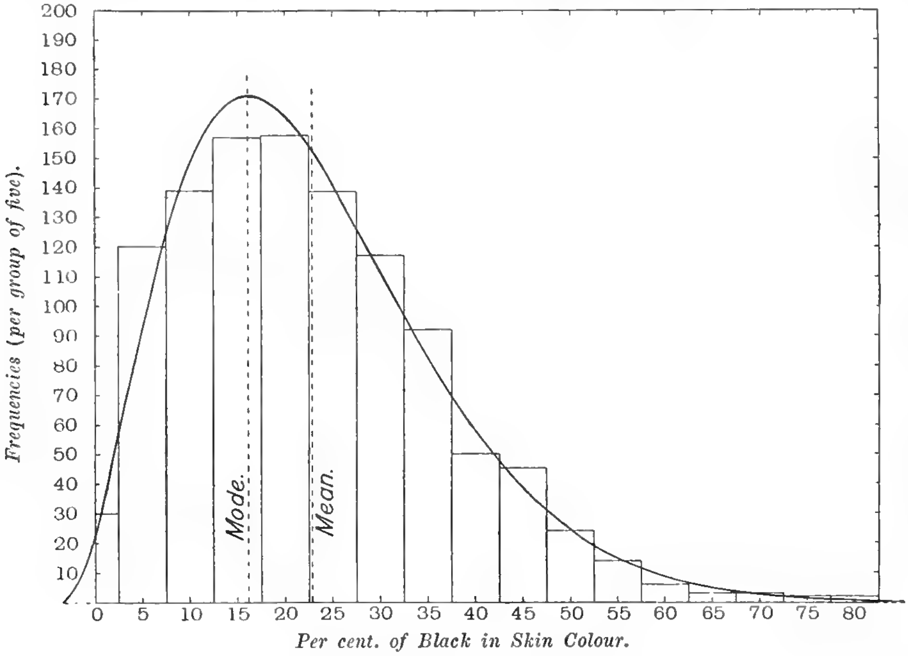

Table XLVIII (pp. 89—97).

Percentage frequency of Occurrences in a Second Sample of m after p Occurrences in a First Sample n. (M. Greenwood, *Biometrika*, Vol. IX. pp. 69—90.)

If we assume the truth of Bayes' Theorem then an event having occurred p times and failed q times in n trials, the chance that it will occur s times and fail $m-s$ times in a second series of m trials is:

$$C_s=\frac{\underline{|m}}{\underline{|s}\,\underline{|m-s}}\,\frac{\int_0^1 x^{p+s}(1-x)^{q+m-s}\,dx}{\int_0^1 x^p(1-x)^q\,dx}.$$

These results can be evaluated as all the indices are integers and the series $C_0+C_1+C_2+\ldots+C_s+\ldots$ expressed in the usual hypergeometrical form:

$$\frac{\underline{|q+m}\,\underline{|n+1}}{\underline{|q}\,\underline{|n+m+1}}\left\{1+\frac{m}{\underline{|1}}\,\frac{p+1}{q+m}+\frac{m(m-1)}{\underline{|2}}\,\frac{(p+1)(p+2)}{(q+m)(q+m-1)}+\frac{m(m-1)(m-2)}{\underline{|3}}\right.$$
$$\left.\times\frac{(p+1)(p+2)(p+3)}{(q+m)(q+m-1)(q+m-2)}+\ldots\ldots\right\}\ldots\ldots\ldots\ldots\ldots(\text{lxxx}).$$

If n be large and p not widely different from q, then results may be obtained from the Gaussian curve, using as S. D. $\sqrt{m\frac{p}{n}\frac{q}{n}}$, but if either p/n or q/n be very small and m or n are commensurable, this no longer holds*. The case, however, of p and q widely different and n and m commensurable and themselves small numbers frequently arises, especially in laboratory work or in the treatment of rare diseases†. The present table gives the evaluation of the hypergeometrical series, formula (lxxx) above, for a series of values of m, n, p and q. It is not sufficiently comprehensive to allow of very accurate interpolation in certain of its ranges, but it has involved a large amount of work, and will undoubtedly be of help till a more complete table can be calculated. Meanwhile if the reader feels in doubt as to any interpolation, it is not a very arduous task to calculate the result required from formula (lxxx) by aid of Table XLIX.

Illustration (i). In a batch of 79 recruits for a certain regiment four were found to be syphilitic. What number of syphilitics may be anticipated in a further batch of 40 recruits?

Here $n=79$, $p=4$, $q=75$ and $m=40$. We must first interpolate in the $p=4$ column on p. 97 between $n=100$, $m=25$, and $n=100$, $m=50$ for $m=40$, i.e. we must go $\frac{15}{25}$ towards the $m=50$ series, or we must add 0·4 times the first series to 0·6 times the second series. We then repeat the same process for the series for $p=4$ and $n=50$, $m=25$ and $n=50$, $m=50$ on p. 95. There results:

Occurrences	$n=50$, $m=40$, $p=4$	$n=100$, $m=40$, $p=4$
0	6·8654	20·7406
1	13·5880	26·7023
2	16·3802	21·5249
3	15·7066	14·1138
4	13·2702	8·2460
5	10·3867	4·4566
6	7·7259	2·2637
7	5·5275	1·0886
8	3·8221	·4977
9	2·5583	·2171
10	1·6581	·0906
11	1·0409	·0362
12	·6332	·0139
13	·3734	·0052
14	·2137	·0019
15	·1188	·0007
16 and over	·1311	·0003

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* For a full discussion of the subject: see Pearson, "On the Influence of Past Experience on Future Expectation," *Philosophical Magazine*, 1907, p. 365.

† Tables recently published by Ross and Stott ("Tables of Statistical Error," *Annals of Tropical Medicine and Parasitology*, Vol. v. No. 3, 1911), appear to be designed to meet such cases, but being based on the Gaussian curve are, I think, very likely to lead the user to fallacious conclusions.

We must interpolate between these two series for $n = 79$, that is we must take 0·42 times the first series and 0·58 times the second series. The results are given below, and set against the direct calculation from formula (lxxx), using Table XLIX.

	By Interpolation.	*Direct Calculation.*
	$n=79,\ p=4,\ m=40$	$n=79,\ p=4,\ m=40$
0	14·9130	12·6143
1	21·1943	21·9379
2	19·3642	22·5152
3	14·7828	17·6667
4	10·3562	11·6727
5	6·9472	6·8143
6	4·5578	3·6137
7	2·9529	1·7713
8	1·8940	·8118
9	1·2004	·3507
10	·7489	·1436
11	·4582	·0560
12	·2740	·0208
13	·1598	·0074
14	·0908	·0025
15	·0503	·0008
16	·0271	·0003
17	·0142	
18 and over	·0140	

The interpolation does not give a result very close to the actual series. For example, not more than three syphilitics might be anticipated in 70 $^{\circ}/_{\circ}$ of samples of 40 by the interpolated series; actually not more than 3 are to be expected in 75 $^{\circ}/_{\circ}$ of samples. At the same time the result is much better than the normal curve theory provides. In the latter case we have

$$\text{Mean} = 40 \times \tfrac{4}{79} = 2{\cdot}025,$$

$$\text{Standard-Deviation} = \sqrt{40 \times \tfrac{4}{79} \times \tfrac{75}{79}} = 1{\cdot}387.$$

Hence $$(3{\cdot}5 - 2{\cdot}025)/1{\cdot}387 = 1{\cdot}064$$

and by Table II this value of x corresponds to $\tfrac{1}{2}(1 + \alpha) = {\cdot}86$, i.e. in 86 $^{\circ}/_{\circ}$ per cent. of samples of 40, we should have not more than 3 cases. It will be seen therefore that (i) the values at the latter end of the Table are not close enough to obtain very accurate results by interpolation, but (ii) that the Gaussian gives a still poorer approximation.

Illustration (ii). Of 10 patients subjected by a first surgeon to a given operation only one dies. A second surgeon in performing the same operation on 7 patients, presumably equally affected, loses 4 cases. Would it be reasonable to assume the second surgeon had inferior operative skill?

On p. 91, we have the series for $p = 2$ when $n = 10$ for the values $m = 5$ and $m = 10$. Taking ·6 of the first series and ·4 of the second we have:

	Interpolation from Table.	*Actual Value from formula.*
	$n=10,\ m=7,\ p=1$	$n=10,\ m=7,\ p=1$
0	37·9762	35·9477
1	30·6704	31·4542
2	17·3366	18·8725
3	8·2114	8·9869
4	3·5248	3·4565
5	1·4446	1·0370
6	·5676	·2200
7	·1996	·0251
8	[·0561]	—
9	[·0114]	—
10	[·0012]	—

The chance that if the two surgeons are of equal skill 4 or more patients will die out of the second surgeon's 7 operations is ·058 by interpolation and ·047 actually. Hence the odds against the occurrence are 16 to 1 by the table and 20 to 1 actually. It will be observed that interpolation gives small values at impossible numbers of deaths, but these have to be reckoned in to obtain the total number 100. That all seven patients should die under the second surgeon, if of equal skill, involves odds of 500 to 1 about in the interpolation result, but 4000 to 1 about actually. On the Gaussian hypothesis in the original problem the mean $= 7 \times \frac{1}{10} = \cdot 7$ and the S. D. $= \sqrt{7 \times \frac{1}{10} \times \frac{9}{10}} = \cdot 7937$, and $(3\cdot 5 - \cdot 7)/\cdot 7937 = 3\cdot 52$ roughly, or this corresponds to odds of about 4545 to 1—which are wholly unreasonable. Thus the Table gives by interpolation odds of approximately the right value, which may serve many useful purposes, for those who are unable to work out the values required from formula (lxxx). At the same time it is clear that a much larger Table with closer values of the quantities involved is desirable.

TABLE XLIX (pp. 98—101)

The Logarithms of Factorials. (Calculated by Julia Bell, published here for the first time.)

This table was obtained by adding up in succession consecutive logarithms in a table of logarithms to 12 figures. Not until the work was completed did we realise the existence of the splendid table of C. F. Degen*, which was then used to confirm our own results. De Morgan in his *Treatise on the Theory of Probabilities* of 1837 published an abridgement to six decimals of Degen's Table of Factorials. His values cannot, however, be trusted to the sixth figure of the mantissa. The

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* *Tabularum ad faciliorem et breviorem probabilitatis computationem utilum.* Havniae, MDCCCXXIV. This gives the logarithms of the factorials up to 1200 with 18 figures in the mantissa.

use of a factorial table is extremely varied, especially in problems in probability involving high numbers.

Illustration. In a certain district the number of children born per month is 662 and the chance of a birth being male is ·51 and of its being female ·49. Evaluate the chance that in a given month there should be an equal number of boys and girls born, and compare it with the chance of the most probable numbers (338 boys and 324 girls) being born.

The chance of equal numbers of boys and girls being born is:

$$C_e = (\cdot 51)^{331}\,(\cdot 49)^{331}\,\frac{\underline{|662}}{\underline{|331}\;\underline{|331}}.$$

Therefore

$$\log C_e = 331\left\{\begin{array}{l}\bar{1}\cdot 707{,}5702\\ +\,\bar{1}\cdot 690{,}1961\end{array}\right\}\begin{array}{l}+\,1581\cdot 714{,}6156\\ -\,1383\cdot 941{,}4114\end{array}$$

where the logs of the factorials are found from Table XLIX. Hence

$$\log C_e = \left.\begin{array}{r}\overline{200}\cdot 660{,}6453\\ +\,197\cdot 773{,}2042\end{array}\right\} = \bar{2}\cdot 433{,}8495$$

or $C_e = \cdot 027155$, or once in about 36·8 months, say once in three years the records may be expected to show equal numbers of boys and girls born in the month*.

The chance of the most probable number of boys and girls is given by

$$C_m = (\cdot 51)^{338}\,(\cdot 49)^{324}\,\frac{\underline{|662}}{\underline{|338}\;\underline{|324}},$$

$$\begin{aligned}\log C_m = {} & 338\times\bar{1}\cdot 707{,}5702 + 1581\cdot 714{,}6156\\ & +324\times\bar{1}\cdot 690{,}1961 - 709\cdot 645{,}9652\\ & \qquad\qquad\qquad\quad\;\; - 674\cdot 359{,}6453\\ = {} & \left.\begin{array}{r}\overline{200}\cdot 782{,}2640\\ +\,197\cdot 709{,}0051\end{array}\right\} = \bar{2}\cdot 491{,}2691.\end{aligned}$$

Or $C_m = \cdot 030993$, or the most probable numbers will only be born once in 32·3 months, or say once in two years and eight months.

We have $C_e/C_m = \cdot 876$, or the chance of equal boys and girls is 88 % of the chance of the most probable numbers of boys and girls.

TABLE L (pp. 102—112)

Tables of Fourth-Moments of Subgroup Frequencies. (Calculated by Alice Lee and P. F. Everitt; published here for the first time.)

In the usual method of determining the raw moments of a frequency, we take moments about an arbitrary origin, which is towards the apparent mode and

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* Actually of course the problem is more complex, because the number of children born per month is not constant.

multiply by plus and minus abscissae increasing by units—the 'working unit.' Thus an error made in an early moment may be carried on to the later moments. To control the results Table L was calculated a number of years ago, and from it the fourth moments for such frequencies as most usually occur can be read off at sight, and the raw fourth moment column thus tested before proceeding further.

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	
Head Length	Frequency	Abscissa	$N\nu_1'$	$N\nu_2'$	$N\nu_3'$	$N\nu_4'$	Table L
171	1	−20	− 20	+ 400	− 8,000	+160,000	160,000
2	1	19	19	361	6,859	130,321	130,321
3	2	18	38	648	11,664	209,952	209,952
4	0	17	—	—	—	—	—
5	3	16	48	768	12,288	196,608	196,608
6	3	15	45	675	10,125	151,875	151,875
7	5	14	70	980	13,720	192,080	192,080
8	7	13	91	1,183	15,379	199,927	199,927
9	12	12	144	1,728	20,736	248,832	248,832
180	13	11	143	1,573	17,303	190,333	190,333
1	17	10	170	1,700	17,000	170,000	170,000
2	28	9	252	2,268	20,412	183,708	183,708
3	24	8	192	1,536	12,288	98,304	98,304
4	43	7	301	2,107	14,749	103,243	103,243
5	53	6	318	1,908	11,448	68,688	68,688
6	57	5	285	1,425	7,125	35,625	35,625
7	55	4	220	880	3,520	14,080	14,080
8	68	3	204	612	1,836	5,508	5,508
9	83	2	166	332	664	1,328	1,328
190	85	− 1	− 85	+ 85	− 85	+ 85	85
1	96	0	—	—		—	—
2	102	+ 1	+102	+ 102	+ 102	+ 102	102
3	79	2	158	316	632	1,264	1,264
4	83	3	249	747	2,241	6,723	6,723
5	66	4	264	1,056	4,224	16,896	16,896
6	66	5	330	1,650	8,250	$\boxed{42,250}$	41,250
7	56	6	336	2,016	12,096	72,576	72,576
8	43	7	301	2,107	14,749	103,243	103,243
9	35	8	280	2,240	17,920	143,360	143,360
200	30	9	270	2,430	21,870	196,830	196,830
1	20	10	200	2,000	20,000	200,000	200,000
2	24	11	264	2,904	31,944	351,384	351,384
3	14	12	168	2,016	24,192	290,304	290,304
4	13	13	169	2,197	28,561	371,293	371,293
5	8	14	112	1,568	21,952	307,328	307,328
6	3	15	45	675	10,125	151,875	151,875
7	6	16	96	1,536	24,576	393,216	393,216
8	0	17	—	—	—	—	—
9	1	18	18	324	5,832	104,976	104,976
210	1	+19	+ 19	+ 361	+ 6,859	+130,321	130,321
Totals	1306	—	+	+	+	+	—

The multiplication can therefore be done very rapidly and it suffices to re-examine not the whole of the arithmetic but only those rows which do not agree with the table.

Illustration. Calculate the first four raw moments of the distribution of head lengths in 1306 non-habitual criminals on the previous page and test whether they are correct.

This was an actually worked out case, and it will be seen that in this instance only one slip was made—that of a wrong multiplication by 5 in the contribution to the fourth moment of the frequency of head lengths 196. Often far more serious blunders are found. Correction would be made and the columns then added up on the adding machine. Two points should be noticed. First it is not in practice necessary to copy out the results from Table L,—they are merely compared on the table itself with the items in column (vii) and any divergence noted. Secondly in actual practice, it would be quite sufficient to take 20 instead of 40 sub-groups in this case. Sheppard's corrections would fully adjust for the difference.

TABLE LI (pp. 113—121)

Tables of the General Term of Poisson's Exponential Expansion ("Law of Small Numbers"). (H. E. Soper, *Biometrika*, Vol. x. p. 25.)

The limit to the binomial series

$$p^n + np^{n-1}q + \frac{n(n-1)}{1.2}p^{n-2}q^2 + \frac{n(n-1)(n-2)}{1.2.3}p^{n-3}q^3 + \dots \dots \text{(lxxxi)},$$

when q is very small, but $nq = m$ is finite, was first shewn by Poisson to be

$$e^{-m}\left(1 + m + \frac{m^2}{1.2} + \frac{m^3}{1.2.3} + \dots + \frac{m^x}{x!} + \dots\right) \quad \dots\dots\dots\text{(lxxxii)}.$$

The present table provides the value of the terms of this series, i.e. $e^{-m}m^x/x!$ to six decimals for $m = 0{\cdot}1$ to $m = 15$ by tenths.

A previous table for $m = 0{\cdot}1$ to $m = 10$ to four decimals has been published by Bortkewitsch*, but his values are not always correct to the fourth decimal. Poisson's exponential limit to the binomial has been termed the "Law of Small Numbers" by Bortkewitsch, but there are objections to the term. The approximation depends on the smallness of q (or, of course, p) and the largeness of n, so that the mean m is finite. Thus 100 murders per annum might be quite a "small number," if they occurred in a population of 40,000,000, for n would be large and q would be small. It is therefore space and time which has limited the present table to $m = 15$, not the idea of m being small of necessity.

Illustration (i). The number of monthly births in the Canton Vaud being taken as 662, and one birth in 114 being that of an imbecile, find the chance of 12 or more imbeciles being born in a month.

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* *Das Gesetz der kleinen Zahlen*, Leipzig, 1898.

The binomial is $\left(\frac{113}{114}+\frac{1}{114}\right)^{662}$. n is accordingly large and q small, while $nq = 5{\cdot}8$ nearly. We look out 5·8 in Table L and sum the terms for 12 and beyond. We find the chance of 12 or more = ·01595. Actually worked from the binomial, it is ·01564. Or about once in five years, we might expect in Canton Vaud a month with 12 imbecile births*.

Illustration (ii). Bortkewitsch (*loc. cit.* p. 25) gives the following deaths from kicks of a horse in ten Prussian Army Corps during 20 years, reached after excluding four corps for special reasons:

Annual Deaths	Frequency Observed	Frequency Poisson's Series
0	109	108·72
1	65	66·22
2	22	20·22
3	3	4·12
4	1	·63
5	—	·08
6 and over	—	·01
Totals ...	200	200

The mean m of the observed frequency is ·61, whence using Table LI (p. 113) and taking ·9 the series for 0·6 and ·1 times the series for 0·7, we reach figures, which multiplied by 200 give us the column headed "Frequency, Poisson's Series" above. Such good agreement, however, is very rare. A good fit to actual data with the Exponential Binomial Limit is not often found. Its chief use lies in theoretical investigations of chance and probable error: see Whitaker, *Biometrika*, Vol. x. p. 36.

TABLE LII (pp. 122—124)

Table of Poisson's Exponential for Cell Frequencies 1 *to* 30. (Lucy Whitaker, *Biometrika*, Vol. x. pp. 36—71.)

Given a cell in which the frequency is n_s corresponding to the population N. Then if n_s and N are very large (or we suppose, without this, the individual to be returned before a second draw), the number in this sth cell will be distributed in M samples of m according to the binomial law

$$M\left\{\left(1-\frac{n_s}{N}\right)+\frac{n_s}{N}\right\}^m.$$

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* See *Eugenics Laboratory Memoirs*, XIII. "A Second Study of the Influence of Parental Alcoholism," p. 22.

The mean will be mn_s/N and the standard deviation $\sqrt{m\frac{n_s}{N}\left(1-\frac{n_s}{N}\right)}$. If we only have a single sample of m and do not know the distribution in the actual population we are compelled to give n_s/N the value m_s/m, where m_s is the number found in the sth cell of the sample. If n_s/N or m_s/m be very small and m large, the binomial will approach Poisson's Exponential Limit, and in such cases the deviations in the samples for the sth cell will be distributed very differently from those following a Gaussian law, and the usual rule for deducing the probability of deviations of a given size by means of the probability integral fails markedly. It is not till we get something like 30 out of 1000 in a cell that we can trust the Gaussian to give us at all a reasonable approach. The present table endeavours to provide material in the case of cell frequencies 1 to 30, which will supply the place of the probability integral.

Illustration (i). Suppose the actual number to be expected in a cell is 17, what is the probability that the observed number will deviate by more than 5 from this result? Looking at p. 123 we see that in 8·467 °/₀ of cases there will be a deviation in defect of 6 or more and in 9·526 °/₀ of cases a deviation in excess of 6 or more. Hence in 17·993 °/₀ say 18 °/₀ of cases we should get values less than 12 or greater than 22. Thus once in every 5 or 6 trials we should get values which differ as widely as 6 or more from the true value.

Now look at the matter from the Gaussian standpoint. The standard deviation is

$$\sqrt{m\frac{17}{m}\left(1-\frac{17}{m}\right)}=\sqrt{17\left(1-\frac{17}{m}\right)}.$$

Here m is supposed large compared with 17, so that the S. D. $=\sqrt{17}=4{\cdot}123$ nearly. But suppose $m=800$, we should have

$$\text{S. D.}=\sqrt{17\,(1-{\cdot}02125)}=\sqrt{17\times{\cdot}97875}=4{\cdot}079.$$

Now we want deviations in excess of 5, i.e. we must take $5{\cdot}5/4{\cdot}079=1{\cdot}348$. If we turn to Table II we find for this argument

$$\tfrac{1}{2}(1+\alpha)={\cdot}9102 \text{ or } \tfrac{1}{2}(1-\alpha)={\cdot}0898.$$

Hence we should conclude that in not more than 17·96 °/₀ of cases would deviations exceed ± 5. Actually such occur in 17·99 °/₀ of cases. Thus the actual percentages are very close, but the Poisson series tells us that 8·47 °/₀ of cases will be in defect and 9·53 °/₀ in excess, while the Gaussian gives 8·98 °/₀ in both excess and defect. We may further ask the percentage of times that 17 itself would occur; according to the Gaussian it will occur in 9·76 °/₀ of trials, actually it will occur in 9·63 °/₀. With values of cell-frequency less than 17, say in the single digits, far greater divergences will be encountered.

Illustration (ii). Consider the fourfold Table below and discuss the relative probabilities that it has arisen from a population which shews 0, 1, 2, 3, etc. individuals for this size of sample in the cell *B*, not-*A*. On the assumption that 0 is really the population of this cell, the probability is unity. Hence we have the following result.

	A	Not-*A*	Totals
B ...	127·5	0	127·5
Not-*B* ...	863·5	87	950·5
	991	87	1078

Population of cell	0	1	2	3	4	5	6	7	8	9	10	11	12	13 & over
Probability of 0 occurring	1	·36788	·13534	·04979	·01832	·00674	·00248	·00091	·00034	·00012	·00005	·00002	·00001	·00000

Sum = 1·58200.

Whence taking the *à priori* probabilities proportional to the probability of 0 occurring on the separate possibilities we have:

Probabilities that the Table arose from a population with x in the B, not-A cell.

x	Probability	x	Probability
0	·632,110	7	·000,575
1	·232,541	8	·000,215
2	·085,550	9	·000,076
3	·031,473	10	·000,032
4	·011,580	11	·000,013
5	·004,260	12	·000,006
6	·001,568	13 and over	·000,000

The "association" of such a Table cannot therefore be considered "perfect," for in 37 °/o of cases it would arise from a Table with a unit or more in the *B*, not-*A* cell. The above is actually a Table of the correlation of stature in father and son. Grave caution is therefore needful in discussing such "perfect association" tables.

TABLE LIII (p. 125)

Angles, Arcs and Decimals of Degrees. (Based on Hutton's *Mathematical Tables.*)

This Table gives degrees in radians for the first two quadrants; it then gives minutes and seconds from 1 to 60 in fractions of a degree and in radians. The need of such a table is very obvious, and arises in too great a variety of circumstances to be specified.

Illustration. It is required to plot the curve*:

$$x = 14{\cdot}9917 \tan \theta,$$

$$y = 235{\cdot}323 \cos^{32{\cdot}8023} \theta e^{-4{\cdot}5696} \theta.$$

Here $\log y = \log 235{\cdot}323 + 32{\cdot}8023 \log \cos \theta - 4{\cdot}5696 \log e \times \theta.$

To cover the whole range of observations we must proceed from $\theta = -45^\circ$ to $\theta = +45^\circ$ roughly. It will be found sufficient to take θ by steps of 3° and ultimately perhaps of 4°. Hence 14·9917 is put on the arithmometer and multiplied in succession by the natural tangents of 3°, 6°, 9° ... etc. Plus and minus signs are given to these values of x. The corresponding values of y are found in three columns. The first is obtained by putting 32·8023 on the arithmometer and multiplying by the logarithmic cosines of 3°, 6°, 9°, etc. The second is obtained by multiplying (taking the third factor from Table LIII)

$$4{\cdot}5696 \times \log e \times {\cdot}017{,}4533 = {\cdot}216{,}7955$$

on the machine and multiplying the result in succession by 3, 6, 9, etc.

The first column is added to $\log 235{\cdot}323 = 2{\cdot}371{,}6644$ and the second column first subtracted and then added to the result to obtain the value of $\log y$ for positive and negative abscissae respectively. The antilogarithms give the ordinates.

Another problem sometimes arises given x to find y. For example: In the above curve the mode is at 117·9998 cms. of stature and the origin at 113·8228, thus the distance between them $= 4{\cdot}1770$ cms. or since the working unit of $x = 2$ cms. and the positive direction of x is towards dwarfs, the mode is at $x = -2{\cdot}0885$. Required to find the maximum ordinate y_{mo}. We have

$$\tan \theta = -2{\cdot}0885/14{\cdot}9917 = -{\cdot}139{,}3104,$$

whence by a table of natural tangents

$$\theta = -7^\circ\ 55'\ {\cdot}851265,$$
$$= -7^\circ\ 55'\ 51''.$$

The log cosine of this value of θ is

$$\bar{1}{\cdot}995{,}8962.$$

Table LIII gives us:

$$\begin{array}{rr}
7^\circ = & {\cdot}122{,}1730 \text{ in arc} \\
55' = & {\cdot}015{,}9989 \ \text{,,} \ \text{,,} \\
{\cdot}851{,}265' = {\cdot}851{,}265 \times {\cdot}000{,}2909 = & \underline{{\cdot}000{,}2476} \ \text{,,} \ \text{,,} \\
\text{Hence } \theta = & -{\cdot}138{,}4195 \ \text{,,} \ \text{,,}
\end{array}$$

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* See *Phil. Trans.* Vol. 186, A, p. 387. Pearson's Type IV frequency curve fitted to the stature of 2192 St Louis School Girls aged 8.

Hence
$$\begin{aligned}\log y_{mo} &= 2{\cdot}371{,}6644 \\ &\quad + 32{\cdot}8023\,(-\,{\cdot}004{,}1038) \\ &\quad + 1{\cdot}984{,}5521 \times {\cdot}138{,}4195 \\ &= 2{\cdot}511{,}7510.\end{aligned}$$

Hence $y_{mo} = 324{\cdot}901.$

TABLE LIV (pp. 126—142)

Tables of the $G(r, \nu)$ Integrals. (Calculated by Alice Lee, D.Sc. *Transactions British Association Report*, Dover, 1899, pp. 65—120.)

The purpose of this table is to obtain the value of the integral

$$G(r, \nu) = \int_0^\pi \sin^r \theta e^{\nu\theta} d\theta \quad \text{..........................(lxxxiii).}$$

In order to obtain small differences in tabulated values two additional functions $F(r, \nu)$ and $H(r, \nu)$ are introduced.

The relations between the three functions are then expressed by the following series of equations:

$$F(r, \nu) = e^{-\frac{1}{2}\nu\pi} G(r, \nu) \quad \text{..............................(lxxxiv),}$$

$$F(r, \nu) = \frac{e^{\nu\phi} (\cos\phi)^{r+1}}{\sqrt{r-1}} H(r, \nu) \quad \text{.....................(lxxxv),}$$

$$G(r, \nu) = e^{\frac{1}{2}\nu\pi} F(r, \nu) \quad \text{.................................(lxxxvi),}$$

$$G(r, \nu) = \frac{e^{\nu\phi + \frac{1}{2}\nu\pi} (\cos\phi)^{r+1}}{\sqrt{r-1}} H(r, \nu) \quad \text{..............(lxxxvii),}$$

$$H(r, \nu) = \frac{\sqrt{r-1}\, e^{-\nu\phi}}{(\cos\phi)^{r+1}} F(r, \nu) \quad \text{.....................(lxxxviii),}$$

$$H(r, \nu) = \frac{\sqrt{r-1}\, e^{-\nu\phi - \frac{1}{2}\nu\pi}}{(\cos\phi)^{r+1}} G(r, \nu) \quad \text{..................(lxxxix),}$$

where $\tan\phi = \nu/r$.

Pearson's Type IV Skew Frequency Curve is of the form

$$y = y_0 \frac{e^{-\nu \tan^{-1}\frac{x}{a}}}{\left\{1 + \left(\frac{x}{a}\right)^2\right\}^{\frac{1}{2}(r+2)}} \quad \text{..............................(xc).}$$

Hence if N be its total area, i.e. the entire population under discussion,

$$N = y_0 a e^{-\frac{1}{2}\nu\pi} \int_0^\pi \sin^r \theta e^{\nu\theta} d\theta,$$

or

$$y_0 = \frac{N}{a} \frac{1}{F(r, \nu)} \quad \text{..............................(xci).}$$

The function $H(r, \nu)$ is introduced because, as a rule, its logarithms have far smaller differences and it is thus capable of more exact determination from a table of double entry. Its physical relation to the curve may be expressed as follows; let the origin be transferred to the mean, then if y_1 be the ordinate at the mean,

$$y_1 = \frac{N}{\sigma} \frac{1}{H(r, \nu)} \quad \text{..............................(xci)}^{\text{bis}},$$

where σ is the standard-deviation of the curve

$$= \frac{a}{\sqrt{r-1} \cos \phi} \quad \text{..............................(xcii)}.$$

The distance of the mean from the origin is given by

$$\mu_1' = -a \tan \phi \quad \text{..............................(xciii)}.$$

When r is fairly large:

$$\frac{1}{F(r, \nu)} = \sqrt{\frac{r}{2\pi}} \frac{e^{\frac{\cos^2 \phi}{3r} - \frac{1}{12r} - \phi r \tan \phi}}{(\cos \phi)^{r+1}} \quad \text{..................(xciv)}.$$

Hence
$$\frac{1}{H(r, \nu)} = \sqrt{\frac{r}{r-1}} \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}q^2} \quad \text{.....................(xcv)},$$

where
$$q = \sqrt{\frac{1 - 4 \cos^2 \phi}{6r}},$$

and thus the evaluation if ϕ be $> 60°$ may be made by aid of Table II*.

Illustration. In the curve fitted to the statures of St Louis School Girls, aged 8 (p. lxxx), we have

$$N = 2192, \qquad a = 14{\cdot}9917,$$
$$r = 30{\cdot}8023, \qquad \nu = 4{\cdot}56967.$$

Find y_0.

We have $\tan \phi = \nu / r = {\cdot}148{,}3548$.

Hence $\phi = 8° \, 26'{\cdot}31315 = 8°{\cdot}43855$.

Turning to the Tables, p. 136, we see the large differences of $\log F(r, \nu)$ at this value of ϕ, and accordingly settle to work with $\log H(r, \nu)$.

We have for $\log H(r, \nu)$,

	$r = 30$	$r = 31$
$\phi = 8°$	·388,2032	·388,5583,
$\phi = 9°$	·388,2278	·388,5822,

$\phi = 8°{\cdot}4386$, $r = 30$:

$$\log H(r, \nu) = {\cdot}388{,}2032 + ({\cdot}4386)[246] - \tfrac{1}{2}({\cdot}4386) \times ({\cdot}5614)[28]$$
$$= {\cdot}388{,}2137.$$

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* For a fuller discussion of these integrals see *Phil. Trans.* Vol. 186, A, pp. 376—381, *B. A. Trans. Report*, Liverpool, 1896, Preliminary Report of Committee..., and the *B.A. Trans. Report*, Dover, 1899, already cited.

$\phi = 8^\circ{\cdot}4386,\ r = 31$:

$$\begin{aligned}\log H(r, \nu) &= {\cdot}388{,}5583 + ({\cdot}4386)[238] - \tfrac{1}{2}({\cdot}4386)({\cdot}5614)[27]\\ &= {\cdot}388{,}5684.\end{aligned}$$

$\phi = 8^\circ{\cdot}4386,\ r = 32$:

$$\begin{aligned}\log H(r, \nu) &= {\cdot}388{,}8910 + ({\cdot}4386)[231] - \tfrac{1}{2}({\cdot}4386)({\cdot}5614)[26]\\ &= {\cdot}388{,}9008.\end{aligned}$$

Hence $\phi = 8^\circ{\cdot}4386,\ r = 30{\cdot}8023$:

$$\begin{aligned}\log H(r, \nu) &= {\cdot}388{,}2137 + {\cdot}8023\,[3547] - \tfrac{1}{2}({\cdot}8023)({\cdot}1977)[-223]\\ &= {\cdot}388{\cdot}5001.\end{aligned}$$

Hence by formula (lxxxv):

$$\log F(r, \nu) = \nu\phi \log e + \overline{r+1}\,\log\cos\phi - \tfrac{1}{2}\log(r-1) + \log H(r, \nu).$$

Or, using Tables LIII and LV, we have

$$\begin{array}{rl} \log F(r, \nu) = & {\cdot}292{,}2901 - {\cdot}737{,}1249\\ + & \bar{1}{\cdot}849{,}6578\\ + & {\cdot}388{,}5001\\ \hline & {\cdot}530{,}4480\\ - & {\cdot}737{,}1249\\ \hline \log F(r, \nu) = & \bar{1}{\cdot}793{,}3231 \end{array}$$

Finally from formula (xci):

$$\begin{array}{rll} \log y_0 = & \log N - \log a & - \bar{1}{\cdot}793{,}3231\\ = & 3{\cdot}340{,}8405 & - 1{\cdot}175{,}8509\\ & & - \bar{1}{\cdot}793{,}3231\\ \cline{3-3} & & - \ {\cdot}969{,}1740\\ = & 2{\cdot}371{,}76665. & \end{array}$$

Or $\qquad y_0 = 235{\cdot}324$*.

Table LV.

This table contains some miscellaneous constants in frequent statistical or biometric use and requires no illustration. It has already been used in the illustrations to previous tables.

I have had the generous assistance of my colleagues Miss E. M. Elderton and Mr H. E. Soper in the preparation of the Illustrations to these Tables. I can hardly hope that arithmetical slips have wholly escaped us in a first edition, and I shall be grateful for the communication of any corrections that my readers may discover are necessary.

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* The value 235·323 obtained in *Phil. Trans.* Vol. 186, A, p. 387, was found by the approximate formula (xciv) before tables were calculated.

Every reader may now see in what way the higher branches of mathematics are concerned in our present subject. They are the abbreviators of long and tedious operations, and it would be perfectly possible, with sufficient time and industry, to do without their use......When both the ordinary and the mathematical result are derived from the same hypothesis, the latter must be the more correct: and in those numerous cases in which the difficulty lies in reducing the original circumstances to a mathematical form, there is nothing to show that we are less liable to error in deducing a common sense result from principles too indefinite for calculation, than we should be in attempting to define more closely, and to apply numerical reasoning.—De Morgan.

TABLE I.

Table of Deviates of the Normal Curve for each Permille of Frequency.

Permille	*·000*	*·001*	*·002*	*·003*	*·004*	*·005*	*·006*	*·007*	*·008*	*·009*	*·010*	
·00	∞	3·0902	2·8782	2·7478	2·6521	2·5758	2·5121	2·4573	2·4089	2·3656	2·3263	*·99*
·01	2·3263	2·2904	2·2571	2·2262	2·1973	2·1701	2·1444	2·1201	2·0969	2·0749	2·0537	*·98*
·02	2·0537	2·0335	2·0141	1·9954	1·9774	1·9600	1·9431	1·9268	1·9110	1·8957	1·8808	*·97*
·03	1·8808	1·8663	1·8522	1·8384	1·8250	1·8119	1·7991	1·7866	1·7744	1·7624	1·7507	*·96*
·04	1·7507	1·7392	1·7279	1·7169	1·7060	1·6954	1·6849	1·6747	1·6646	1·6546	1·6449	*·95*
·05	1·6449	1·6352	1·6258	1·6164	1·6072	1·5982	1·5893	1·5805	1·5718	1·5632	1·5548	*·94*
·06	1·5548	1·5464	1·5382	1·5301	1·5220	1·5141	1·5063	1·4985	1·4909	1·4833	1·4758	*·93*
·07	1·4758	1·4684	1·4611	1·4538	1·4466	1·4395	1·4325	1·4255	1·4187	1·4118	1·4051	*·92*
·08	1·4051	1·3984	1·3917	1·3852	1·3787	1·3722	1·3658	1·3595	1·3532	1·3469	1·3408	*·91*
·09	1·3408	1·3346	1·3285	1·3225	1·3165	1·3106	1·3047	1·2988	1·2930	1·2873	1·2816	*·90*
·10	1·2816	1·2759	1·2702	1·2646	1·2591	1·2536	1·2481	1·2426	1·2372	1·2319	1·2265	*·89*
·11	1·2265	1·2212	1·2160	1·2107	1·2055	1·2004	1·1952	1·1901	1·1850	1·1800	1·1750	*·88*
·12	1·1750	1·1700	1·1650	1·1601	1·1552	1·1503	1·1455	1·1407	1·1359	1·1311	1·1264	*·87*
·13	1·1264	1·1217	1·1170	1·1123	1·1077	1·1031	1·0985	1·0939	1·0893	1·0848	1·0803	*·86*
·14	1·0803	1·0758	1·0714	1·0669	1·0625	1·0581	1·0537	1·0494	1·0450	1·0407	1·0364	*·85*
·15	1·0364	1·0322	1·0279	1·0237	1·0194	1·0152	1·0110	1·0069	1·0027	0·9986	0·9945	*·84*
·16	0·9945	0·9904	0·9863	0·9822	0·9782	0·9741	0·9701	0·9661	0·9621	0·9581	0·9542	*·83*
·17	0·9542	0·9502	0·9463	0·9424	0·9385	0·9346	0·9307	0·9269	0·9230	0·9192	0·9154	*·82*
·18	0·9154	0·9116	0·9078	0·9040	0·9002	0·8965	0·8927	0·8890	0·8853	0·8816	0·8779	*·81*
·19	0·8779	0·8742	0·8705	0·8669	0·8633	0·8596	0·8560	0·8524	0·8488	0·8452	0·8416	*·80*
·20	0·8416	0·8381	0·8345	0·8310	0·8274	0·8239	0·8204	0·8169	0·8134	0·8099	0·8064	*·79*
·21	0·8064	0·8030	0·7995	0·7961	0·7926	0·7892	0·7858	0·7824	0·7790	0·7756	0·7722	*·78*
·22	0·7722	0·7688	0·7655	0·7621	0·7588	0·7554	0·7521	0·7488	0·7454	0·7421	0·7388	*·77*
·23	0·7388	0·7356	0·7323	0·7290	0·7257	0·7225	0·7192	0·7160	0·7128	0·7095	0·7063	*·76*
·24	0·7063	0·7031	0·6999	0·6967	0·6935	0·6903	0·6871	0·6840	0·6808	0·6776	0·6745	*·75*
·25	0·6745	0·6713	0·6682	0·6651	0·6620	0·6588	0·6557	0·6526	0·6495	0·6464	0·6433	*·74*
·26	0·6433	0·6403	0·6372	0·6341	0·6311	0·6280	0·6250	0·6219	0·6189	0·6158	0·6128	*·73*
·27	0·6128	0·6098	0·6068	0·6038	0·6008	0·5978	0·5948	0·5918	0·5888	0·5858	0·5828	*·72*
·28	0·5828	0·5799	0·5769	0·5740	0·5710	0·5681	0·5651	0·5622	0·5592	0·5563	0·5534	*·71*
·29	0·5534	0·5505	0·5476	0·5446	0·5417	0·5388	0·5359	0·5330	0·5302	0·5273	0·5244	*·70*
·30	0·5244	0·5215	0·5187	0·5158	0·5129	0·5101	0·5072	0·5044	0·5015	0·4987	0·4959	*·69*
·31	0·4959	0·4930	0·4902	0·4874	0·4845	0·4817	0·4789	0·4761	0·4733	0·4705	0·4677	*·68*
·32	0·4677	0·4649	0·4621	0·4593	0·4565	0·4538	0·4510	0·4482	0·4454	0·4427	0·4399	*·67*
·33	0·4399	0·4372	0·4344	0·4316	0·4289	0·4261	0·4234	0·4207	0·4179	0·4152	0·4125	*·66*
·34	0·4125	0·4097	0·4070	0·4043	0·4016	0·3989	0·3961	0·3934	0·3907	0·3880	0·3853	*·65*
·35	0·3853	0·3826	0·3799	0·3772	0·3745	0·3719	0·3692	0·3665	0·3638	0·3611	0·3585	*·64*
·36	0·3585	0·3558	0·3531	0·3505	0·3478	0·3451	0·3425	0·3398	0·3372	0·3345	0·3319	*·63*
·37	0·3319	0·3292	0·3266	0·3239	0·3213	0·3186	0·3160	0·3134	0·3107	0·3081	0·3055	*·62*
·38	0·3055	0·3029	0·3002	0·2976	0·2950	0·2924	0·2898	0·2871	0·2845	0·2819	0·2793	*·61*
·39	0·2793	0·2767	0·2741	0·2715	0·2689	0·2663	0·2637	0·2611	0·2585	0·2559	0·2533	*·60*
·40	0·2533	0·2508	0·2482	0·2456	0·2430	0·2404	0·2378	0·2353	0·2327	0·2301	0·2275	*·59*
·41	0·2275	0·2250	0·2224	0·2198	0·2173	0·2147	0·2121	0·2096	0·2070	0·2045	0·2019	*·58*
·42	0·2019	0·1993	0·1968	0·1942	0·1917	0·1891	0·1866	0·1840	0·1815	0·1789	0·1764	*·57*
·43	0·1764	0·1738	0·1713	0·1687	0·1662	0·1637	0·1611	0·1586	0·1560	0·1535	0·1510	*·56*
·44	0·1510	0·1484	0·1459	0·1434	0·1408	0·1383	0·1358	0·1332	0·1307	0·1282	0·1257	*·55*
·45	0·1257	0·1231	0·1206	0·1181	0·1156	0·1130	0·1105	0·1080	0·1055	0·1030	0·1004	*·54*
·46	0·1004	0·0979	0·0954	0·0929	0·0904	0·0878	0·0853	0·0828	0·0803	0·0778	0·0753	*·53*
·47	0·0753	0·0728	0·0702	0·0677	0·0652	0·0627	0·0602	0·0577	0·0552	0·0527	0·0502	*·52*
·48	0·0502	0·0476	0·0451	0·0426	0·0401	0·0376	0·0351	0·0326	0·0301	0·0276	0·0251	*·51*
·49	0·0251	0·0226	0·0201	0·0175	0·0150	0·0125	0·0100	0·0075	0·0050	0·0025	0·0000	*·50*
	·010	*·009*	*·008*	*·007*	*·006*	*·005*	*·004*	*·003*	*·002*	*·001*	*·000*	Permille

TABLE II. *Area and Ordinate in terms of Abscissa.*

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −	z	Δ −	Δ^2 −
·00	·5000000	39894	0	·3989423	199	399
·01	·5039894	39890	4	·3989223	598	399
·02	·5079783	39882	8	·3988625	997	399
·03	·5119665	39870	12	·3987628	1395	398
·04	·5159534	39854	16	·3986233	1793	398
·05	·5199388	39834	20	·3984439	2191	397
·06	·5239222	39810	24	·3982248	2588	397
·07	·5279032	39782	28	·3979661	2984	396
·08	·5318814	39750	32	·3976677	3379	395
·09	·5358564	39714	36	·3973298	3773	394
·10	·5398278	39675	40	·3969525	4166	393
·11	·5437953	39631	44	·3965360	4558	392
·12	·5477584	39584	48	·3960802	4948	390
·13	·5517168	39532	51	·3955854	5337	389
·14	·5556700	39477	55	·3950517	5724	387
·15	·5596177	39418	59	·3944793	6110	386
·16	·5635595	39355	63	·3938684	6493	384
·17	·5674949	39288	67	·3932190	6875	382
·18	·5714237	39217	71	·3925315	7255	380
·19	·5753454	39143	74	·3918060	7633	378
·20	·5792597	39065	78	·3910427	8008	375
·21	·5831662	38983	82	·3902419	8381	373
·22	·5870644	38897	86	·3894038	8752	371
·23	·5909541	38808	89	·3885286	9120	368
·24	·5948349	38715	93	·3876166	9485	365
·25	·5987063	38618	97	·3866681	9847	362
·26	·6025681	38518	100	·3856834	10207	360
·27	·6064199	38414	104	·3846627	10564	357
·28	·6102612	38306	107	·3836063	10917	354
·29	·6140919	38195	111	·3825146	11268	350
·30	·6179114	38081	114	·3813878	11615	347
·31	·6217195	37963	118	·3802264	11958	344
·32	·6255158	37842	121	·3790305	12298	340
·33	·6293000	37717	125	·3778007	12635	337
·34	·6330717	37589	128	·3765372	12968	333
·35	·6368307	37458	131	·3752403	13297	329
·36	·6405764	37323	135	·3739106	13623	325
·37	·6443088	37185	138	·3725483	13944	322
·38	·6480273	37044	141	·3711539	14262	318
·39	·6517317	36900	144	·3697277	14575	313
·40	·6554217	36753	147	·3682701	14885	309
·41	·6590970	36602	150	·3667817	15190	305
42	·6627573	36449	153	·3652627	15491	301
·43	·6664022	36293	156	·3637136	15787	296
·44	·6700314	36133	159	·3621349	16079	292
·45	·6736448	35971	162	·3605270	16367	288
·46	·6772419	35806	165	·3588903	16650	283
·47	·6808225	35638	168	·3572253	16928	278
·48	·6843863	35467	171	·3555325	17202	274
·49	·6879331	35294	173	·3538124	17470	269
·50	·6914625		176	·3520653		264

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −
·50	·6914625	35118	176
·51	·6949743	34939	179
·52	·6984682	34758	181
·53	·7019440	34574	184
·54	·7054015	34388	186
·55	·7088403	34200	189
·56	·7122603	34009	191
·57	·7156612	33815	193
·58	·7190427	33620	196
·59	·7224047	33422	198
·60	·7257469	33222	200
·61	·7290691	33020	202
·62	·7323711	32816	204
·63	·7356527	32610	206
·64	·7389137	32402	208
·65	·7421539	32192	210
·66	·7453731	31980	212
·67	·7485711	31767	214
·68	·7517478	31551	215
·69	·7549029	31334	217
·70	·7580363	31116	219
·71	·7611479	30896	220
·72	·7642375	30674	222
·73	·7673049	30451	223
·74	·7703500	30226	225
·75	·7733726	30001	226
·76	·7763727	29773	227
·77	·7793501	29545	228
·78	·7823046	29316	230
·79	·7852361	29085	231
·80	·7881446	28853	232
·81	·7910299	28620	233
·82	·7938919	28387	234
·83	·7967306	28152	235
·84	·7995458	27917	235
·85	·8023375	27680	236
·86	·8051055	27443	237
·87	·8078498	27205	238
·88	·8105703	26967	238
·89	·8132671	26728	239
·90	·8159399	26489	239
·91	·8185887	26249	240
·92	·8212136	26008	240
·93	·8238145	25768	241
·94	·8263912	25527	241
·95	·8289439	25285	241
·96	·8314724	25044	242
·97	·8339768	24802	242
·98	·8364569	24560	242
·99	·8389129	24318	242
1·00	·8413447		242

TABLE II.—(*continued*).

z	Δ −	Δ^2 −
·3520653	17734	264
·3502919	17994	259
·3484925	18248	254
·3466677	18497	249
·3448180	18741	244
·3429439	18981	239
·3410458	19215	234
·3391243	19444	229
·3371799	19667	224
·3352132	19886	219
·3332246	20099	213
·3312147	20307	208
·3291840	20510	203
·3271330	20707	197
·3250623	20899	192
·3229724	21086	187
·3208638	21267	181
·3187371	21442	176
·3165929	21613	170
·3144317	21777	165
·3122539	21936	159
·3100603	22090	154
·3078513	22239	148
·3056274	22381	143
·3033893	22519	137
·3011374	22650	132
·2988724	22777	126
·2965948	22897	121
·2943050	23013	115
·2920038	23122	110
·2896916	23227	104
·2873689	23325	99
·2850364	23419	93
·2826945	23507	88
·2803438	23589	83
·2779849	23666	77
·2756182	23738	72
·2732444	23805	66
·2708640	23866	61
·2684774	23922	56
·2660852	23972	51
·2636880	24017	45
·2612863	24058	40
·2588805	24093	35
·2564713	24122	30
·2540591	24147	25
·2516443	24167	20
·2492277	24182	15
·2468095	24191	10
·2443904	24196	5
·2419707		0

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +
1·00	·8413447	24076	242	·2419707	24196	0
1·01	·8437524	23834	242	·2395511	24191	5
1·02	·8461358	23592	242	·2371320	24182	10
1·03	·8484950	23351	242	·2347138	24168	14
1·04	·8508300	23109	242	·2322970	24149	19
1·05	·8531409	22868	241	·2298821	24125	24
1·06	·8554277	22626	241	·2274696	24097	28
1·07	·8576903	22386	241	·2250599	24064	33
1·08	·8599289	22145	240	·2226535	24027	37
1·09	·8621434	21905	240	·2202508	23986	41
1·10	·8643339	21665	240	·2178522	23940	46
1·11	·8665005	21426	239	·2154582	23890	50
1·12	·8686431	21188	239	·2130691	23836	54
1·13	·8707619	20950	238	·2106856	23778	58
1·14	·8728568	20712	237	·2083078	23715	62
1·15	·8749281	20475	237	·2059363	23649	66
1·16	·8769756	20239	236	·2035714	23578	70
1·17	·8789995	20004	235	·2012135	23504	74
1·18	·8809999	19769	235	·1988631	23426	78
1·19	·8829768	19535	234	·1965205	23344	82
1·20	·8849303	19302	233	·1941861	23259	85
1·21	·8868606	19070	232	·1918602	23170	89
1·22	·8887676	18839	231	·1895432	23077	93
1·23	·8906514	18609	230	·1872354	22981	96
1·24	·8925123	18379	229	·1849373	22882	99
1·25	·8943502	18151	228	·1826491	22779	103
1·26	·8961653	17924	227	·1803712	22673	106
1·27	·8979577	17697	226	·1781038	22564	109
1·28	·8997274	17472	225	·1758474	22452	112
1·29	·9014747	17248	224	·1736022	22337	115
1·30	·9031995	17026	223	·1713686	22218	118
1·31	·9049021	16804	222	·1691468	22097	121
1·32	·9065825	16584	220	·1669370	21973	124
1·33	·9082409	16365	219	·1647397	21847	127
1·34	·9098773	16147	218	·1625551	21717	129
1·35	·9114920	15930	217	·1603833	21585	132
1·36	·9130850	15715	215	·1582248	21451	134
1·37	·9146565	15501	214	·1560797	21314	137
1·38	·9162067	15289	212	·1539483	21175	139
1·39	·9177356	15078	211	·1518308	21033	142
1·40	·9192433	14868	210	·1497275	20890	144
1·41	·9207302	14660	208	·1476385	20744	146
1·42	·9221962	14453	207	·1455641	20596	148
1·43	·9236415	14248	205	·1435046	20446	150
1·44	·9250663	14044	204	·1414600	20294	152
1·45	·9264707	13842	202	·1394306	20140	154
1·46	·9278550	13642	201	·1374165	19985	155
1·47	·9292191	13443	199	·1354181	19828	157
1·48	·9305634	13245	197	·1334353	19669	159
1·49	·9318879	13049	196	·1314684	19508	160
1·50	·9331928		194	·1295176		162

TABLE II. *Area and Ordinate in terms of Abscissa.*

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +
1·50	·9331928	12855	194	·1295176	19346	162
1·51	·9344783	12662	193	·1275830	19183	163
1·52	·9357445	12471	191	·1256646	19018	165
1·53	·9369916	12282	189	·1237628	18853	166
1·54	·9382198	12094	188	·1218775	18685	167
1·55	·9394292	11908	186	·1200090	18517	168
1·56	·9406201—	11724	184	·1181573	18348	169
1·57	·9417924	11541	183	·1163225	18177	170
1·58	·9429466	11360	181	·1145048	18006	171
1·59	·9440826	11181	179	·1127042	17834	172
1·60	·9452007	11004	177	·1109208	17661	173
1·61	·9463011	10828	176	·1091548	17487	174
1·62	·9473839	10654	174	·1074061	17312	174
1·63	·9484493	10482	172	·1056748	17137	175
1·64	·9494974	10311	170	·1039611	16962	176
1·65	·9505285	10142	169	·1022649	16786	176
1·66	·9515428	9975	167	·1005864	16609	177
1·67	·9525403	9810	165	·0989255	16432	177
1·68	·9535213	9647	163	·0972823	16255	177
1·69	·9544860	9485	162	·0956568	16077	178
1·70	·9554345	9325	160	·0940491	15899	178
1·71	·9563671	9167	158	·0924591	15722	178
1·72	·9572838	9011	156	·0908870	15544	178
1·73	·9581849	8856	155	·0893326	15366	178
1·74	·9590705	8704	153	·0877961	15188	178
1·75	·9599408	8553	151	·0862773	15010	178
1·76	·9607961	8403	149	·0847764	14832	178
1·77	·9616364	8256	147	·0832932	14654	178
1·78	·9624620	8110	146	·0818278	14477	177
1·79	·9632730	7966	144	·0803801	14300	177
1·80	·9640697	7824	142	·0789502	14123	177
1·81	·9648521	7684	140	·0775379	13946	176
1·82	·9656205	7545	139	·0761433	13770	176
1·83	·9663750	7409	137	·0747663	13594	176
1·84	·9671159	7273	135	·0734068	13419	175
1·85	·9678432	7140	133	·0720649	13245	175
1·86	·9685572	7009	132	·0707404	13071	174
1·87	·9692581	6879	130	·0694333	12897	173
1·88	·9699460	6751	128	·0681436	12725	173
1·89	·9706210	6624	126	·0668711	12553	172
1·90	·9712834	6500	125	·0656158	12382	171
1·91	·9719334	6377	123	·0643777	12211	170
1·92	·9725711	6255	121	·0631566	12041	170
1·93	·9731966	6136	120	·0619524	11873	169
1·94	·9738102	6018	118	·0607652	11705	168
1·95	·9744119	5902	116	·0595947	11538	167
1·96	·9750021	5787	115	·0584409	11372	166
1·97	·9755808	5674	113	·0573038	11206	165
1·98	·9761482	5563	111	·0561831	11042	164
1·99	·9767045	5453	110	·0550789	10879	163
2·00	·9772499		108	·0539910		162

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −
2·00	·9772499	5345	108
2·01	·9777844	5239	106
2·02	·9783083	5134	105
2·03	·9788217	5031	103
2·04	·9793248	4929	102
2·05	·9798178	4829	100
2·06	·9803007	4731	98
2·07	·9807738	4634	97
2·08	·9812372	4539	95
2·09	·9816911	4445	94
2·10	·9821356	4352	92
2·11	·9825708	4262	91
2·12	·9829970	4172	89
2·13	·9834142	4084	88
2·14	·9838226	3998	86
2·15	·9842224	3913	85
2·16	·9846137	3829	84
2·17	·9849966	3747	82
2·18	·9853713	3666	81
2·19	·9857379	3587	79
2·20	·9860966	3509	78
2·21	·9864474	3432	77
2·22	·9867906	3357	75
2·23	·9871263	3283	74
2·24	·9874545	3210	73
2·25	·9877755	3138	71
2·26	·9880894	3068	70
2·27	·9883962	2999	69
2·28	·9886962	2932	68
2·29	·9889893	2865	66
2·30	·9892759	2800	65
2·31	·9895559	2736	64
2·32	·9898296	2674	63
2·33	·9900969	2612	62
2·34	·9903581	2552	60
2·35	·9906133	2492	59
2·36	·9908625	2434	58
2·37	·9911060	2377	57
2·38	·9913437	2321	56
2·39	·9915758	2267	55
2·40	·9918025	2213	54
2·41	·9920237	2160	53
2·42	·9922397	2108	52
2·43	·9924506	2058	51
2·44	·9926564	2008	50
2·45	·9928572	1960	49
2·46	·9930531	1912	48
2·47	·9932443	1865	47
2·48	·9934309	1820	46
2·49	·9936128	1775	45
2·50	·9937903		44

TABLE II.—(*continued*).

z	Δ −	Δ^2 +
·0539910	10717	162
·0529192	10557	161
·0518636	10397	160
·0508239	10238	159
·0498001	10081	157
·0487920	9924	156
·0477996	9769	155
·0468226	9616	154
·0458611	9463	153
·0449148	9312	151
·0439836	9162	150
·0430674	9013	149
·0421661	8866	147
·0412795	8720	146
·0404076	8575	145
·0395500	8432	143
·0387069	8290	142
·0378779	8149	140
·0370629	8010	139
·0362619	7873	138
·0354746	7737	136
·0347009	7602	135
·0339408	7468	133
·0331939	7337	132
·0324603	7206	130
·0317397	7077	129
·0310319	6950	127
·0303370	6824	126
·0296546	6699	125
·0289847	6576	123
·0283270	6455	122
·0276816	6335	120
·0270481	6216	119
·0264265	6099	117
·0258166	5984	116
·0252182	5870	114
·0246313	5757	113
·0240556	5646	111
·0234910	5536	110
·0229374	5428	108
·0223945	5322	107
·0218624	5217	105
·0213407	5113	104
·0208294	5011	102
·0203284	4910	101
·0198374	4811	99
·0193563	4713	98
·0188850	4617	96
·0184233	4522	95
·0179711	4428	93
·0175283		92

x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +
2·50	·9937903	1731	44	·0175283	4336	92
2·51	·9939634	1688	43	·0170947	4246	91
2·52	·9941323	1646	42	·0166701	4157	89
2·53	·9942969	1605	41	·0162545	4069	88
2·54	·9944574	1565	40	·0158476	3982	86
2·55	·9946139	1525	39	·0154493	3897	85
2·56	·9947664	1487	39	·0150596	3814	84
2·57	·9949151	1449	38	·0146782	3731	82
2·58	·9950600	1412	37	·0143051	3650	81
2·59	·9952012	1376	36	·0139401	3571	80
2·60	·9953388	1341	35	·0135830	3493	78
2·61	·9954729	1306	35	·0132337	3416	77
2·62	·9956035	1272	34	·0128921	3340	76
2·63	·9957308	1239	33	·0125581	3266	74
2·64	·9958547	1207	32	·0122315	3193	73
2·65	·9959754	1176	32	·0119122	3121	72
2·66	·9960930	1145	31	·0116001	3051	70
2·67	·9962074	1115	30	·0112951	2981	69
2·68	·9963189	1085	29	·0109969	2913	68
2·69	·9964274	1056	29	·0107056	2847	67
2·70	·9965330	1028	28	·0104209	2781	66
2·71	·9966358	1001	27	·0101428	2717	64
2·72	·9967359	974	27	·0098712	2654	63
2·73	·9968333	948	26	·0096058	2592	62
2·74	·9969280	922	26	·0093466	2531	61
2·75	·9970202	897	25	·0090936	2471	60
2·76	·9971099	873	24	·0088465	2413	59
2·77	·9971972	849	24	·0086052	2355	57
2·78	·9972821	825	23	·0083697	2299	56
2·79	·9973646	803	23	·0081398	2244	55
2·80	·9974449	781	22	·0079155	2189	54
2·81	·9975229	759	22	·0076965	2136	53
2·82	·9975988	738	21	·0074829	2084	52
2·83	·9976726	717	21	·0072744	2033	51
2·84	·9977443	697	20	·0070711	1983	50
2·85	·9978140	678	20	·0068728	1934	49
2·86	·9978818	658	19	·0066793	1886	48
2·87	·9979476	640	19	·0064907	1839	47
2·88	·9980116	622	18	·0063067	1793	46
2·89	·9980738	604	18	·0061274	1748	45
2·90	·9981342	587	17	·0059525	1704	44
2·91	·9981929	570	17	·0057821	1661	43
2·92	·9982498	553	16	·0056160	1619	42
2·93	·9983052	537	16	·0054541	1578	41
2·94	·9983589	522	16	·0052963	1537	40
2·95	·9984111	507	15	·0051426	1497	40
2·96	·9984618	492	15	·0049929	1459	39
2·97	·9985110	478	14	·0048470	1421	38
2·98	·9985588	464	14	·0047050	1384	37
2·99	·9986051	450	14	·0045666	1347	36
3·00	·9986501		13	·0044318		35

TABLE II. *Area and Ordinate in terms of Abscissa.*

x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +
3·00	·9986501	437	13	·0044318	1312	35
3·01	·9986938	424	13	·0043007	1277	35
3·02	·9987361	411	13	·0041729	1243	34
3·03	·9987772	399	12	·0040486	1210	33
3·04	·9988171	387	12	·0039276	1178	32
3·05	·9988558	375	12	·0038098	1146	32
3·06	·9988933	364	11	·0036951	1115	31
3·07	·9989297	353	11	·0035836	1085	30
3·08	·9989650	342	11	·0034751	1056	29
3·09	·9989992	332	10	·0033695	1027	29
3·10	·9990324	322	10	·0032668	999	28
3·11	·9990646	312	10	·0031669	971	27
3·12	·9990957	302	10	·0030698	944	27
3·13	·9991260	293	9	·0029754	918	26
3·14	·9991553	284	9	·0028835	893	26
3·15	·9991836	275	9	·0027943	868	25
3·16	·9992112	267	9	·0027075	843	24
3·17	·9992378	258	8	·0026231	820	24
3·18	·9992636	250	8	·0025412	797	23
3·19	·9992886	242	8	·0024615	774	23
3·20	·9993129	235	8	·0023841	752	22
3·21	·9993363	227	7	·0023089	731	21
3·22	·9993590	220	7	·0022358	710	21
3·23	·9993810	213	7	·0021649	689	20
3·24	·9994024	206	7	·0020960	669	20
3·25	·9994230	200	7	·0020290	650	19
3·26	·9994429	193	6	·0019641	631	19
3·27	·9994623	187	6	·0019010	612	18
3·28	·9994810	181	6	·0018397	595	18
3·29	·9994991	175	6	·0017803	577	17
3·30	·9995166	169	6	·0017226	560	17
3·31	·9995335	164	6	·0016666	543	17
3·32	·9995499	159	5	·0016122	527	16
3·33	·9995658	153	5	·0015595	512	16
3·34	·9995811	148	5	·0015084	496	15
3·35	·9995959	143	5	·0014587	481	15
3·36	·9996103	139	5	·0014106	467	15
3·37	·9996242	134	5	·0013639	453	14
3·38	·9996376	130	4	·0013187	439	14
3·39	·9996505	125	4	·0012748	426	13
3·40	·9996631	121	4	·0012322	413	13
3·41	·9996752	117	4	·0011910	400	13
3·42	·9996869	113	4	·0011510	388	12
3·43	·9996982	109	4	·0011122	376	12
3·44	·9997091	106	4	·0010747	364	12
3·45	·9997197	102	4	·0010383	353	11
3·46	·9997299	99	3	·0010030	342	11
3·47	·9997398	95	3	·0009689	331	11
3·48	·9997493	92	3	·0009358	320	10
3·49	·9997585	89	3	·0009037	310	10
3·50	·9997674		3	·0008727		10

x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −
3·50	·9997674	86	3
3·51	·9997759	83	3
3·52	·9997842	80	3
3·53	·9997922	77	3
3·54	·9997999	74	3
3·55	·9998074	72	3
3·56	·9998146	69	3
3·57	·9998215	67	2
3·58	·9998282	65	2
3·59	·9998347	62	2
3·60	9998409	60	2
3·61	9998469	58	2
3·62	·9998527	56	2
3·63	·9998583	54	2
3·64	·9998637	52	2
3·65	·9998689	50	2
3·66	·9998739	48	2
3·67	·9998787	47	2
3·68	·9998834	45	2
3·69	·9998879	43	2
3·70	·9998922	42	2
3·71	·9998964	40	2
3·72	·9999004	39	1
3·73	·9999043	37	1
3·74	·9999080	36	1
3·75	·9999116	35	1
3·76	·9999150	33	1
3·77	·9999184	32	1
3·78	·9999216	31	1
3·79	·9999247	30	1
3·80	·9999277	29	1
3·81	·9999305	28	1
3·82	·9999333	27	1
3·83	·9999359	26	1
3·84	·9999385	25	1
3·85	·9999409	24	1
3·86	·9999433	23	1
3·87	·9999456	22	1
3·88	·9999478	21	1
3·89	·9999499	20	1
3·90	·9999519	19	1
3·91	·9999539	19	1
3·92	·9999557	18	1
3·93	·9999575	17	1
3·94	·9999593	17	1
3·95	·9999609	16	1
3·96	·9999625	15	1
3·97	·9999641	15	1
3·98	·9999655	14	1
3·99	·9999670	14	1
4·00	·9999683		1

TABLE II.—(*continued*).

z	Δ −	Δ^2 +
·0008727	301	10
·0008426	291	10
·0008135	282	9
·0007853	273	9
·0007581	264	9
·0007317	256	8
·0007061	247	8
·0006814	239	8
·0006575	232	8
·0006343	224	8
·0006119	217	7
·0005902	210	7
·0005693	203	7
·0005490	196	7
·0005294	189	6
·0005105	183	6
·0004921	177	6
·0004744	171	6
·0004573	165	6
·0004408	160	6
·0004248	155	5
·0004093	149	5
·0003944	144	5
·0003800	139	5
·0003661	135	5
·0003526	130	5
·0003396	125	4
·0003271	121	4
·0003149	117	4
·0003032	113	4
·0002919	109	4
·0002810	105	4
·0002705	102	4
·0002604	98	4
·0002506	95	3
·0002411	91	3
·0002320	88	3
·0002232	85	3
·0002147	82	3
·0002065	79	3
·0001987	76	3
·0001910	73	3
·0001837	71	3
·0001766	68	3
·0001698	66	2
·0001633	63	2
·0001569	61	2
·0001508	59	2
·0001449	57	2
·0001393	55	2
·0001338		2

x	$\frac{1}{2}(1+a)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +
4·00	·9999683	13	1	·0001338	53	2
4·01	·9999696	13	1	·0001286	51	2
4·02	·9999709	12	0	·0001235	49	2
4·03	·9999721	12		·0001186	47	2
4·04	·9999733	11		·0001140	45	2
4·05	·9999744	11		·0001094	43	2
4·06	·9999755	10		·0001051	42	2
4·07	·9999765	10		·0001009	40	2
4·08	·9999775	9		·0000969	39	2
4·09	·9999784	9		·0000930	37	1
4·10	·9999793	9		·0000893	36	1
4·11	·9999802	8		·0000857	35	1
4·12	·9999811	8		·0000822	33	1
4·13	·9999819	8		·0000789	32	1
4·14	·9999826	7		·0000757	31	1
4·15	·9999834	7		·0000726	30	1
4·16	·9999841	7		·0000697	28	1
4·17	·9999848	7		·0000668	27	1
4·18	·9999854	6		·0000641	26	1
4·19	·9999861	6		·0000615	25	1
4·20	·9999867	6		·0000589	24	1
4·21	·9999872	6		·0000565	23	1
4·22	·9999878	5		·0000542	22	1
4·23	·9999883	5		·0000519	22	1
4·24	·9999888	5		·0000498	21	1
4·25	·9999893	5		·0000477	20	1
4·26	·9999898	4		·0000457	19	1
4·27	·9999902	4		·0000438	18	1
4·28	·9999907	4		·0000420	18	1
4·29	·9999911	4		·0000402	17	1
4·30	·9999915	4		·0000385	16	1
4·31	·9999918	4		·0000369	16	1
4·32	·9999922	3		·0000354	15	1
4·33	·9999925	3		·0000339	14	1
4·34	·9999929	3		·0000324	14	1
4·35	·9999932	3		·0000310	13	1
4·36	·9999935	3		·0000297	13	1
4·37	·9999938	3		·0000284	12	1
4·38	·9999941	3		·0000272	12	0
4·39	·9999943	3		·0000261	11	
4·40	·9999946	2		·0000249	11	
4·41	·9999948	2		·0000239	10	
4·42	·9999951	2		·0000228	10	
4·43	·9999953	2		·0000218	9	
4·44	·9999955	2		·0000209	9	
4·45	·9999957	2		·0000200	9	
4·46	·9999959	2		·0000191	8	
4·47	·9999961	2		·0000183	8	
4·48	·9999963	2		·0000175	8	
4·49	·9999964	2		·0000167	7	
4·50	·9999966			·0000160		

TABLE II. *Area and Ordinate in terms of Abscissa*[*].

x	$\frac{1}{2}(1+a)$	z
4·50	66023	159837
4·51	67586	152797
4·52	69080	146051
4·53	70508	139590
4·54	71873	133401
4·55	73177	127473
4·56	74423	121797
4·57	75614	116362
4·58	76751	111159
4·59	77838	106177
4·60	78875	101409
4·61	79867	96845
4·62	80813	92477
4·63	81717	88297
4·64	82580	84298
4·65	83403	80472
4·66	84190	76812
4·67	84940	73311
4·68	85656	69962
4·69	86340	66760
4·70	86992	63698
4·71	87614	60771
4·72	88208	57972
4·73	88774	55296
7·74	89314	52739
4·75	89829	50295
4·76	90320	47960
4·77	90789	45728
4·78	91235	43596
4·79	91661	41559
4·80	92067	39613
4·81	92453	37755
4·82	92822	35980
4·83	93173	34285
4·84	93508	32667
4·85	93827	31122
4·86	94131	29647
4·87	94420	28239
4·88	94696	26895
4·89	94958	25613
4·90	95208	24390
4·91	95446	23222
4·92	95673	22108
4·93	95889	21046
4·94	96094	20033
4·95	96289	19066
4·96	96475	18144
4·97	96652	17265
4·98	96821	16428
4·99	96981	15629
5·00	97133	14867
5·01	97278	14141
5·02	97416	13450
5·03	97548	12791
5·04	97672	12162
5·05	97791	11564
5·06	97904	10994
5·07	98011	10451
5·08	98113	9934
5·09	98210	9441
5·10	98302	8972
5·11	98389	8526
5·12	98472	8101
5·13	98551	7696
5·14	98626	7311
5·15	98698	6944
5·16	98765	6595
5·17	98830	6263
5·18	98891	5947
5·19	98949	5647
5·20	99004	5361
5·21	99056	5089
5·22	99105	4831
5·23	99152	4585
5·24	99197	4351
5·25	99240	4128
5·26	99280	3917
5·27	99318	3716
5·28	99354	3525
5·29	99388	3344
5·30	99421	3171
5·31	99452	3007
5·32	99481	2852
5·33	99509	2704
5·34	99535	2563
5·35	99560	2430
5·36	99584	2303
5·37	99606	2183
5·38	99628	2069
5·39	99648	1960
5·40	99667	1857
5·41	99685	1760
5·42	99702	1667
5·43	99718	1579
5·44	99734	1495
5·45	99748	1416
5·46	99762	1341
5·47	99775	1270
5·48	99787	1202
5·49	99799	1138
5·50	99810	1077
5·51	99821	1019
5·52	99831	965
5·53	99840	913
5·54	99849	864
5·55	99857	817
5·56	99865	773
5·57	99873	731
5·58	99880	691
5·59	99886	654
5·60	99893	618
5·61	99899	585
5·62	99905	553
5·63	99910	522
5·64	99915	494
5·65	99920	467
5·66	99924	441
5·67	99929	417
5·68	99933	394
5·69	99936	372
5·70	99940	351
5·71	99944	332
5·72	99947	313
5·73	99950	296
5·74	99953	280
5·75	99955	264
5·76	99958	249
5·77	99960	235
5·78	99963	222
5·79	99965	210
5·80	99967	198
5·81	99969	187
5·82	99971	176
5·83	99972	166
5·84	99974	157
5·85	99975	148
5·86	99977	139
5·87	99978	131
5·88	99979	124
5·89	99981	117
5·90	99982	110
5·91	99983	104
5·92	99984	98
5·93	99985	92
5·94	99986	87
5·95	99987	82
5·96	99987	77
5·97	99988	73
5·98	99989	68
5·99	99990	65
6·00	99990	61

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[*] Prefix ·99999 to each entry.

TABLE III. *Abscissa and Ordinate in terms of difference of Areas.*

α	x	Δ +	Δ^2 +	Δ^3 +	z	Δ −	Δ^2 −	Δ^3 −
·00	·0000000	125335	0	20	·3989423	313	627	0
·01	·0125335	125354	20	20	·3989109	940	627	0
·02	·0250689	125394	39	20	·3988169	1567	627	0
·03	·0376083	125453	59	20	·3986603	2194	627	0
·04	·0501536	125532	79	20	·3984408	2821	627	0
·05	·0627068	125631	99	20	·3981587	3449	628	1
·06	·0752699	125750	119	20	·3978138	4078	628	1
·07	·0878448	125889	139	20	·3974060	4707	629	1
·08	·1004337	126048	159	20	·3969353	5337	630	1
·09	·1130385	126228	180	21	·3964016	5967	631	1
·10	·1256613	126429	201	21	·3958049	6599	632	1
·11	·1383042	126650	221	21	·3951450	7232	633	1
·12	·1509692	126893	243	21	·3944218	7866	634	1
·13	·1636585	127157	264	22	·3936352	8501	635	1
·14	·1763742	127443	286	22	·3927852	9137	636	1
·15	·1891184	127751	308	22	·3918715	9775	638	2
·16	·2018935	128081	330	23	·3908939	10415	640	2
·17	·2147016	128434	353	23	·3898525	11056	641	2
·18	·2275450	128811	376	24	·3887469	11699	643	2
·19	·2404260	129211	400	24	·3875769	12344	645	2
·20	·2533471	129635	424	25	·3863425	12991	647	2
·21	·2663106	130084	449	25	·3850434	13641	649	2
·22	·2793190	130559	474	26	·3836794	14292	652	2
·23	·2923749	131059	500	27	·3822501	14946	654	3
·24	·3054808	131586	527	27	·3807555	15603	657	3
·25	·3186394	132140	554	28	·3791952	16262	659	3
·26	·3318533	132722	582	29	·3775690	16924	662	3
·27	·3451255	133333	611	30	·3758766	17589	665	3
·28	·3584588	133973	640	30	·3741177	18258	668	3
·29	·3718561	134644	671	31	·3722919	18929	672	3
·30	·3853205	135346	702	32	·3703990	19604	675	4
·31	·3988551	136081	735	34	·3684386	20283	679	4
·32	·4124631	136849	768	35	·3664103	20965	682	4
·33	·4261480	137652	803	36	·3643138	21651	686	4
·34	·4399132	138490	839	37	·3621487	22342	690	4
·35	·4537622	139366	876	39	·3599146	23036	695	4
·36	·4676988	140281	914	40	·3576109	23735	699	5
·37	·4817268	141235	954	42	·3552374	24439	704	5
·38	·4958503	142231	996	43	·3527935	25148	709	5
·39	·5100735	143271	1039	45	·3502788	25861	714	5
·40	·5244005	144355	1085	47	·3476926	26580	719	6
·41	·5388360	145487	1132	49	·3450346	27305	725	6
·42	·5533847	146668	1181	51	·3423041	28035	730	6
·43	·5680515	147900	1232	54	·3395005	28772	736	6
·44	·5828415	149186	1286	56	·3366233	29514	743	7
·45	·5977601	150529	1342	59	·3336719	30264	749	7
·46	·6128130	151930	1402	62	·3306455	31020	756	7
·47	·6280060	153394	1464	65	·3275435	31783	763	7
·48	·6433454	154923	1529	69	·3243652	32554	771	8
·49	·6588377	156521	1598	72	·3211098	33333	779	8
·50	·6744898		1670		·3177766		787	

TABLE III. *Abscissa and Ordinate in terms of difference of Areas.*

a	*x*	Δ +	Δ^2 +	Δ^3 +	*z*	Δ −	Δ^2 −	Δ^3 −
·50	·6744898		1670		·3177766		787	
		158191		76		34119		9
·51	·6903088		1747		·3143646		795	
		159937		81		34915		9
·52	·7063026		1828		·3108732		804	
		161765		86		35719		9
·53	·7224791		1913		·3073013		814	
		163678		91		36532		10
·54	·7388468		2004		·3036481		823	
		165682		96		37356		10
·55	·7554150		2100		·2999125		834	
		167782		102		38189		11
·56	·7721932		2203		·2960936		844	
		169984		109		39034		11
·57	·7891917		2312		·2921902		856	
		172296		116		39889		12
·58	·8064212		2428		·2882013		867	
		174724		124		40757		12
·59	·8238936		2552		·2841256		880	
		177276		133		41637		13
·60	·8416212		2685		·2799619		893	
		179961		143		42530		14
·61	·8596174		2828		·2757089		907	
		182789		153		43437		15
·62	·8778963		2981		·2713653		921	
		185771		165		44358		15
·63	·8964734		3147		·2669295		937	
		188917		178		45295		16
·64	·9153651		3325		·2624000		953	
		192242		193		46247		17
·65	·9345893		3518		·2577753		970	
		195760		209		47217		18
·66	·9541653		3727		·2530535		988	
		199486		227		48205		19
·67	·9741139		3954		·2482330		1007	
		203440		248		49213		20
·68	·9944579		4201		·2433117		1028	
		207641		271		50240		22
·69	1·0152220		4472		·2382877		1049	
		212114		297		51289		23
·70	1·0364334		4769		·2331588		1072	
		216882		326		52362		25
·71	1·0581216		5095		·2279226		1097	
		221977		360		53459		26
·72	1·0803193		5455		·2225767		1123	
		227432		399		54582		28
·73	1·1030626		5854		·2171185		1152	
		233286		443		55734		30
·74	1·1263911		6297		·2115451		1182	
		239583		495		56916		33
·75	1·1503494		6792		·2058535		1215	
		246374		555		58130		35
·76	1·1749868		7347		·2000405		1250	
		253721		625		59380		38
·77	1·2003589		7972		·1941024		1288	
		261693		709		60669		42
·78	1·2265281		8681		·1880356		1330	
		270373		808		61999		45
·79	1·2535654		9488		·1818357		1375	
		279861		926		63374		50
·80	1·2815516		10414		·1754983		1425	

TABLE IV.

Extension of Table of the Probability Integral $F = \frac{1}{2}(1 - \alpha)$.

$F = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}x^2} dx.$ *The table gives* $(-\log F)$ *for* x.

x	$-\log F$	x	$-\log F$	x	$-\log F$
5	6·54265	30	197·30921	50	544·96634
6	9·00586	31	210·56940	60	783·90743
7	11·89285	32	224·26344	70	1066·26576
8	15·20614	33	238·39135	80	1392·04459
9	18·94746	34	252·95315	90	1761·24604
10	23·11805	35	267·94888	100	2173·87154
11	27·71882	36	283·37855	150	4888·38812
12	32·75044	37	299·24218	200	8688·58977
13	38·21345	38	315·53979	250	13574·49960
14	44·10827	39	332·27139	300	19546·12790
15	50·43522	40	349·43701	350	26603·48018
16	57·19458	41	367·03664	400	34746·55970
17	64·38658	42	385·07032	450	43975·36860
18	72·01140	43	403·53804	500	54289·40830
19	80·06919	44	422·43983		
20	88·56010	45	441·77568		N.B. To obtain anything but a rough appreciation after $x = 50$, the table would require much extension, but for many practical problems it suffices to take after $x = 50$: $F = \frac{1}{\sqrt{2\pi}} \frac{1}{x} e^{-\frac{1}{2}x^2}$.
21	97·48422	46	461·54561		
22	106·84167	47	481·74964		
23	116·63253	48	502·38776		
24	126·85686	49	523·45999		
25	137·51475	50	544·96634		
26	148·60624				
27	160·13139				
28	172·09024				
29	184·48283				
30	197·30921				

From each of the values in this table ·30103 must be subtracted, if we wish to obtain the probability $2F$, then given by $(-\log 2F)$, that the value is greater than x, without regard to sign.

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
1	·67449	·47694
2	·47694	·33724
3	·38942	·27536
4	·33724	·23847
5	·30164	·21329
6	·27536	·19471
7	·25493	·18026
8	·23847	·16862
9	·22483	·15898
10	·21329	·15082
11	·20337	·14380
12	·19471	·13768
13	·18707	·13228
14	·18026	·12747
15	·17415	·12314
16	·16862	·11923
17	·16359	·11567
18	·15898	·11241
19	·15474	·10942
20	·15082	·10665
21	·14719	·10408
22	·14380	·10168
23	·14064	·09945
24	·13768	·09735
25	·13490	·09539
26	·13228	·09353
27	·12981	·09179
28	·12747	·09013
29	·12525	·08856
30	·12314	·08708
31	·12114	·08566
32	·11923	·08431
33	·11741	·08302
34	·11567	·08179
35	·11401	·08062
36	·11241	·07949
37	·11088	·07841
38	·10942	·07737
39	·10800	·07637
40	·10665	·07541
41	·10534	·07448
42	·10408	·07359
43	·10286	·07273
44	·10168	·07190
45	·10055	·07110
46	·09945	·07032
47	·09838	·06957
48	·09735	·06884
49	·09636	·06813
50	·09539	·06745
51	·09445	·06678
52	·09353	·06614
53	·09265	·06551
54	·09179	·06490
55	·09095	·06431
56	·09013	·06373
57	·08934	·06317
58	·08856	·06262
59	·08781	·06209
60	·08708	·06157
61	·08636	·06107
62	·08566	·06057
63	·08498	·06009
64	·08431	·05962
65	·08366	·05916
66	·08302	·05871
67	·08240	·05827
68	·08179	·05784
69	·08120	·05742
70	·08062	·05700
71	·08005	·05660
72	·07949	·05621
73	·07894	·05582
74	·07841	·05544
75	·07788	·05507
76	·07737	·05471
77	·07687	·05435
78	·07637	·05400
79	·07589	·05366
80	·07541	·05332
81	·07494	·05299
82	·07448	·05267
83	·07403	·05235
84	·07359	·05204
85	·07316	·05173
86	·07273	·05143
87	·07231	·05113
88	·07190	·05084
89	·07150	·05056
90	·07110	·05027
91	·07071	·05000
92	·07032	·04972
93	·06994	·04946
94	·06957	·04919
95	·06920	·04893
96	·06884	·04868
97	·06848	·04843
98	·06813	·04818
99	·06779	·04793
100	·06745	·04769
101	·06711	·04746
102	·06678	·04722
103	·06646	·04699
104	·06614	·04677
105	·06582	·04654
106	·06551	·04632
107	·06521	·04611
108	·06490	·04589
109	·06460	·04568
110	·06431	·04547
111	·06402	·04527
112	·06373	·04507
113	·06345	·04487
114	·06317	·04467
115	·06290	·04447
116	·06262	·04428
117	·06236	·04409
118	·06209	·04391
119	·06183	·04372
120	·06157	·04354
121	·06132	·04336
122	·06107	·04318
123	·06082	·04300
124	·06057	·04283
125	·06033	·04266
126	·06009	·04249
127	·05985	·04232
128	·05962	·04216
129	·05939	·04199
130	·05916	·04183
131	·05893	·04167
132	·05871	·04151
133	·05849	·04136
134	·05827	·04120
135	·05805	·04105
136	·05784	·04090
137	·05763	·04075
138	·05742	·04060
139	·05721	·04045
140	·05700	·04031
141	·05680	·04017
142	·05660	·04002
143	·05640	·03988
144	·05621	·03974
145	·05601	·03961
146	·05582	·03947
147	·05563	·03934
148	·05544	·03920
149	·05526	·03907
150	·05507	·03894

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2	n	χ_1	χ_2	n	χ_1	χ_2
151	·05489	·03881	201	·04757	·03364	251	·04257	·03010
152	·05471	·03868	202	·04746	·03356	252	·04249	·03004
153	·05453	·03856	203	·04734	·03347	253	·04240	·02998
154	·05435	·03843	204	·04722	·03339	254	·04232	·02993
155	·05418	·03831	205	·04711	·03331	255	·04224	·02987
156	·05400	·03819	206	·04699	·03323	256	·04216	·02981
157	·05383	·03806	207	·04688	·03315	257	·04207	·02975
158	·05366	·03794	208	·04677	·03307	258	·04199	·02969
159	·05349	·03782	209	·04666	·03299	259	·04191	·02964
160	·05332	·03771	210	·04654	·03291	260	·04183	·02958
161	·05316	·03759	211	·04643	·03283	261	·04175	·02952
162	·05299	·03747	212	·04632	·03276	262	·04167	·02947
163	·05283	·03736	213	·04622	·03268	263	·04159	·02941
164	·05267	·03724	214	·04611	·03260	264	·04151	·02935
165	·05251	·03713	215	·04600	·03253	265	·04143	·02930
166	·05235	·03702	216	·04589	·03245	266	·04136	·02924
167	·05219	·03691	217	·04579	·03238	267	·04128	·02919
168	·05204	·03680	218	·04568	·03230	268	·04120	·02913
169	·05188	·03669	219	·04558	·03223	269	·04112	·02908
170	·05173	·03658	220	·04547	·03216	270	·04105	·02903
171	·05158	·03647	221	·04537	·03208	271	·04097	·02897
172	·05143	·03637	222	·04527	·03201	272	·04090	·02892
173	·05128	·03626	223	·04517	·03194	273	·04082	·02887
174	·05113	·03616	224	·04507	·03187	274	·04075	·02881
175	·05099	·03605	225	·04497	·03180	275	·04067	·02876
176	·05084	·03595	226	·04487	·03173	276	·04060	·02871
177	·05070	·03585	227	·04477	·03166	277	·04053	·02866
178	·05056	·03575	228	·04467	·03159	278	·04045	·02860
179	·05041	·03565	229	·04457	·03152	279	·04038	·02855
180	·05027	·03555	230	·04447	·03145	280	·04031	·02850
181	·05013	·03545	231	·04438	·03138	281	·04024	·02845
182	·05000	·03535	232	·04428	·03131	282	·04017	·02840
183	·04986	·03526	233	·04419	·03125	283	·04009	·02835
184	·04972	·03516	234	·04409	·03118	284	·04002	·02830
185	·04959	·03507	235	·04400	·03111	285	·03995	·02825
186	·04946	·03497	236	·04391	·03105	286	·03988	·02820
187	·04932	·03488	237	·04381	·03098	287	·03981	·02815
188	·04919	·03478	238	·04372	·03092	288	·03974	·02810
189	·04906	·03469	239	·04363	·03085	289	·03968	·02806
190	·04893	·03460	240	·04354	·03079	290	·03961	·02801
191	·04880	·03451	241	·04345	·03172	291	·03954	·02796
192	·04868	·03442	242	·04336	·03066	292	·03947	·02791
193	·04855	·03433	243	·04327	·03060	293	·03940	·02786
194	·04843	·03424	244	·04318	·03053	294	·03934	·02782
195	·04830	·03415	245	·04309	·03047	295	·03927	·02777
196	·04818	·03407	246	·04300	·03041	296	·03920	·02772
197	·04806	·03398	247	·04292	·03035	297	·03913	·02767
198	·04793	·03389	248	·04283	·03029	298	·03907	·02763
199	·04781	·03381	249	·04274	·03022	299	·03901	·02758
200	·04769	·03372	250	·04266	·03016	300	·03894	·02754

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
301	·03888	·02749
302	·03881	·02744
303	·03875	·02740
304	·03868	·02735
305	·03862	·02731
306	·03856	·02726
307	·03850	·02722
308	·03843	·02718
309	·03837	·02713
310	·03831	·02709
311	·03825	·02704
312	·03819	·02700
313	·03812	·02696
314	·03806	·02692
315	·03800	·02687
316	·03794	·02683
317	·03788	·02679
318	·03782	·02675
319	·03776	·02670
320	·03771	·02666
321	·03765	·02662
322	·03759	·02658
323	·03753	·02654
324	·03747	·02650
325	·03741	·02646
326	·03736	·02642
327	·03730	·02637
328	·03724	·02633
329	·03719	·02629
330	·03713	·02625
331	·03707	·02621
332	·03702	·02618
333	·03696	·02614
334	·03691	·02610
335	·03685	·02606
336	·03680	·02602
337	·03674	·02598
338	·03669	·02594
339	·03663	·02590
340	·03658	·02587
341	·03653	·02583
342	·03647	·02579
343	·03642	·02575
344	·03637	·02571
345	·03631	·02568
346	·03626	·02564
347	·03621	·02560
348	·03616	·02557
349	·03610	·02553
350	·03605	·02549
351	·03600	·02546
352	·03595	·02542
353	·03590	·02538
354	·03585	·02535
355	·03580	·02531
356	·03575	·02528
357	·03570	·02524
358	·03565	·02521
359	·03560	·02517
360	·03555	·02514
361	·03550	·02510
362	·03545	·02507
363	·03540	·02503
364	·03535	·02500
365	·03530	·02496
366	·03526	·02493
367	·03521	·02490
368	·03516	·02486
369	·03511	·02483
370	·03507	·02479
371	·03502	·02476
372	·03497	·02473
373	·03492	·02469
374	·03488	·02466
375	·03483	·02463
376	·03478	·02460
377	·03474	·02456
378	·03469	·02453
379	·03465	·02450
380	·03460	·02447
381	·03456	·02443
382	·03451	·02440
383	·03446	·02437
384	·03442	·02434
385	·03438	·02431
386	·03433	·02428
387	·03429	·02424
388	·03424	·02421
389	·03420	·02418
390	·03415	·02415
391	·03411	·02412
392	·03407	·02409
393	·03402	·02406
394	·03398	·02403
395	·03394	·02400
396	·03389	·02397
397	·03385	·02394
398	·03381	·02391
399	·03377	·02388
400	·03372	·02385
401	·03368	·02382
402	·03364	·02379
403	·03360	·02376
404	·03356	·02373
405	·03352	·02370
406	·03347	·02367
407	·03343	·02364
408	·03339	·02361
409	·03335	·02358
410	·03331	·02355
411	·03327	·02353
412	·03323	·02350
413	·03319	·02347
414	·03315	·02344
415	·03311	·02341
416	·03307	·02338
417	·03303	·02336
418	·03299	·02333
419	·03295	·02330
420	·03291	·02327
421	·03287	·02324
422	·03283	·02322
423	·03279	·02319
424	·03276	·02316
425	·03272	·02313
426	·03268	·02311
427	·03264	·02308
428	·03260	·02305
429	·03256	·02303
430	·03253	·02300
431	·03249	·02297
432	·03245	·02295
433	·03241	·02292
434	·03238	·02289
435	·03234	·02287
436	·03230	·02284
437	·03227	·02281
438	·03223	·02279
439	·03219	·02276
440	·03216	·02274
441	·03212	·02271
442	·03208	·02269
443	·03205	·02266
444	·03201	·02263
445	·03197	·02261
446	·03194	·02258
447	·03190	·02256
448	·03187	·02253
449	·03183	·02251
450	·03180	·02248

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
451	·03176	·02246
452	·03173	·02243
453	·03169	·02241
454	·03166	·02238
455	·03162	·02236
456	·03159	·02233
457	·03155	·02231
458	·03152	·02229
459	·03148	·02226
460	·03145	·02224
461	·03141	·02221
462	·03138	·02219
463	·03135	·02217
464	·03131	·02214
465	·03128	·02212
466	·03125	·02209
467	·03121	·02207
468	·03118	·02205
469	·03115	·02202
470	·03111	·02200
471	·03108	·02198
472	·03105	·02195
473	·03101	·02193
474	·03098	·02191
475	·03095	·02188
476	·03092	·02186
477	·03088	·02184
478	·03085	·02181
479	·03082	·02179
480	·03079	·02177
481	·03075	·02175
482	·03072	·02172
483	·03069	·02170
484	·03066	·02168
485	·03063	·02166
486	·03060	·02163
487	·03056	·02161
488	·03053	·02159
489	·03050	·02157
490	·03047	·02155
491	·03044	·02152
492	·03041	·02150
493	·03038	·02148
494	·03035	·02146
495	·03032	·02144
496	·03029	·02142
497	·03026	·02139
498	·03022	·02137
499	·03019	·02135
500	·03016	·02133
501	·03013	·02131
502	·03010	·02129
503	·03007	·02127
504	·03004	·02124
505	·03001	·02122
506	·02998	·02120
507	·02996	·02118
508	·02993	·02116
509	·02990	·02114
510	·02987	·02112
511	·02984	·02110
512	·02981	·02108
513	·02978	·02106
514	·02975	·02104
515	·02972	·02102
516	·02969	·02100
517	·02966	·02098
518	·02964	·02096
519	·02961	·02094
520	·02958	·02092
521	·02955	·02089
522	·02952	·02087
523	·02949	·02085
524	·02947	·02084
525	·02944	·02082
526	·02941	·02080
527	·02938	·02078
528	·02935	·02076
529	·02933	·02074
530	·02930	·02072
531	·02927	·02070
532	·02924	·02068
533	·02922	·02066
534	·02919	·02064
535	·02916	·02062
536	·02913	·02060
537	·02911	·02058
538	·02908	·02056
539	·02905	·02054
540	·02903	·02052
541	·02900	·02051
542	·02897	·02049
543	·02895	·02047
544	·02892	·02045
545	·02889	·02043
546	·02887	·02041
547	·02884	·02039
548	·02881	·02037
549	·02879	·02036
550	·02876	·02034
551	·02873	·02032
552	·02871	·02030
553	·02868	·02028
554	·02866	·02026
555	·02863	·02024
556	·02860	·02023
557	·02858	·02021
558	·02855	·02019
559	·02853	·02017
560	·02850	·02015
561	·02848	·02014
562	·02845	·02012
563	·02843	·02010
564	·02840	·02008
565	·02838	·02006
566	·02835	·02005
567	·02833	·02003
568	·02830	·02001
569	·02828	·01999
570	·02825	·01998
571	·02823	·01996
572	·02820	·01994
573	·02818	·01992
574	·02815	·01991
575	·02813	·01990
576	·02810	·01987
577	·02808	·01986
578	·02806	·01984
579	·02803	·01982
580	·02801	·01980
581	·02798	·01978
582	·02796	·01977
583	·02793	·01975
584	·02791	·01974
585	·02789	·01972
586	·02786	·01970
587	·02784	·01969
588	·02782	·01967
589	·02779	·01965
590	·02777	·01964
591	·02774	·01962
592	·02772	·01960
593	·02770	·01959
594	·02767	·01957
595	·02765	·01955
596	·02763	·01954
597	·02761	·01952
598	·02758	·01950
599	·02756	·01949
600	·02754	·01947

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2
601	·02751	·01945
602	·02749	·01944
603	·02747	·01942
604	·02744	·01941
605	·02742	·01939
606	·02740	·01937
607	·02738	·01936
608	·02735	·01934
609	·02733	·01933
610	·02731	·01931
611	·02729	·01929
612	·02726	·01928
613	·02724	·01926
614	·02722	·01925
615	·02720	·01923
616	·02718	·01922
617	·02715	·01920
618	·02713	·01919
619	·02711	·01917
620	·02709	·01915
621	·02707	·01914
622	·02704	·01912
623	·02702	·01911
624	·02700	·01909
625	·02698	·01908
626	·02696	·01906
627	·02694	·01905
628	·02692	·01903
629	·02689	·01902
630	·02687	·01900
631	·02685	·01899
632	·02683	·01897
633	·02681	·01896
634	·02679	·01894
635	·02677	·01893
636	·02675	·01891
637	·02672	·01890
638	·02670	·01888
639	·02668	·01887
640	·02666	·01885
641	·02664	·01884
642	·02662	·01822
643	·02660	·01881
644	·02658	·01879
645	·02656	·01878
646	·02654	·01876
647	·02652	·01875
648	·02650	·01874
649	·02648	·01872
650	·02646	·01871

n	χ_1	χ_2
651	·02644	·01869
652	·02642	·01868
653	·02639	·01866
654	·02637	·01865
655	·02635	·01864
656	·02633	·01862
657	·02631	·01861
658	·02629	·01859
659	·02627	·01858
660	·02625	·01856
661	·02623	·01855
662	·02621	·01854
663	·02620	·01852
664	·02618	·01851
665	·02616	·01849
666	·02614	·01848
667	·02612	·01847
668	·02610	·01845
669	·02608	·01844
670	·02606	·01843
671	·02604	·01841
672	·02602	·01840
673	·02600	·01838
674	·02598	·01837
675	·02596	·01836
676	·02594	·01834
677	·02592	·01833
678	·02590	·01832
679	·02588	·01830
680	·02587	·01829
681	·02585	·01828
682	·02583	·01826
683	·02581	·01825
684	·02579	·01824
685	·02577	·01822
686	·02575	·01821
687	·02573	·01820
688	·02571	·01818
689	·02570	·01817
690	·02568	·01816
691	·02566	·01814
692	·02564	·01813
693	·02562	·01812
694	·02560	·01810
695	·02558	·01809
696	·02557	·01808
697	·02555	·01807
698	·02553	·01805
699	·02551	·01804
700	·02549	·01803

n	χ_1	χ_2
701	·02548	·01801
702	·02546	·01800
703	·02544	·01799
704	·02542	·01798
705	·02540	·01796
706	·02538	·01795
707	·02537	·01794
708	·02535	·01792
709	·02533	·01791
710	·02531	·01790
711	·02530	·01789
712	·02528	·01787
713	·02526	·01786
714	·02524	·01785
715	·02522	·01784
716	·02521	·01782
717	·02519	·01781
718	·02517	·01780
719	·02515	·01779
720	·02514	·01777
721	·02512	·01776
722	·02510	·01775
723	·02508	·01774
724	·02507	·01773
725	·02505	·01771
726	·02503	·01770
727	·02502	·01769
728	·02500	·01768
729	·02498	·01766
730	·02496	·01765
731	·02495	·01764
732	·02493	·01763
733	·02491	·01762
734	·02490	·01760
735	·02488	·01759
736	·02486	·01758
737	·02485	·01757
738	·02483	·01756
739	·02481	·01754
740	·02479	·01753
741	·02478	·01752
742	·02476	·01751
743	·02474	·01750
744	·02473	·01749
745	·02471	·01747
746	·02469	·01746
747	·02468	·01745
748	·02466	·01744
749	·02465	·01743
750	·02463	·01742

TABLE V. *Probable Errors of Means and Standard Deviations.*

n	χ_1	χ_2	n	χ_1	χ_2	n	χ_1	χ_2
751	·02461	·01740	*801*	·02383	·01685	*851*	·02312	·01635
752	·02460	·01739	*802*	·02382	·01684	*852*	·02311	·01634
753	·02458	·01738	*803*	·02380	·01683	*853*	·02309	·01633
754	·02456	·01737	*804*	·02379	·01682	*854*	·02308	·01632
755	·02455	·01736	*805*	·02377	·01681	*855*	·02307	·01631
756	·02453	·01735	*806*	·02376	·01680	*856*	·02305	·01630
757	·02451	·01733	*807*	·02374	·01679	*857*	·02304	·01629
758	·02450	·01732	*808*	·02373	·01678	*858*	·02303	·01628
759	·02448	·01731	*809*	·02371	·01677	*859*	·02301	·01627
760	·02447	·01730	*810*	·02370	·01676	*860*	·02300	·01626
761	·02445	·01729	*811*	·02368	·01675	*861*	·02299	·01625
762	·02443	·01728	*812*	·02367	·01674	*862*	·02297	·01624
763	·02442	·01727	*813*	·02366	·01673	*863*	·02296	·01624
764	·02440	·01725	*814*	·02364	·01672	*864*	·02295	·01623
765	·02439	·01724	*815*	·02363	·01671	*865*	·02293	·01622
766	·02437	·01723	*816*	·02361	·01670	*866*	·02292	·01621
767	·02435	·01722	*817*	·02360	·01669	*867*	·02291	·01620
768	·02434	·01721	*818*	·02358	·01668	*868*	·02289	·01619
769	·02432	·01720	*819*	·02357	·01667	*869*	·02288	·01618
770	·02431	·01719	*820*	·02355	·01666	*870*	·02287	·01617
771	·02429	·01718	*821*	·02354	·01665	*871*	·02285	·01616
772	·02428	·01717	*822*	·02353	·01664	*872*	·02284	·01615
773	·02426	·01715	*823*	·02351	·01662	*873*	·02283	·01614
774	·02424	·01714	*824*	·02350	·01661	*874*	·02281	·01613
775	·02423	·01713	*825*	·02348	·01660	*875*	·02280	·01612
776	·02421	·01712	*826*	·02347	·01659	*876*	·02279	·01611
777	·02420	·01711	*827*	·02345	·01658	*877*	·02278	·01610
778	·02418	·01710	*828*	·02344	·01657	*878*	·02276	·01610
779	·02417	·01709	*829*	·02343	·01656	*879*	·02275	·01609
780	·02415	·01708	*830*	·02341	·01655	*880*	·02274	·01608
781	·02414	·01707	*831*	·02340	·01654	*881*	·02272	·01607
782	·02412	·01706	*832*	·02338	·01653	*882*	·02271	·01606
783	·02410	·01704	*833*	·02337	·01652	*883*	·02270	·01605
784	·02409	·01703	*834*	·02336	·01651	*884*	·02269	·01604
785	·02407	·01702	*835*	·02334	·01651	*885*	·02267	·01603
786	·02406	·01701	*836*	·02333	·01650	*886*	·02266	·01602
787	·02404	·01700	*837*	·02331	·01649	*887*	·02265	·01601
788	·02403	·01699	*838*	·02330	·01648	*888*	·02263	·01600
789	·02401	·01698	*839*	·02329	·01647	*889*	·02262	·01600
790	·02400	·01697	*840*	·02327	·01646	*890*	·02261	·01599
791	·02398	·01696	*841*	·02326	·01645	*891*	·02260	·01598
792	·02397	·01695	*842*	·02324	·01644	*892*	·02258	·01597
793	·02395	·01694	*843*	·02323	·01643	*893*	·02257	·01596
794	·02394	·01693	*844*	·02322	·01642	*894*	·02256	·01595
795	·02392	·01692	*845*	·02320	·01641	*895*	·02255	·01594
796	·02391	·01690	*846*	·02319	·01640	*896*	·02253	·01593
797	·02389	·01689	*847*	·02318	·01639	*897*	·02252	·01592
798	·02388	·01688	*848*	·02316	·01638	*898*	·02251	·01592
799	·02386	·01687	*849*	·02315	·01637	*899*	·02250	·01591
800	·02385	·01686	*850*	·02313	·01636	*900*	·02248	·01590

TABLE V.

Probable Errors of Means and Standard Deviations.

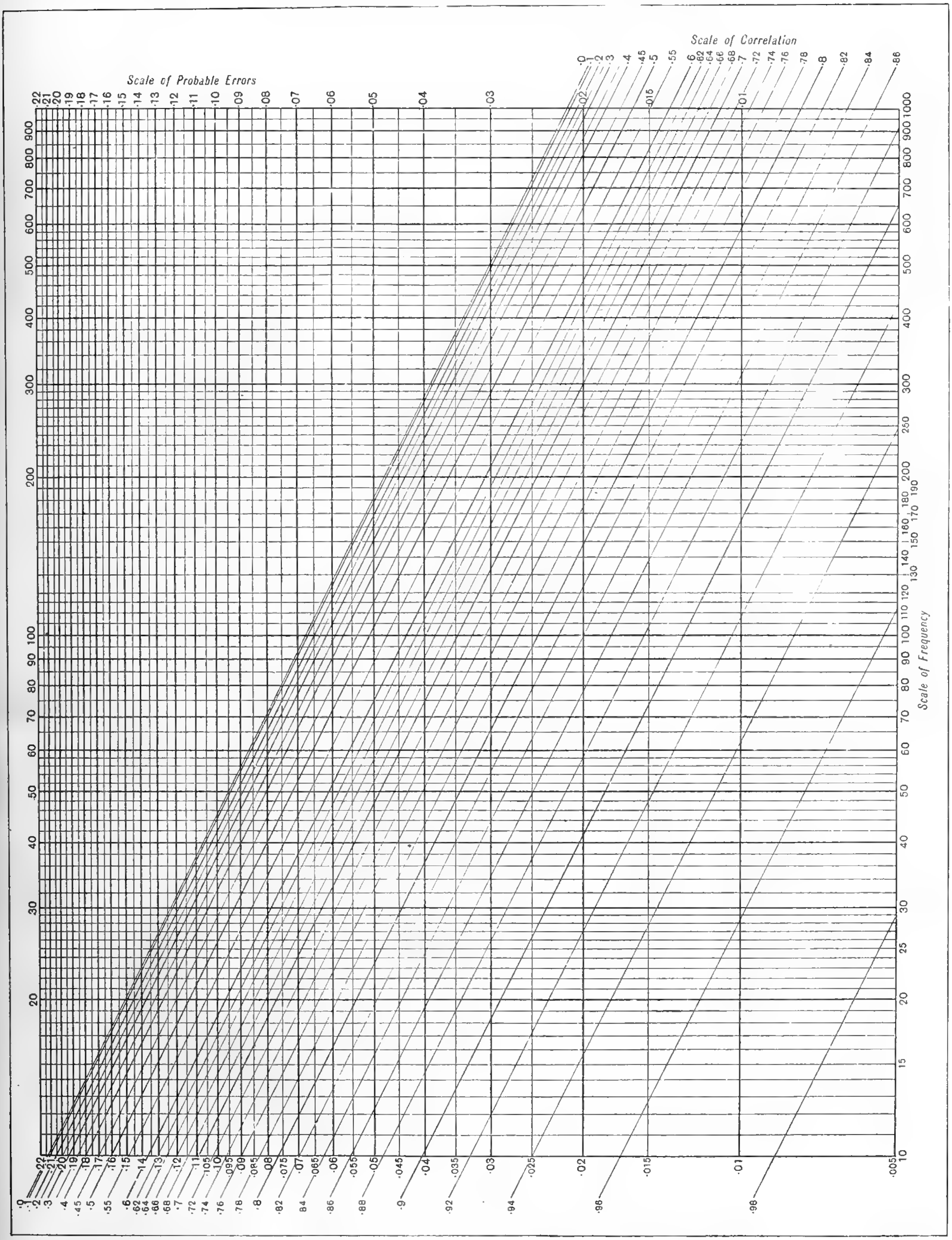
n	χ_1	χ_2	n	χ_1	χ_2
901	·02247	·01589	951	·02187	·01547
902	·02246	·01588	952	·02186	·01546
903	·02245	·01587	953	·02185	·01545
904	·02243	·01586	954	·02184	·01544
905	·02242	·01585	955	·02183	·01543
906	·02241	·01585	956	·02181	·01543
907	·02240	·01584	957	·02180	·01542
908	·02238	·01583	958	·02179	·01541
909	·02237	·01582	959	·02178	·01540
910	·02236	·01581	960	·02177	·01539
911	·02235	·01580	961	·02176	·01539
912	·02233	·01579	962	·02175	·01538
913	·02232	·01578	963	·02174	·01537
914	·02231	·01578	964	·02172	·01536
915	·02230	·01577	965	·02171	·01535
916	·02229	·01576	966	·02170	·01535
917	·02227	·01575	967	·02169	·01534
918	·02226	·01574	968	·02168	·01533
919	·02225	·01573	969	·02167	·01532
920	·02224	·01572	970	·02166	·01531
921	·02223	·01572	971	·02165	·01531
922	·02221	·01571	972	·02163	·01530
923	·02220	·01570	973	·02162	·01529
924	·02219	·01569	974	·02161	·01528
925	·02218	·01568	975	·02160	·01527
926	·02217	·01567	976	·02159	·01527
927	·02215	·01566	977	·02158	·01526
928	·02214	·01566	978	·02157	·01525
929	·02213	·01565	979	·02156	·01524
930	·02212	·01564	980	·02155	·01524
931	·02211	·01563	981	·02153	·01523
932	·02209	·01562	982	·02152	·01522
933	·02208	·01561	983	·02151	·01521
934	·02207	·01561	984	·02150	·01520
935	·02206	·01560	985	·02149	·01520
936	·02205	·01559	986	·02148	·01519
937	·02203	·01558	987	·02147	·01518
938	·02202	·01557	988	·02146	·01517
939	·02201	·01556	989	·02145	·01517
940	·02200	·01556	990	·02144	·01516
941	·02199	·01555	991	·02143	·01515
942	·02198	·01554	992	·02142	·01514
943	·02196	·01553	993	·02140	·01514
944	·02195	·01552	994	·02139	·01513
945	·02191	·01551	995	·02138	·01512
946	·02193	·01551	996	·02137	·01511
947	·02192	·01550	997	·02136	·01510
948	·02191	·01549	998	·02135	·01510
949	·02189	·01548	999	·02134	·01509
950	·02188	·01547	1000	·02133	·01508

TABLE VI.

Probable Errors of Coefficient of Variation.

V	ψ	Δ +	Δ^2 +	Δ^3 +
0	0·00000	1·00010	0	60
1	1·00010	1·00070	60	60
2	2·00080	1·00190	120	60
3	3·00270	1·00370	180	60
4	4·00639	1·00609	239	59
5	5·01248	1·00908	299	59
6	6·02156	1·01266	358	59
7	7·03422	1·01682	417	58
8	8·05104	1·02157	475	58
9	9·07261	1·02690	533	57
10	10·09950	1·03280	590	57
11	11·13230	1·03927	647	56
12	12·17157	1·04630	703	56
13	13·21787	1·05389	759	55
14	14·27176	1·06202	814	54
15	15·33379	1·07070	868	53
16	16·40449	1·07991	921	53
17	17·48440	1·08965	974	52
18	18·57405	1·09990	1025	51
19	19·67395	1·11066	1076	50
20	20·78461	1·12192	1126	49
21	21·90653	1·13368	1175	48
22	23·04021	1·14591	1223	47
23	24·18612	1·15861	1270	46
24	25·34473	1·17177	1316	45
25	26·51650	1·18539	1362	44
26	27·70190	1·19945	1406	43
27	28·90135	1·21395	1449	42
28	30·11530	1·22886	1491	41
29	31·34416	1·24418	1533	40
30	32·58834	1·25991	1573	39
31	33·84825	1·27603	1612	38
32	35·12428	1·29253	1650	37
33	36·41681	1·30940	1687	36
34	37·72621	1·32664	1723	35
35	39·05285	1·34422	1758	34
36	40·39707	1·36215	1793	33
37	41·75922	1·38041	1826	32
38	43·13962	1·39899	1858	31
39	44·53861	1·41789	1890	30
40	45·95650	1·43709	1920	30
41	47·39359	1·45658	1950	29
42	48·85017	1·47636	1978	28
43	50·32654	1·49642	2006	27
44	51·82296	1·51675	2033	26
45	53·33971	1·53734	2059	25
46	54·87706	1·55818	2084	24
47	56·43524	1·57927	2109	24
48	58·01451	1·60059	2132	23
49	59·61510	1·62214	2155	22
50	61·23724		2177	

TABLE VII. *Abac for Probable Errors of r.*



Abac for determining the Probable Errors of Correlation Coefficients.

TABLE VIII. *Values of* $1-r^2$ *for* $r=$ ·001 *to* ·999.

Values of $1-r^2$.

r	*·000*	*·001*	*·002*	*·003*	*·004*	*·005*	*·006*	*·007*	*·008*	*·009*
·000	1·000 000	·999 999	·999 996	·999 991	·999 984	·999 975	·999 964	·999 951	·999 936	·999 919
·010	·999 900	·999 879	·999 856	·999 831	·999 804	·999 775	·999 744	·999 711	·999 676	·999 639
·020	·999 600	·999 559	·999 516	·999 471	·999 424	·999 375	·999 324	·999 271	·999 216	·999 159
·030	·999 100	·999 039	·998 976	·998 911	·998 844	·998 775	·998 704	·998 631	·998 556	·998 479
·040	·998 400	·998 319	·998 236	·998 151	·998 064	·997 975	·997 884	·997 791	·997 696	·997 599
·050	·997 500	·997 399	·997 296	·997 191	·997 084	·996 975	·996 864	·996 751	·996 636	·996 519
·060	·996 400	·996 279	·996 156	·996 031	·995 904	·995 775	·995 644	·995 511	·995 376	·995 239
·070	·995 100	·994 959	·994 816	·994 671	·994 524	·994 375	·994 224	·994 071	·993 916	·993 759
·080	·993 600	·993 439	·993 276	·993 111	·992 944	·992 775	·992 604	·992 431	·992 256	·992 079
·090	·991 900	·991 719	·991 536	·991 351	·991 164	·990 975	·990 784	·990 591	·990 396	·990 199
·100	·990 000	·989 799	·989 596	·989 391	·989 184	·988 975	·988 764	·988 551	·988 336	·988 119
·110	·987 900	·987 679	·987 456	·987 231	·987 004	·986 775	·986 544	·986 311	·986 076	·985 839
·120	·985 600	·985 359	·985 116	·984 871	·984 624	·984 375	·984 124	·983 871	·983 616	·983 359
·130	·983 100	·982 839	·982 576	·982 311	·982 044	·981 775	·981 504	·981 231	·980 956	·980 679
·140	·980 400	·980 119	·979 836	·979 551	·979 264	·978 975	·978 684	·978 391	·978 096	·977 799
·150	·977 500	·977 199	·976 896	·976 591	·976 284	·975 975	·975 664	·975 351	·975 036	·974 719
·160	·974 400	·974 079	·973 756	·973 431	·973 104	·972 775	·972 444	·972 111	·971 776	·971 439
·170	·971 100	·970 759	·970 416	·970 071	·969 724	·969 375	·969 024	·968 671	·968 316	·967 959
·180	·967 600	·967 239	·966 876	·966 511	·966 144	·965 775	·965 404	·965 031	·964 656	·964 279
·190	·963 900	·963 519	·963 136	·962 751	·962 364	·961 975	·961 584	·961 191	·960 796	·960 399
·200	·960 000	·959 599	·959 196	·958 791	·958 384	·957 975	·957 564	·957 151	·956 736	·956 319
·210	·955 900	·955 479	·955 056	·954 631	·954 204	·953 775	·953 344	·952 911	·952 476	·952 039
·220	·951 600	·951 159	·950 716	·950 271	·949 824	·949 375	·948 924	·948 471	·948 016	·947 559
·230	·947 100	·946 639	·946 176	·945 711	·945 244	·944 775	·944 304	·943 831	·943 356	·942 879
·240	·942 400	·941 919	·941 436	·940 951	·940 464	·939 975	·939 484	·938 991	·938 496	·937 999
·250	·937 500	·936 999	·936 496	·935 991	·935 484	·934 975	·934 464	·933 951	·933 436	·932 919
·260	·932 400	·931 879	·931 356	·930 831	·930 304	·929 775	·929 244	·928 711	·928 176	·927 639
·270	·927 100	·926 559	·926 016	·925 471	·924 924	·924 375	·923 824	·923 271	·922 716	·922 159
·280	·921 600	·921 039	·920 476	·919 911	·919 344	·918 775	·918 204	·917 631	·917 056	·916 479
·290	·915 900	·915 319	·914 736	·914 151	·913 564	·912 975	·912 384	·911 791	·911 196	·910 599
·300	·910 000	·909 399	·908 796	·908 191	·907 584	·906 975	·906 364	·905 751	·905 136	·904 519
·310	·903 900	·903 279	·902 656	·902 031	·901 404	·900 775	·900 144	·899 511	·898 876	·898 239
·320	·897 600	·896 959	·896 316	·895 671	·895 024	·894 375	·893 724	·893 071	·892 416	·891 759
·330	·891 100	·890 439	·889 776	·889 111	·888 444	·887 775	·887 104	·886 431	·885 756	·885 079
·340	·884 400	·883 719	·883 036	·882 351	·881 664	·880 975	·880 284	·879 591	·878 896	·878 199
·350	·877 500	·876 799	·876 096	·875 391	·874 684	·873 975	·873 264	·872 551	·871 836	·871 119
·360	·870 400	·869 679	·868 956	·868 231	·867 504	·866 775	·866 044	·865 311	·864 576	·863 839
·370	·863 100	·862 359	·861 616	·860 871	·860 124	·859 375	·858 624	·857 871	·857 116	·856 359
·380	·855 600	·854 839	·854 076	·853 311	·852 544	·851 775	·851 004	·850 231	·849 456	·848 679
·390	·847 900	·847 119	·846 336	·845 551	·844 764	·843 975	·843 184	·842 391	·841 596	·840 799
·400	·840 000	·839 199	·838 396	·837 591	·836 784	·835 975	·835 164	·834 351	·833 536	·832 719
·410	·831 900	·831 079	·830 256	·829 431	·828 604	·827 775	·826 944	·826 111	·825 276	·824 439
·420	·823 600	·822 759	·821 916	·821 071	·820 224	·819 375	·818 524	·817 671	·816 816	·815 959
·430	·815 100	·814 239	·813 376	·812 511	·811 644	·810 775	·809 904	·809 031	·808 156	·807 279
·440	·806 400	·805 519	·804 636	·803 751	·802 864	·801 975	·801 084	·800 191	·799 296	·798 399
·450	·797 500	·796 599	·795 696	·794 791	·793 884	·792 975	·792 064	·791 151	·790 236	·789 319
·460	·788 400	·787 479	·786 556	·785 631	·784 704	·783 775	·782 844	·781 911	·780 976	·780 039
·470	·779 100	·778 159	·777 216	·776 271	·775 324	·774 375	·773 424	·772 471	·771 516	·770 559
·480	·769 600	·768 639	·767 676	·766 711	·765 744	·764 775	·763 804	·762 831	·761 856	·760 879
·490	·759 900	·758 919	·757 936	·756 951	·755 964	·754 975	·753 984	·752 991	·751 996	·750 999

TABLE VIII. *Values of* $1-r^2$ *for* $r=\cdot001$ *to* $\cdot999$.

Values of $1-r^2$.

r	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009
·500	·750 000	·748 999	·747 996	·746 991	·745 984	·744 975	·743 964	·742 951	·741 936	·740 919
·510	·739 900	·738 879	·737 856	·736 831	·735 804	·734 775	·733 744	·732 711	·731 676	·730 639
·520	·729 600	·728 559	·727 516	·726 471	·725 424	·724 375	·723 324	·722 271	·721 216	·720 159
·530	·719 100	·718 039	·716 976	·715 911	·714 844	·713 775	·712 704	·711 631	·710 556	·709 479
·540	·708 400	·707 319	·706 236	·705 151	·704 064	·702 975	·701 884	·700 791	·699 696	·698 599
·550	·697 500	·696 399	·695 296	·694 191	·693 084	·691 975	·690 864	·689 751	·688 636	·687 519
·560	·686 400	·685 279	·684 156	·683 031	·681 904	·680 775	·679 644	·678 511	·677 376	·676 239
·570	·675 100	·673 959	·672 816	·671 671	·670 524	·669 375	·668 224	·667 071	·665 916	·664 759
·580	·663 600	·662 439	·661 276	·660 111	·658 944	·657 775	·656 604	·655 431	·654 256	·653 079
·590	·651 900	·650 719	·649 536	·648 351	·647 164	·645 975	·644 784	·643 591	·642 396	·641 199
·600	·640 000	·638 799	·637 596	·636 391	·635 184	·633 975	·632 764	·631 551	·630 336	·629 119
·610	·627 900	·626 679	·625 456	·624 231	·623 004	·621 775	·620 544	·619 311	·618 076	·616 839
·620	·615 600	·614 359	·613 116	·611 871	·610 624	·609 375	·608 124	·606 871	·605 616	·604 359
·630	·603 100	·601 839	·600 576	·599 311	·598 044	·596 775	·595 504	·594 231	·592 956	·591 679
·640	·590 400	·589 119	·587 836	·586 551	·585 264	·583 975	·582 684	·581 391	·580 096	·578 799
·650	·577 500	·576 199	·574 896	·573 591	·572 284	·570 975	·569 664	·568 351	·567 036	·565 719
·660	·564 400	·563 079	·561 756	·560 431	·559 104	·557 775	·556 444	·555 111	·553 776	·552 439
·670	·551 100	·549 759	·548 416	·547 071	·545 724	·544 375	·543 024	·541 671	·540 316	·538 959
·680	·537 600	·536 239	·534 876	·533 511	·532 144	·530 775	·529 404	·528 031	·526 656	·525 279
·690	·523 900	·522 519	·521 136	·519 751	·518 364	·516 975	·515 584	·514 191	·512 796	·511 399
·700	·510 000	·508 599	·507 196	·505 791	·504 384	·502 975	·501 564	·500 151	·498 736	·497 319
·710	·495 900	·494 479	·493 056	·491 631	·490 204	·488 775	·487 344	·485 911	·484 476	·483 039
·720	·481 600	·480 159	·478 716	·477 271	·475 824	·474 375	·472 924	·471 471	·470 016	·468 559
·730	·467 100	·465 639	·464 176	·462 711	·461 244	·459 775	·458 304	·456 831	·455 356	·453 879
·740	·452 400	·450 919	·449 436	·447 951	·446 464	·444 975	·443 484	·441 991	·440 496	·438 999
·750	·437 500	·435 999	·434 496	·432 991	·431 484	·429 975	·428 464	·426 951	·425 436	·423 919
·760	·422 400	·420 879	·419 356	·417 831	·416 304	·414 775	·413 244	·411 711	·410 176	·408 639
·770	·407 100	·405 559	·404 016	·402 471	·400 924	·399 375	·397 824	·396 271	·394 716	·393 159
·780	·391 600	·390 039	·388 476	·386 911	·385 344	·383 775	·382 204	·380 631	·379 056	·377 479
·790	·375 900	·374 319	·372 736	·371 151	·369 564	·367 975	·366 384	·364 791	·363 196	·361 599
·800	·360 000	·358 399	·356 796	·355 191	·353 584	·351 975	·350 364	·348 751	·347 136	·345 519
·810	·343 900	·342 279	·340 656	·339 031	·337 404	·335 775	·334 144	·332 511	·330 876	·329 239
·820	·327 600	·325 959	·324 316	·322 671	·321 024	·319 375	·317 724	·316 071	·314 416	·312 759
·830	·311 100	·309 439	·307 776	·306 111	·304 444	·302 775	·301 104	·299 431	·297 756	·296 079
·840	·294 400	·292 719	·291 036	·289 351	·287 664	·285 975	·284 284	·282 591	·280 896	·279 199
·850	·277 500	·275 799	·274 096	·272 391	·270 684	·268 975	·267 264	·265 551	·263 836	·262 119
·860	·260 400	·258 679	·256 956	·255 231	·253 504	·251 775	·250 044	·248 311	·246 576	·244 839
·870	·243 100	·241 359	·239 616	·237 871	·236 124	·234 375	·232 624	·230 871	·229 116	·227 359
·880	·225 600	·223 839	·222 076	·220 311	·218 544	·216 775	·215 004	·213 231	·211 456	·209 679
·890	·207 900	·206 119	·204 336	·202 551	·200 764	·198 975	·197 184	·195 391	·193 596	·191 799
·900	·190 000	·188 199	·186 396	·184 591	·182 784	·180 975	·179 164	·177 351	·175 536	·173 719
·910	·171 900	·170 079	·168 256	·166 431	·164 604	·162 775	·160 944	·159 111	·157 276	·155 439
·920	·153 600	·151 759	·149 916	·148 071	·146 224	·144 375	·142 524	·140 671	·138 816	·136 959
·930	·135 100	·133 239	·131 376	·129 511	·127 644	·125 775	·123 904	·122 031	·120 156	·118 279
·940	·116 400	·114 519	·112 636	·110 751	·108 864	·106 975	·105 084	·103 191	·101 296	·099 399
·950	·097 500	·095 599	·093 696	·091 791	·089 884	·087 975	·086 064	·084 151	·082 236	·080 319
·960	·078 400	·076 479	·074 556	·072 631	·070 704	·068 775	·066 844	·064 911	·062 976	·061 039
·970	·059 100	·057 159	·055 216	·053 271	·051 324	·049 375	·047 424	·045 471	·043 516	·041 559
·980	·039 600	·037 639	·035 676	·033 711	·031 744	·029 775	·027 804	·025 831	·023 856	·021 879
·990	019 900	·017 919	·015 936	·013 951	·011 964	·009 975	·007 984	·005 991	·003 996	·001 999

TABLE IX. *Values of the Incomplete Normal Moment Function* $\mu_n(x)$.

A. *Odd Moments* $m_n(x) = \mu_n(x)/\{(n-1)(n-3)(n-5)\dots 2\}$.

x	$m_1(x)$	$m_3(x)$	$m_5(x)$	$m_7(x)$	$m_9(x)$
0·0	·0000000	·0000000	·0000000	·0000000	·0000000
0·1	·0019897	·0000050	·0000000	·0000000	·0000000
0·2	·0078996	·0000787	·0000005	·0000000	·0000000
0·3	·0175545	·0003920	·0000059	·0000001	·0000000
0·4	·0306721	·0012105	·0000321	·0000006	·0000000
0·5	·0468770	·0028688	·0001183	·0000037	·0000001
0·6	·0657177	·0057372	·0003390	·0000151	·0000005
0·7	·0866883	·0101861	·0008146	·0000493	·0000024
0·8	·1092507	·0165494	·0017172	·0001350	·0000086
0·9	·1328570	·0250925	·0032702	·0003242	·0000259
1·0	·1569716	·0359862	·0057399	·0006988	·0000687
1·1	·1810901	·0492895	·0094199	·0013795	·0001634
1·2	·2047562	·0649423	·0146092	·0025293	·0003549
1·3	·2275737	·0827672	·0215865	·0043539	·0007135
1·4	·2492148	·1024819	·0305828	·0070957	·0013414
1·5	·2694247	·1237174	·0417570	·0110219	·0023776
1·6	·2880214	·1460428	·0551764	·0164068	·0040005
1·7	·3048932	·1689923	·0708039	·0235098	·0064248
1·8	·3199921	·1920929	·0884945	·0325513	·0098944
1·9	·3333265	·2148899	·1080009	·0436894	·0146688
2·0	·3449513	·2369694	·1289874	·0569995	·0210055
2·1	·3549587	·2579749	·1510502	·0724606	·0291380
2·2	·3634677	·2776192	·1737425	·0899486	·0392533
2·3	·3706152	·2956902	·1966019	·1092390	·0514703
2·4	·3765478	·3120515	·2191769	·1300173	·0658224
2·5	·3814140	·3266380	·2410506	·1518971	·0822459
2·6	·3853593	·3394489	·2618602	·1744437	·1005767
2·7	·3885213	·3505370	·2813106	·1972006	·1205553
2·8	·3910268	·3599983	·2991823	·2197160	·1418391
2·9	·3929897	·3679593	·3153329	·2415682	·1640231
3·0	·3945104	·3745671	·3296946	·2623860	·1866637
3·1	·3956755	·3799784	·3422662	·2818638	·2093055
3·2	·3965582	·3843517	·3531029	·2997718	·2315079
3·3	·3972197	·3878403	·3623049	·3159582	·2528687
3·4	·3977101	·3905878	·3700046	·3303476	·2730432
3·5	·3980696	·3927244	·3763548	·3429335	·2917571
3·6	·3983304	·3943653	·3815183	·3537687	·3088145
3·7	·3985175	·3956099	·3856585	·3629529	·3240979
3·8	·3986503	·3965425	·3889331	·3706199	·3375646
3·9	·3987436	·3972329	·3914881	·3769253	·3492376
4·0	·3988085	·3977378	·3934552	·3820351	·3591947
4·1	·3988530	·3981028	·3949499	·3861165	·3675554
4·2	·3988833	·3983635	·3960708	·3893304	·3744677
4·3	·3989037	·3985475	·3969007	·3918258	·3800964
4·4	·3989173	·3986759	·3975073	·3937367	·3846117
4·5	·3989263	·3987645	·3979452	·3951801	·3881809
4·6	·3989321	·3988248	·3982573	·3962557	·3909614
4·7	·3989359	·3988656	·3984770	·3970466	·3930967
4·8	·3989383	·3988927	·3986298	·3976205	·3947135
4·9	·3989398	·3989106	·3987348	·3980315	·3959207
5·0	·3989408	·3989222	·3988061	·3983221	·3968097
∞	·3989423	·3989423	·3989423	·3989423	·3989423

TABLE IX. *Values of the Incomplete Normal Moment Function.*

B. *Even Moments* $m_n(x) = \mu_n(x)/\{(n-1)(n-3)(n-5)\ldots 1\}$.

x	$m_2(x)$	$m_4(x)$	$m_6(x)$	$m_8(x)$	$m_{10}(x)$
0·0	·0000000	·0000000	·0000000	·0000000	·0000000
0·1	·0001325	·0000002	·0000000	·0000000	·0000000
0·2	·0010512	·0000084	·0000000	·0000000	·0000000
0·3	·0034951	·0000626	·0000008	·0000000	·0000000
0·4	·0081136	·0002572	·0000058	·0000001	·0000000
0·5	·0154298	·0007604	·0000270	·0000008	·0000001
0·6	·0258121	·0018200	·0000925	·0000037	·0000001
0·7	·0394585	·0037575	·0002588	·0000139	·0000006
0·8	·0563914	·0069507	·0006223	·0000437	·0000025
0·9	·0764632	·0118045	·0013297	·0001177	·0000086
1·0	·0993740	·0187171	·0025857	·0002812	·0000251
1·1	·1246965	·0280428	·0046525	·0006094	·0000658
1·2	·1519070	·0400559	·0078427	·0012160	·0001558
1·3	·1804203	·0549214	·0125028	·0022617	·0003386
1·4	·2096248	·0726741	·0189894	·0039577	·0006842
1·5	·2389164	·0932091	·0276408	·0065653	·0012964
1·6	·2677274	·1162835	·0387442	·0103869	·0023209
1·7	·2955511	·1415300	·0525059	·0157516	·0039494
1·8	·3219594	·1684803	·0690258	·0229926	·0064207
1·9	·3466134	·1965937	·0882796	·0324204	·0100147
2·0	·3692680	·2252921	·1101113	·0442938	·0150415
2·1	·3897700	·2539927	·1342371	·0587910	·0218224
2·2	·4080525	·2821413	·1602593	·0759866	·0306667
2·3	·4241237	·3092387	·1876903	·0958345	·0418437
2·4	·4380556	·3348616	·2159821	·1181613	·0555560
2·5	·4499695	·3586763	·2445598	·1426700	·0719132
2·6	·4600231	·3804450	·2728554	·1689546	·0909136
2·7	·4683965	·4000247	·3003387	·1965228	·1124320
2·8	·4752816	·4173616	·3265431	·2248263	·1362197
2·9	·4808719	·4324798	·3510842	·2532933	·1619132
3·0	·4853546	·4454679	·3736720	·2813629	·1890538
3·1	·4889053	·4564647	·3941138	·3085150	·2171145
3·2	·4916838	·4656432	·4123121	·3342962	·2455315
3·3	·4938321	·4731975	·4282552	·3583379	·2737379
3·4	·4954736	·4793298	·4420056	·3803672	·3011962
3·5	·4967130	·4842409	·4536843	·4002102	·3274261
3·6	·4976381	·4881218	·4634555	·4177877	·3520261
3·7	·4983205	·4911484	·4715111	·4331061	·3746880
3·8	·4988183	·4934784	·4780568	·4462441	·3952025
3·9	·4991771	·4952491	·4833001	·4573366	·4134583
4·0	·4994330	·4965779	·4874418	·4665592	·4294345
4·1	·4996133	·4975627	·4906683	·4741120	·4431886
4·2	·4997391	·4982835	·4931479	·4802063	·4548407
4·3	·4998258	·4988045	·4950279	·4850521	·4645574
4·4	·4998849	·4991766	·4964343	·4888500	·4725352
4·5	·4999247	·4994392	·4974729	·4917846	·4789861
4·6	·4999512	·4996222	·4982298	·4940207	·4841246
4·7	·4999688	·4997483	·4987744	·4957010	·4881574
4·8	·4999802	·4998342	·4991613	·4969464	·4912765
4·9	·4999876	·4998919	·4994326	·4978572	·4936544
5·0	·4999923	·4999303	·4996206	·4985144	·4954417
∞	·5000000	·5000000	·5000000	·5000000	·5000000

TABLE X. *Diagram of Generalised 'Probable Error.'*

Table of Generalised 'Probable Errors.'

Number of Variables	Probable Error
1	0·674,4898
2	1·177,4062
3	1·538,1667
4	1·832,1239
5	2·086,0146
6	2·312,5982
7	2·519,0869
8	2·710,0022
9	2·888,3962
10	3·056,4366
11	3·215,7402

Diagram for Value of Probable Error for *n* Variables.

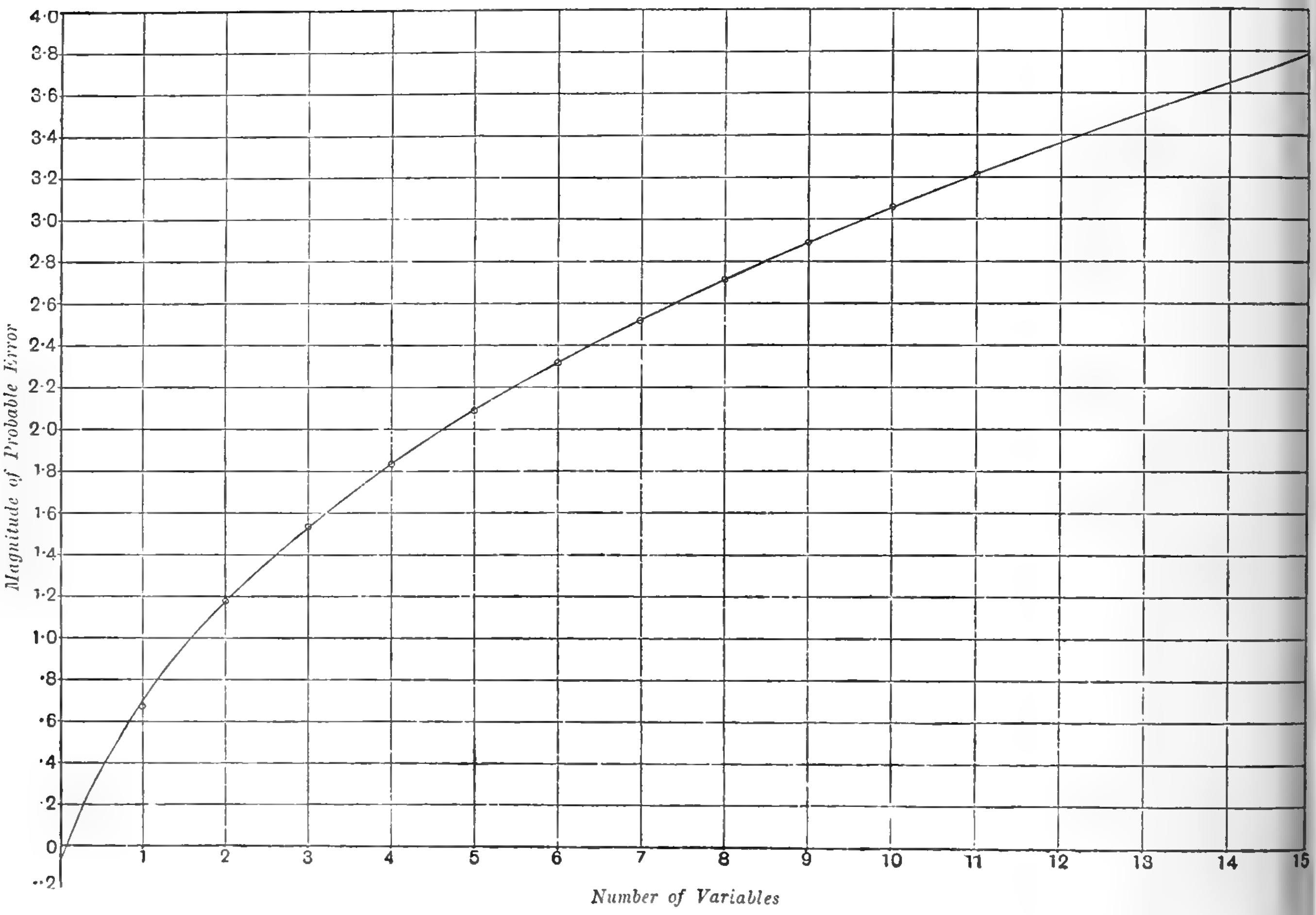


TABLE XI. *Constants of Normal Curve from Moments of Tail about Stump.*

Values of the Functions ψ_1 and ψ_2 required to determine the Constants of a Normal Frequency Distribution from the Moments of its Truncated Tail.

h'	ψ_1	ψ_2	ψ_3	h'	ψ_1	ψ_2	ψ_3
0·00	·571	1·253	2·000	1·1	·734	1·977	7·371
0·01	·573	1·259	2·016	1·2	·746	2·051	8·690
0·02	·574	1·265	2·032	1·3	·757	2·126	10·331
0·03	·576	1·271	2·049	1·4	·767	2·202	12·383
0·04	·578	1·276	2·066	1·5	·777	2·280	14·968
0·05	·580	1·282	2·083	1·6	·787	2·358	18·248
0·06	·581	1·288	2·100	1·7	·796	2·437	22·439
0·07	·583	1·294	2·118	1·8	·804	2·517	27·832
0·08	·585	1·300	2·136	1·9	·813	2·598	34·823
0·09	·587	1·305	2·155	2·0	·820	2·679	43·956
0·1	·588	1·311	2·173	2·1	·828	2·762	55·977
0·2	·605	1·371	2·377	2·2	·835	2·845	71·925
0·3	·622	1·432	2·617	2·3	·842	2·929	93·248
0·4	·638	1·495	2·902	2·4	·848	3·013	121·988
0·5	·653	1·560	3·241	2·5	·854	3·098	161·038
0·6	·668	1·626	3·646	2·6	·860	3·184	214·537
0·7	·682	1·693	4·133	2·7	·866	3·270	288·434
0·8	·696	1·762	4·720	2·8	·871	3·357	391·374
0·9	·709	1·833	5·433	2·9	·876	3·445	535·963
1·0	·722	1·904	6·303	3·0	·880	3·532	740·796
1·1	·734	1·977	7·371	3·5	·888	3·969	[4299·226]

Let d equal distance of centroid of tail from stump, Σ = standard deviation of the tail about its mean, and n = its area.

(i) Find ψ_1 from $\psi_1 = \Sigma^2/d^2$. Hence from table determine h'.

(ii) From this value of h' find ψ_2, then $\sigma = d \times \psi_2$ gives the standard deviation of the uncurtailed normal curve.

(iii) $h = h' \times \sigma$ gives the origin of the uncurtailed normal curve.

(iv) Knowing h', Table II gives $\frac{1}{2}(1+\alpha)$ and therefore the ratio $\frac{1}{2}(1-\alpha)$ of tail to total area of curve N, or $N = n/\frac{1}{2}(1-\alpha)$. For many purposes it is sufficient to use $N = n \times \psi_3$.

TABLE XII. *Test for Goodness of Fit. Values of P.*

χ^2	$n'=3$	$n'=4$	$n'=5$	$n'=6$	$n'=7$	$n'=8$	$n'=9$	$n'=10$	$n'=11$
1	·606531	·801253	·909796	·962566	·985612	·994829	·998249	·999438	·999828
2	·367879	·572407	·735759	·849146	·919699	·959840	·981012	·991468	·996340
3	·223130	·391625	·557825	·699986	·808847	·885002	·934357	·964295	·981424
4	·135335	·261464	·406006	·549416	·676676	·779778	·857123	·911413	·947347
5	·082085	·171797	·287298	·415880	·543813	·659963	·757576	·834308	·891178
6	·049787	·111610	·199148	·306219	·423190	·539750	·647232	·739919	·815263
7	·030197	·071897	·135888	·220640	·320847	·428880	·536632	·637119	·725444
8	·018316	·046012	·091578	·156236	·238103	·332594	·433470	·534146	·628837
9	·011109	·029291	·061099	·109064	·173578	·252656	·342296	·437274	·532104
10	·006738	·018566	·040428	·075235	·124652	·188573	·265026	·350485	·440493
11	·004087	·011726	·026564	·051380	·088376	·138619	·201699	·275709	·357518
12	·002479	·007383	·017351	·034787	·061969	·100558	·151204	·213308	·285057
13	·001503	·004637	·011276	·023379	·043036	·072109	·111850	·162607	·223672
14	·000912	·002905	·007295	·015609	·029636	·051181	·081765	·122325	·172992
15	·000553	·001817	·004701	·010363	·020256	·036000	·059145	·090937	·132061
16	·000335	·001134	·003019	·006844	·013754	·025116	·042380	·066881	·099632
17	·000203	·000707	·001933	·004500	·009283	·017396	·030109	·048716	·074364
18	·000123	·000440	·001234	·002947	·006232	·011970	·021226	·035174	·054964
19	·000075	·000273	·000786	·001922	·004164	·008187	·014860	·025193	·040263
20	·000045	·000170	·000499	·001250	·002769	·005570	·010336	·017913	·029253
21	·000028	·000105	·000317	·000810	·001835	·003770	·007147	·012650	·021093
22	·000017	·000065	·000200	·000524	·001211	·002541	·004916	·008880	·015105
23	·000010	·000040	·000127	·000338	·000796	·001705	·003364	·006197	·010747
24	·000006	·000025	·000080	·000217	·000522	·001139	·002292	·004301	·007600
25	·000004	·000016	·000050	·000139	·000341	·000759	·001554	·002971	·005345
26	·000002	·000010	·000032	·000090	·000223	·000504	·001050	·002043	·003740
27	·000001	·000006	·000020	·000057	·000145	·000333	·000707	·001399	·002604
28	·000001	·000004	·000012	·000037	·000094	·000220	·000474	·000954	·001805
29	·000001	·000002	·000008	·000023	·000061	·000145	·000317	·000648	·001246
30	·000000	·000001	·000005	·000015	·000039	·000095	·000211	·000439	·000857
40	·000000	·000000	·000000	·000000	·000001	·000001	·000003	·000008	·000017
50	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000
60	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000
70	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000

TABLE XII.—(*continued*).

χ^2	$n'=12$	$n'=13$	$n'=14$	$n'=15$	$n'=16$	$n'=17$	$n'=18$	$n'=19$	$n'=20$
1	·999950	·999986	·999997	·999999	1·	1·	1·	1·	1·
2	·998496	·999406	·999774	·999917	·999970	·999990	·999997	·999999	1·
3	·990726	·995544	·997934	·999074	·999598	·999830	·999931	·999972	·999989
4	·969917	·983436	·991191	·995466	·997737	·998903	·999483	·999763	·999894
5	·931167	·957979	·975193	·985813	·992127	·995754	·997771	·998860	·999431
6	·873365	·916082	·946153	·966491	·979749	·988095	·993187	·996197	·997929
7	·799073	·857613	·902151	·934711	·957650	·973260	·983549	·990125	·994213
8	·713304	·785131	·843601	·889327	·923783	·948867	·966547	·978637	·986671
9	·621892	·702931	·772943	·831051	·877517	·913414	·940261	·959743	·973479
10	·530387	·615960	·693934	·762183	·819739	·866628	·903610	·931906	·952946
11	·443263	·528919	·610817	·686036	·752594	·809485	·856564	·894357	·923839
12	·362642	·445680	·527643	·606303	·679028	·743980	·800136	·847237	·885624
13	·293326	·369041	·447812	·526524	·602298	·672758	·736186	·791573	·838571
14	·232993	·300708	·373844	·449711	·525529	·598714	·667102	·729091	·783691
15	·182498	·241436	·307354	·378154	·451418	·524638	·595482	·661967	·722598
16	·141130	·191236	·249129	·313374	·382051	·452961	·523834	·592547	·657277
17	·107876	·149597	·199304	·256178	·318864	·385597	·454366	·523105	·589868
18	·081581	·115691	·157520	·206781	·262666	·323897	·388841	·455653	·522438
19	·061094	·088529	·123104	164949	·213734	·268663	·328532	·391823	·456836
20	·045341	·067086	·095210	·130141	·171932	·220220	·274229	·332819	·394578
21	·033371	·050380	·072929	·101632	·136830	·178510	·226291	·279413	·336801
22	·024374	·037520	·055362	·078614	·107804	·143191	·184719	·231985	·284256
23	·017676	·027726	·041677	·060270	·084140	·113735	·149251	·190590	·237342
24	·012733	·020341	·031130	·045822	·065093	·089504	·119435	·155028	·196152
25	·009117	·014822	·023084	·034566	·049943	·069824	·094710	·124915	·160542
26	·006490	·010734	·017001	·025887	·038023	·054028	·074461	·099758	·130189
27	·004595	·007727	·012441	·019254	·028736	·041483	·058068	·078995	·104653
28	·003238	·005532	·009050	·014228	·021569	·031620	·044938	·062055	·083428
29	·002270	·003940	·006546	·010450	·016085	·023936	·034526	·048379	·065985
30	·001585	·002792	·004710	·007632	·011921	·018002	·026345	·037446	·051798
40	·000036	·000072	·000138	·000255	·000453	·000778	·001294	·002087	·003272
50	·000001	·000001	·000003	·000006	·000012	·000023	·000042	·000075	·000131
60	·000000	·000000	·000000	·000000	·000000	·000001	·000001	·000002	·000004
70	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000

TABLE XII. *Test for Goodness of Fit. Values of* **P**.

χ^2	$n'=21$	$n'=22$	$n'=23$	$n'=24$	$n'=25$	$n'=26$	$n'=27$	$n'=28$	$n'=29$	$n'=30$
1	1·	1·	1·	1·	1·	1·	1·	1·	1·	1·
2	1·	1·	1·	1·	1·	1·	1·	1·	1·	1·
3	·999996	·999998	·999999	1·	1·	1·	1·	1·	1·	1·
4	·999954	·999980	·999992	·999997	·999999	1·	1·	1·	1·	1·
5	·999722	·999868	·999939	·999972	·999987	·999994	·999998	·999999	1·	1·
6	·998898	·999427	·999708	·999855	·999929	·999966	·999984	·999993	·999997	·999999
7	·996685	·998142	·998980	·999452	·999711	·999851	·999924	·999962	·999981	·999991
8	·991868	·995143	·997160	·998371	·999085	·999494	·999726	·999853	·999924	·999960
9	·982907	·989214	·993331	·995957	·997595	·998596	·999194	·999546	·999748	·999863
10	·968171	·978912	·986304	·991277	·994547	·996653	·997981	·998803	·999302	·999599
11	·946223	·962787	·974749	·983189	·989012	·992946	·995549	·997239	·998315	·998988
12	·916076	·939617	·957379	·970470	·979908	·986567	·991173	·994294	·996372	·997728
13	·877384	·908624	·933161	·951990	·966121	·976501	·983974	·989247	·992900	·995384
14	·830496	·869599	·901479	·926871	·946650	·961732	·973000	·981254	·987189	·991377
15	·776408	·822952	862238	·894634	·920759	·941383	·957334	·969432	·978436	·985015
16	·716624	·769650	·815886	·855268	·888076	·914828	·936203	·952947	·965819	·975536
17	·652974	·711106	·763362	·809251	·848662	·881793	·909083	·931122	·948589	·962181
18	·587408	·649004	·705988	·757489	·803008	·842390	·875773	·903519	·926149	·944272
19	·521826	·585140	·645328	·701224	·751990	·797120	·836430	·870001	·898136	·921288
20	·457930	·521261	·583040	·641912	·696776	·746825	·791556	·830756	·864464	·892927
21	·397132	·458944	·520738	·581087	·638725	·692609	·741964	·786288	·825349	·859149
22	·340511	·399510	·459889	·520252	·579267	·635744	·688697	·737377	·781291	·820189
23	·288795	·343979	·401730	·460771	·519798	·577564	·632947	·685013	·733041	·776543
24	·242392	·293058	·347229	·403808	·461597	·519373	·575965	·630316	·681535	·728932
25	·201431	·247164	·297075	·350285	405760	·462373	·518975	·574462	·627835	·678248
26	·165812	·206449	·251682	·300866	·353165	·407598	·463105	·518600	·573045	·625491
27	·135264	·170853	·211226	·255967	·304453	·355884	·409333	·463794	·518247	·571705
28	·109399	·140151	·175681	·215781	·260040	·307853	·358458	·410973	·464447	·517913
29	·087759	·114002	·144861	·180310	·220131	·263916	·311082	·360899	·412528	·465066
30	·069854	·091988	·118464	·149402	·184752	·224289	·267611	·314154	·363218	·414004
40	·004995	·007437	·010812	·015369	·021387	·029164	·039012	·051237	·066128	·083937
50	·000221	·000365	·000586	·000921	·001416	·002131	·003144	·004551	·006467	·009032
60	·000007	·000013	·000022	·000038	·000064	·000104	·000168	·000264	·000407	·000618
70	·000000	·000000	·000001	·000001	·000002	·000004	·000007	·000011	·000019	·000030

TABLE XIII. *Auxiliary Table A.*

χ^2	$\log\left\{\chi\sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\chi^2}\right\}$	$\log e^{-\frac{1}{2}\chi^2}$	χ^2	$\log\left\{\chi\sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\chi^2}\right\}$	$\log e^{-\frac{1}{2}\chi^2}$
1	$\overline{1}\cdot68479282$	$\overline{1}\cdot78285276$	51	$\overline{11}\cdot68121586$	$\overline{12}\cdot92549071$
2	$\overline{1}\cdot61816058$	$\overline{1}\cdot56570552$	52	$\overline{11}\cdot46828520$	$\overline{12}\cdot70834347$
3	$\overline{1}\cdot48905897$	$\overline{1}\cdot34855828$	53	$\overline{11}\cdot25527422$	$\overline{12}\cdot49119623$
4	$\overline{1}\cdot33438109$	$\overline{1}\cdot13141104$	54	$\overline{11}\cdot04218593$	$\overline{12}\cdot27404899$
5	$\overline{1}\cdot16568886$	$\overline{2}\cdot91426380$	55	$\overline{12}\cdot82902315$	$\overline{12}\cdot05690175$
6	$\overline{2}\cdot98813224$	$\overline{2}\cdot69711655$	56	$\overline{12}\cdot61578858$	$\overline{13}\cdot83975450$
7	$\overline{2}\cdot80445839$	$\overline{2}\cdot47996931$	57	$\overline{12}\cdot40248475$	$\overline{13}\cdot62260726$
8	$\overline{2}\cdot61630713$	$\overline{2}\cdot26282207$	58	$\overline{12}\cdot18911408$	$\overline{13}\cdot40546002$
9	$\overline{2}\cdot42473615$	$\overline{2}\cdot04567483$	59	$\overline{13}\cdot97567885$	$\overline{13}\cdot18831278$
10	$\overline{2}\cdot23046765$	$\overline{3}\cdot82852759$	60	$\overline{13}\cdot76218123$	$\overline{14}\cdot97116554$
11	$\overline{2}\cdot03401675$	$\overline{3}\cdot61138035$	61	$\overline{13}\cdot54862328$	$\overline{14}\cdot75401830$
12	$\overline{3}\cdot83576379$	$\overline{3}\cdot39423311$	62	$\overline{13}\cdot33500696$	$\overline{14}\cdot53687106$
13	$\overline{3}\cdot63599760$	$\overline{3}\cdot17708587$	63	$\overline{13}\cdot12133415$	$\overline{14}\cdot31972382$
14	$\overline{3}\cdot43494271$	$\overline{4}\cdot95993863$	64	$\overline{14}\cdot90760662$	$\overline{14}\cdot10257658$
15	$\overline{3}\cdot23277708$	$\overline{4}\cdot74279139$	65	$\overline{14}\cdot69382607$	$\overline{15}\cdot88542934$
16	$\overline{3}\cdot02964420$	$\overline{4}\cdot52564414$	66	$\overline{14}\cdot48099412$	$\overline{15}\cdot66828209$
17	$\overline{4}\cdot82566143$	$\overline{4}\cdot30849690$	67	$\overline{14}\cdot26611232$	$\overline{15}\cdot45113485$
18	$\overline{4}\cdot62092598$	$\overline{4}\cdot09134966$	68	$\overline{14}\cdot05218213$	$\overline{15}\cdot23398761$
19	$\overline{4}\cdot41551928$	$\overline{5}\cdot87420242$	69	$\overline{15}\cdot83820498$	$\overline{15}\cdot01684037$
20	$\overline{4}\cdot20951024$	$\overline{5}\cdot65705518$	70	$\overline{15}\cdot62418221$	$\overline{16}\cdot79969313$
21	$\overline{4}\cdot00295765$	$\overline{5}\cdot43990794$	71	$\overline{15}\cdot41011512$	$\overline{16}\cdot58254589$
22	$\overline{5}\cdot79591210$	$\overline{5}\cdot22276070$	72	$\overline{15}\cdot19600496$	$\overline{16}\cdot36539865$
23	$\overline{5}\cdot58841744$	$\overline{5}\cdot00561346$	73	$\overline{16}\cdot98185290$	$\overline{16}\cdot14825141$
24	$\overline{5}\cdot38051190$	$\overline{6}\cdot78846622$	74	$\overline{16}\cdot76766009$	$\overline{17}\cdot93110417$
25	$\overline{5}\cdot17222904$	$\overline{6}\cdot57131898$	75	$\overline{16}\cdot55342762$	$\overline{17}\cdot71395693$
26	$\overline{6}\cdot96359847$	$\overline{6}\cdot35417173$	76	$\overline{16}\cdot33915654$	$\overline{17}\cdot49680968$
27	$\overline{6}\cdot75464644$	$\overline{6}\cdot13702449$	77	$\overline{16}\cdot12484787$	$\overline{17}\cdot27966244$
28	$\overline{6}\cdot54539633$	$\overline{7}\cdot91987725$	78	$\overline{17}\cdot91050256$	$\overline{17}\cdot06251520$
29	$\overline{6}\cdot33586907$	$\overline{7}\cdot70273001$	79	$\overline{17}\cdot69612157$	$\overline{18}\cdot84536796$
30	$\overline{6}\cdot12608346$	$\overline{7}\cdot48558277$	80	$\overline{17}\cdot48170578$	$\overline{18}\cdot62822072$
31	$\overline{7}\cdot91605644$	$\overline{7}\cdot26843553$	81	$\overline{17}\cdot26725605$	$\overline{18}\cdot41107348$
32	$\overline{7}\cdot70580334$	$\overline{7}\cdot05128829$	82	$\overline{17}\cdot05277323$	$\overline{18}\cdot19392624$
33	$\overline{7}\cdot49533808$	$\overline{8}\cdot83414105$	83	$\overline{18}\cdot83825810$	$\overline{19}\cdot97677900$
34	$\overline{7}\cdot28467333$	$\overline{8}\cdot61699381$	84	$\overline{18}\cdot62371146$	$\overline{19}\cdot75963176$
35	$\overline{7}\cdot07382065$	$\overline{8}\cdot39984657$	85	$\overline{18}\cdot40913404$	$\overline{19}\cdot54248452$
36	$\overline{8}\cdot86279064$	$\overline{8}\cdot18269932$	86	$\overline{18}\cdot19452656$	$\overline{19}\cdot32533727$
37	$\overline{8}\cdot65159301$	$\overline{9}\cdot96555208$	87	$\overline{19}\cdot97988972$	$\overline{19}\cdot10819003$
38	$\overline{8}\cdot44023670$	$\overline{9}\cdot74840484$	88	$\overline{19}\cdot76522419$	$\overline{20}\cdot89104279$
39	$\overline{8}\cdot22872997$	$\overline{9}\cdot53125760$	89	$\overline{19}\cdot55053062$	$\overline{20}\cdot67389555$
40	$\overline{8}\cdot01708042$	$\overline{9}\cdot31411036$	90	$\overline{19}\cdot33580963$	$\overline{20}\cdot45674831$
41	$\overline{9}\cdot80529511$	$\overline{9}\cdot09696312$	91	$\overline{19}\cdot12106183$	$\overline{20}\cdot23960107$
42	$\overline{9}\cdot59338058$	$\overline{10}\cdot87981588$	92	$\overline{20}\cdot90628780$	$\overline{20}\cdot02245383$
43	$\overline{9}\cdot38134293$	$\overline{10}\cdot66266864$	93	$\overline{20}\cdot69148812$	$\overline{21}\cdot80530659$
44	$\overline{9}\cdot16918780$	$\overline{10}\cdot44552140$	94	$\overline{20}\cdot47666333$	$\overline{21}\cdot58815935$
45	$\overline{10}\cdot95692047$	$\overline{10}\cdot22837416$	95	$\overline{20}\cdot26181397$	$\overline{21}\cdot37101211$
46	$\overline{10}\cdot74454589$	$\overline{10}\cdot01122691$	96	$\overline{20}\cdot04694054$	$\overline{21}\cdot15386486$
47	$\overline{10}\cdot53206866$	$\overline{11}\cdot79407967$	97	$\overline{21}\cdot83204355$	$\overline{22}\cdot93671762$
48	$\overline{10}\cdot31949311$	$\overline{11}\cdot57693243$	98	$\overline{21}\cdot61712348$	$\overline{22}\cdot71957038$
49	$\overline{10}\cdot10682329$	$\overline{11}\cdot35978519$	99	$\overline{21}\cdot40218080$	$\overline{22}\cdot50242314$
50	$\overline{11}\cdot89406301$	$\overline{11}\cdot14263795$	100	$\overline{21}\cdot18721596$	$\overline{22}\cdot28527590$

TABLES XIV—XVI. *Auxiliary Tables* B, C *and* D.

TABLE XIV (B).

Table of colog $[n]$:—$[n] = n(n-2)(n-4)\ldots\ldots$

n odd nos.	colog $[n]$	n even nos.	colog $[n]$
1	·00000000	2	$\overline{1}$·69897000
3	$\overline{1}$·52287875	4	$\overline{1}$·09691001
5	$\overline{2}$·82390874	6	$\overline{2}$·31875876
7	$\overline{3}$·97881070	8	$\overline{3}$·41566878
9	$\overline{3}$·02456819	10	$\overline{4}$·41566878
11	$\overline{5}$·98317551	12	$\overline{5}$·33648753
13	$\overline{6}$·86923215	14	$\overline{6}$·19035949
15	$\overline{7}$·69314089	16	$\overline{8}$·98623951
17	$\overline{8}$·46269197	18	$\overline{9}$·73096701
19	$\overline{9}$·18393837	20	$\overline{10}$·42993701
21	$\overline{11}$·86171908	22	$\overline{11}$·08751433
23	$\overline{12}$·49999124	24	$\overline{13}$·70730309
25	$\overline{13}$·10205123	26	$\overline{14}$·29232974
27	$\overline{15}$·67068747	28	$\overline{16}$·84517171
29	$\overline{16}$·20828947	30	$\overline{17}$·36805045
31	$\overline{18}$·71692778	32	$\overline{19}$·86290048
33	$\overline{19}$·19841384	34	$\overline{20}$·33142156
35	$\overline{21}$·65434579	36	$\overline{22}$·77511906
37	$\overline{22}$·08614407	38	$\overline{23}$·19533546
39	$\overline{24}$·49507946	40	$\overline{25}$·59327547
41	$\overline{26}$·88229561	42	$\overline{27}$·97002618
43	$\overline{27}$·24882715	44	$\overline{28}$·32657350
45	$\overline{29}$·59561464	46	$\overline{30}$·66381567
47	$\overline{31}$·92351678	48	$\overline{32}$·98257443
49	$\overline{32}$·23332070	50	$\overline{33}$·28360443
51	$\overline{34}$·52575052	52	$\overline{35}$·56760109
53	$\overline{36}$·80147465	54	$\overline{37}$·83520733
55	$\overline{37}$·06111196	56	$\overline{38}$·08701930
57	$\overline{39}$·30523711	58	$\overline{40}$·32359131
59	$\overline{41}$·53438510	60	$\overline{42}$·54544006
61	$\overline{43}$·74905526	62	$\overline{44}$·75304837
63	$\overline{45}$·94971471	64	$\overline{46}$·94686839
65	$\overline{46}$·13680135	66	$\overline{47}$·12732446
67	$\overline{48}$·31072655	68	$\overline{49}$·29481554
69	$\overline{50}$·47187746	70	$\overline{51}$·44971750
71	$\overline{52}$·62061911	72	$\overline{53}$·59238501
73	$\overline{54}$·75729625	74	$\overline{55}$·72315329
75	$\overline{56}$·88223499	76	$\overline{57}$·84233970
77	$\overline{58}$·99574426	78	$\overline{59}$·95024509
79	$\overline{59}$·09811717	80	$\overline{60}$·04715511
81	$\overline{61}$·18963215	82	$\overline{62}$·13334125
83	$\overline{63}$·27055406	84	$\overline{64}$·20906197
85	$\overline{65}$·34113514	86	$\overline{66}$·27456352
87	$\overline{67}$·40161588	88	$\overline{68}$·33008084
89	$\overline{69}$·45222588	90	$\overline{70}$·37583833
91	$\overline{71}$·49318448	92	$\overline{72}$·41205051
93	$\overline{73}$·52470154	94	$\overline{74}$·43892265
95	$\overline{75}$·54697793	96	$\overline{76}$·45665142
97	$\overline{77}$·56020620	98	$\overline{78}$·46542534
99	$\overline{79}$·56457100	100	$\overline{80}$·46542534

TABLE XV (C).

χ^2	$\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\,d\chi$
1	·3173106
2	·1572992
3	·0832646
4	·0455003
5	·0253474
6	·0143060
7	·0081506
8	·0046776
9	·0026998
10	·0015654
11	·0009112
12	·0005321
13	·0003115
14	·0001828
15	·0001076
16	·0000634
17	·0000374
18	·0000221
19	·0000132
20	·0000078
21	·0000046
22	·0000027
23	·0000016
24	·0000011
25	·0000007
26	·0000004
27	·0000003
28	·0000002
29	·0000001
30	·0000000

TABLE XVI (D).

Function	Log. Function
$e^{-\frac{1}{2}}$	$\overline{1}$·7828527590
$\sqrt{\frac{2}{\pi}}$	$\overline{1}$·9019400615

TABLE XVII.

Values of ($-\log P$) corresponding to given values of χ^2 in a fourfold table.

(Extension of Table XII for $n' = 4$.)

χ^2	$-\log P$	χ^2	$-\log P$	χ^2	$-\log P$	χ^2	$-\log P$	χ^2	$-\log P$	χ^2	$-\log P$
1	0·096	*26*	5·021	*50*	10·097	*1100*	237·439	*2600*	562·973	*13500*	2929·521
2	0·242	*27*	5·230	*60*	12·231	*1150*	248·287	*2700*	584·680	*14000*	3038·086
3	0·407	*28*	5·440	*70*	14·370	*1200*	259·135	*2800*	606·387	*14500*	3146·652
4	0·583	*29*	5·650	*80*	16·513	*1250*	269·983	*2900*	628·094	*15000*	3255·219
5	0·765	*30*	5·860	*90*	18·659	*1300*	280·832	*3000*	649·801	*15500*	3363·785
6	0·952	*31*	6·071	*100*	20·809	*1350*	291·681	*3500*	758·341	*16000*	3472·352
7	1·143	*32*	6·281	*150*	31·579	*1400*	302·531	*4000*	866·886	*16500*	3580·919
8	1·337	*33*	6·492	*200*	42·375	*1450*	313·381	*4500*	975·434	*17000*	3689·486
9	1·533	*34*	6·703	*250*	53·184	*1500*	324·231	*5000*	1083·995	*17500*	3798·053
10	1·731	*35*	6·914	*300*	64·002	*1550*	335·081	*5500*	1192·538	*18000*	3906·621
11	1·931	*36*	7·126	*350*	74·826	*1600*	345·931	*6000*	1301·092	*18500*	4015·188
12	2·132	*37*	7·337	*400*	85·655	*1650*	356·782	*6500*	1409·649	*19000*	4123·756
13	2·334	*38*	7·549	*450*	96·487	*1700*	367·633	*7000*	1518·206	*19500*	4232·324
14	2·537	*39*	7·761	*500*	107·321	*1750*	378·484	*7500*	1626·765	*20000*	4340·892
15	2·741	*40*	7·972	*550*	118·158	*1800*	389·335	*8000*	1735·324	*20500*	4449·461
16	2·945	*41*	8·184	*600*	128·997	*1850*	400·187	*8500*	1843·885	*21000*	4558·029
17	3·151	*42*	8·397	*650*	139·837	*1900*	411·038	*9000*	1952·446	*21500*	4666·597
18	3·357	*43*	8·609	*700*	150·678	*1950*	421·890	*9500*	2061·008	*22000*	4775·166
19	3·564	*44*	8·821	*750*	161·520	*2000*	432·742	*10000*	2169·570	*22500*	4883·735
20	3·770	*45*	9·034	*800*	172·364	*2050*	443·594	*10500*	2278·133	*23000*	4992·304
21	3·978	*46*	9·246	*850*	183·208	*2100*	454·446	*11000*	2386·697	*23500*	5100·873
22	4·186	*47*	9·459	*900*	194·053	*2200*	476·151	*11500*	2495·261	*24000*	5209·442
23	4·394	*48*	9·672	*950*	204·899	*2300*	497·856	*12000*	2603·825	*24500*	5318·011
24	4·602	*49*	9·885	*1000*	215·745	*2400*	519·561	*12500*	2712·390	*25000*	5426·580
25	4·811	*50*	10·097	*1050*	226·592	*2500*	541·267	*13000*	2820·955		
26	5·021			*1100*	237·439	*2600*	562·973	*13500*	2929·521		

TABLE XVIII.

Values of ($-\log P$), entering with r and ${}_0\sigma_r$.

Values of ${}_0\sigma_r$.

Values of r.	*·01*	*·02*	*·03*	*·04*	*·05*	*·06*	*·07*	*·08*
0·05	6·248	1·907	1·020	0·675	0·498	0·392	0·322	0·273
0·075	13·228	3·760	1·908	1·217	0·874	0·674	0·545	0·456
0·1	22·924	6·267	3·076	1·910	1·343	1·019	0·814	0·675
0·15	50·687	13·329	6·298	3·784	2·586	1·916	1·498	1·218
0·2	90·035	23·254	10·771	6·343	4·259	3·100	2·384	1·924
0·3	206·348	52·453	23·836	13·758	9·057	6·478	4·906	3·903
0·4	380·266	96·013	43·254	24·726	16·112	11·407	8·552	6·686
0·5	626·428	157·607	70·669	40·177	26·025	18·312	13·642	10·597
0·6	970·879	243·753	108·980	61·747	39·845	27·922	20·713	16·020
0·7	1463·946	367·033	163·781	92·579	59·584	41·634	30·792	23·740
0·8	2220·267	556·100	247·801	139·832	89·819	62·625	46·209	35·539
0·9	3607·924	902·949	401·907	226·479	145·241	101·085	74·442	57·134
0·95	5056·547	1265·013	562·757	316·904	203·069	141·207	103·886	79·671

TABLE XIX.

Values of χ^2 corresponding to the values of $(-\log P)$ in Table XVIII.

Values of $_0\sigma_r$.

Values of r.	·01	·02	·03	·04	·05	·06	·07	·08
0·05	31·84	10·88	6·36	4·51	3·52	2·91	2·48	2·19
0·075	64·66	19·95	10·89	7·38	5·58	4·51	3·78	3·28
0·1	109·82	31·93	16·64	10·90	8·03	6·35	5·26	4·51
0·15	238·45	65·13	32·08	20·07	14·24	10·93	8·82	7·39
0·2	422·29	111·35	53·16	32·29	22·35	16·75	13·25	10·97
0·3	956·68	246·62	114·05	67·14	45·11	32·93	25·45	20·64
0·4	1758·21	447·81	204·07	118·18	78·13	56·14	42·73	33·92
0·5	2892·33	731·95	330·80	189·82	124·22	88·38	66·60	52·34
0·6	4479·02	1129·10	507·65	289·58	188·28	133·02	99·55	77·70
0·7	6750·09	1697·24	760·43	431·96	279·58	196·57	146·35	113·61
0·8	10233·49	2568·34	1147·76	649·98	419·22	293·64	217·74	168·34
0·9	16624·37	4166·12	1857·93	1049·48	674·92	471·22	348·23	268·26
0·95	23295·86	5833·82	2599·00	1466·24	941·56	656·32	484·15	372·37

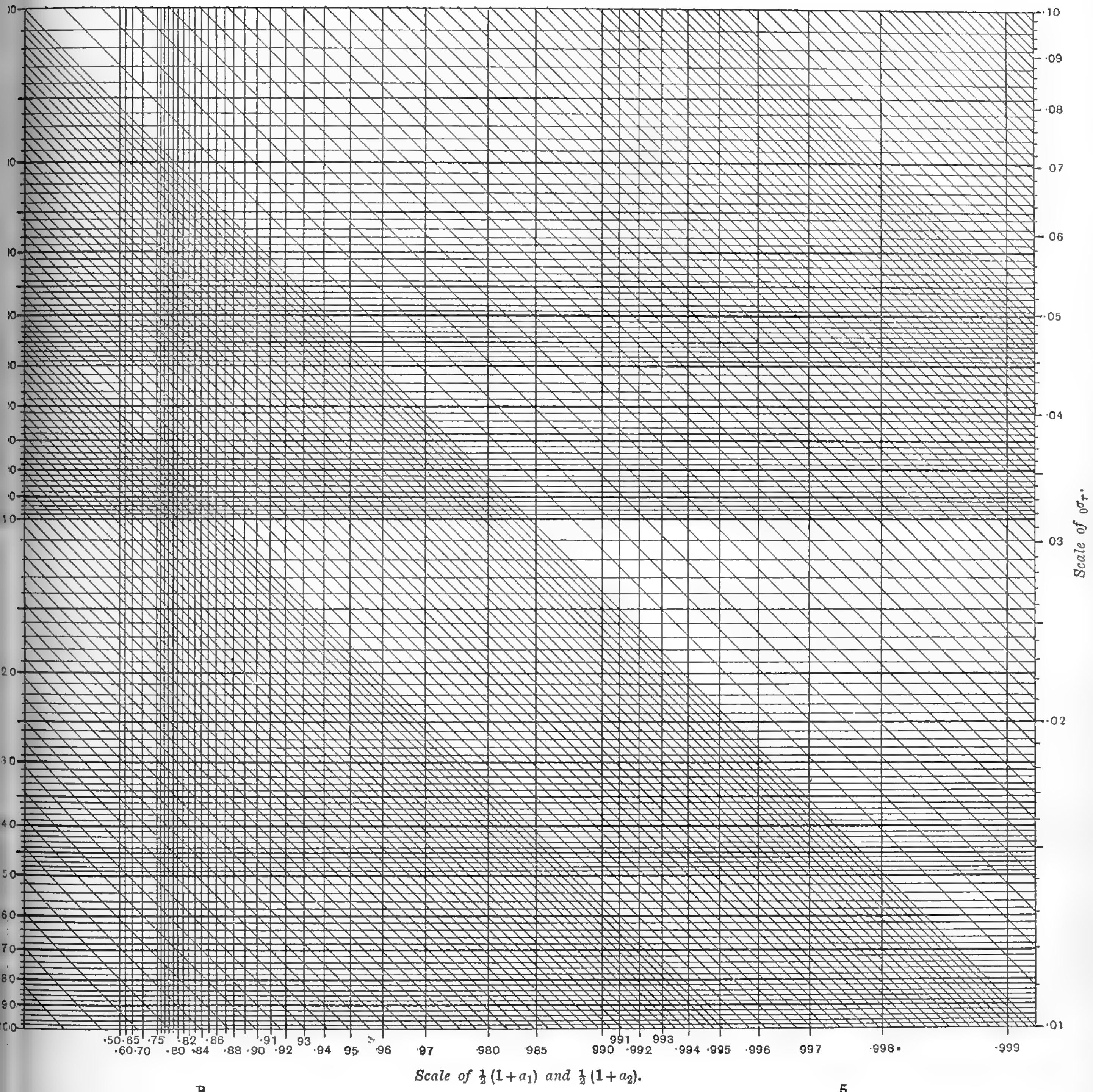
TABLE XX.

Values of $\log\chi^2$ corresponding to values of r and $_0\sigma_r$ in Tables XVII and XVIII.

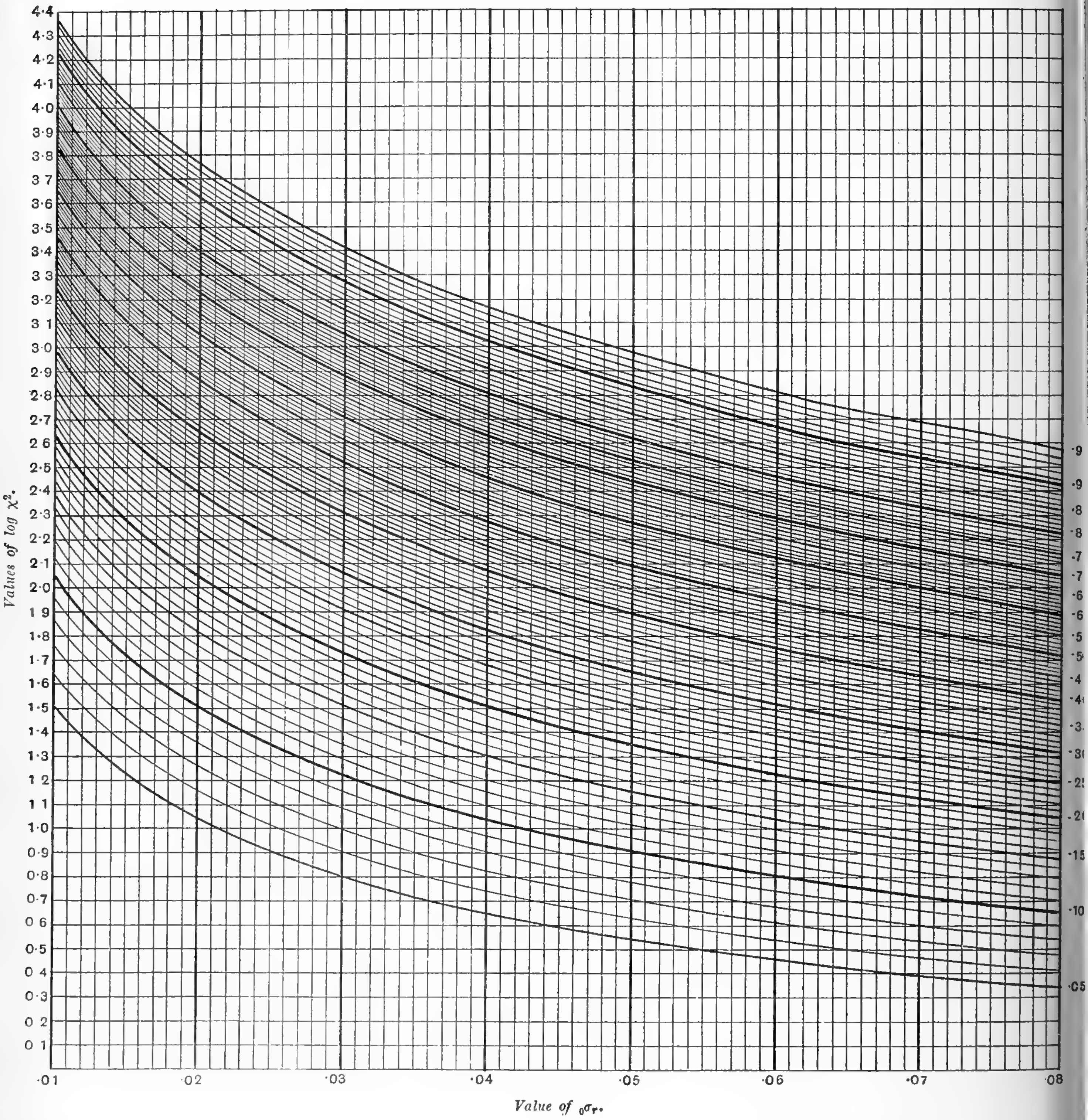
Values of $_0\sigma_r$.

Values of r.	·01	·02	·03	·04	·05	·06	·07	·08
0·05	1·5030	1·0366	0·8035	0·6542	0·5465	0·4639	0·3945	0·3404
0·075	1·8106	1·2999	1·0370	0·8681	0·7466	0·6542	0·5775	0·5159
0·1	2·0407	1·5042	1·2212	1·0374	0·9047	0·8028	0·7210	0·6522
0·15	2·3774	1·8138	1·5062	1·3025	1·1535	1·0386	0·9455	0·8686
0·2	2·6256	2·0467	1·7256	1·5091	1·3493	1·2240	1·1222	1·0400
0·3	2·9808	2·3920	2·0571	1·8270	1·6543	1·5176	1·4057	1·3148
0·4	3·2451	2·6511	2·3098	2·0725	1·8928	1·7493	1·6307	1·5305
0·5	3·4612	2·8645	2·5196	2·2783	2·0942	1·9464	1·8235	1·7188
0·6	3·6512	3·0527	2·7056	2·4618	2·2748	2·1239	1·9980	1·8904
0·7	3·8293	3·2297	2·8811	2·6354	2·4465	2·2935	2·1654	2·0554
0·8	4·0100	3·4097	3·0598	2·8129	2·6224	2·4678	2·3379	2·2262
0·9	4·2207	3·6197	3·2690	3·0210	2·8293	2·6732	2·5419	2·4286
0·95	4·3673	3·7660	3·4148	3·1662	2·9738	2·8171	2·6850	2·5710

XXI. *Abac to determine* ${}_0\sigma_r$.



XXII. *Abac to determine r_P.*



Approximate values of Probable Error of r from a four-fold Correlation Table (to be used with χ_1 of Table V).

TABLE XXIII. *Values of χ_r for Values of r.*

r	χ_r	r	χ_r	r	χ_r	r	χ_r	r	χ_r
·00	1·0000	·20	·9717	·40	·8845	·60	·7298	·80	·4843
·01	·9999	·21	·9688	·41	·8785	·61	·7200	·81	·4687
·02	·9997	·22	·9657	·42	·8723	·62	·7099	·82	·4526
·03	·9994	·23	·9625	·43	·8659	·63	·6997	·83	·4362
·04	·9989	·24	·9591	·44	·8594	·64	·6892	·84	·4192
·05	·9982	·25	·9556	·45	·8527	·65	·6785	·85	·4018
·06	·9975	·26	·9520	·46	·8458	·66	·6675	·86	·3838
·07	·9966	·27	·9482	·47	·8388	·67	·6563	·87	·3652
·08	·9955	·28	·9442	·48	·8315	·68	·6448	·88	·3461
·09	·9943	·29	·9401	·49	·8241	·69	·6331	·89	·3262
·10	·9930	·30	·9358	·50	·8165	·70	·6211	·90	·3057
·11	·9915	·31	·9314	·51	·8087	·71	·6088	·91	·2843
·12	·9899	·32	·9268	·52	·8007	·72	·5962	·92	·2620
·13	·9881	·33	·9221	·53	·7926	·73	·5834	·93	·2387
·14	·9862	·34	·9172	·54	·7842	·74	·5702	·94	·2142
·15	·9841	·35	·9122	·55	·7756	·75	·5568	·95	·1882
·16	·9819	·36	·9070	·56	·7669	·76	·5430	·96	·1605
·17	·9796	·37	·9016	·57	·7579	·77	·5288	·97	·1305
·18	·9771	·38	·8961	·58	·7488	·78	·5144	·98	·0972
·19	·9745	·39	·8904	·59	·7394	·79	·4995	·99	·0585
								1·00	·0000

TABLE XXIV.

Values of χ_a for Values of $\frac{1}{2}(1+\alpha)$.

$\frac{1}{2}(1+\alpha)$	χ_a	$\frac{1}{2}(1+\alpha)$	χ_a	$\frac{1}{2}(1+\alpha)$	χ_a	$\frac{1}{2}(1+\alpha)$	χ_a
·50	1·2533	·65	1·2877	·80	1·4288	·95	2·1132
·51	1·2535	·66	1·2928	·81	1·4457	·96	2·2740
·52	1·2539	·67	1·2984	·82	1·4641	·97	2·5071
·53	1·2546	·68	1·3044	·83	1·4844	·98	2·8915
·54	1·2556	·69	1·3109	·84	1·5067	·985	3·2097
·55	1·2569	·70	1·3180	·85	1·5315	·990	3·7333
·56	1·2585	·71	1·3256	·86	1·5590	·991	3·8854
·57	1·2604	·72	1·3338	·87	1·5897	·992	4·0639
·58	1·2626	·73	1·3427	·88	1·6245	·993	4·2784
·59	1·2652	·74	1·3523	·89	1·6640	·994	4·5419
·60	1·2680	·75	1·3626	·90	1·7094	·995	4·8779
·61	1·2712	·76	1·3738	·91	1·7623	·996	5·3278
·62	1·2748	·77	1·3859	·92	1·8249	·997	5·9776
·63	1·2787	·78	1·3990	·93	1·9003	·998	7·0465
·64	1·2830	·79	1·4133	·94	1·9937	·999	9·3870

TABLE XXV.

$$\text{Values of } y = \frac{n-2}{n-3} \cdot \frac{n-4}{n-5} \cdots \left\{ \begin{matrix} \cdots \frac{4}{3} \cdot \frac{2}{\pi} \text{ if } n \text{ be even} \\ \cdots \frac{3}{2} \cdot \frac{1}{2} \text{ if } n \text{ be odd} \end{matrix} \right\} \int_{-\infty}^{z} (1+z^2)^{-\frac{n}{2}} \, dz.$$

Or, the probability that the mean of a sample of n, drawn at random from a normal population, will not exceed (in algebraic sense) the mean of the population by more than z times the standard deviation of the sample.

z	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
·1	·5633	·5745	·5841	·5928	·6006	·60787	·61462
·2	·6241	·6458	·6634	·6798	·6936	·70705	·71846
·3	·6804	·7096	·7340	·7549	·7733	·78961	·80423
·4	·7309	·7657	·7939	·8175	·8376	·85465	·86970
·5	·7749	·8131	·8428	·8667	·8863	·90251	·91609
·6	·8125	·8518	·8813	·9040	·9218	·93600	·94732
·7	·8440	·8830	·9109	·9314	·9468	·95851	·96747
·8	·8701	·9076	·9332	·9512	·9640	·97328	·98007
·9	·8915	·9269	·9498	·9652	·9756	·98279	·98780
1·0	·9092	·9419	·9622	·9751	·9834	·98890	·99252
1·1	·9236	·9537	·9714	·9821	·9887	·99280	·99539
1·2	·9354	·9628	·9782	·9870	·9922	·99528	·99713
1·3	·9451	·9700	·9832	·9905	·9946	·99688	·99819
1·4	·9531	·9756	·9870	·9930	·9962	·99791	·99885
1·5	·9598	·9800	·9899	·9948	·9973	·99859	·99926
1·6	·9653	·9836	·9920	·9961	·9981	·99903	·99951
1·7	·9699	·9864	·9937	·9970	·9986	·99933	·99968
1·8	·9737	·9886	·9950	·9977	·9990	·99953	·99978
1·9	·9770	·9904	·9959	·9983	·9992	·99967	·99985
2·0	·9797	·9919	·9967	·9986	·9994	·99976	·99990
2·1	·9821	·9931	·9973	·9989	·9996	·99983	·99993
2·2	·9841	·9941	·9978	·9992	·9997	·99987	·99995
2·3	·9858	·9950	·9982	·9993	·9998	·99991	·99996
2·4	·9873	·9957	·9985	·9995	·9998	·99993	·99997
2·5	·9886	·9963	·9987	·9996	·9998	·99995	·99998
2·6	·9898	·9967	·9989	·9996	·9999	·99996	·99999
2·7	·9908	·9972	·9991	·9997	·9999	·99997	·99999
2·8	·9916	·9975	·9992	·9998	·9999	·99998	·99999
2·9	·9924	·9978	·9993	·9998	·9999	·99998	·99999
3·0	·9931	·9981	·9994	·9998	—	·99999	—

TABLE XXVI.

Table for use in plotting Type III Curves, i.e.

$$y = y_0 e^{-p\frac{x}{a}}\left(1 + \frac{x}{a}\right)^p.$$

X	$\log_{10}(1+X) - X\log_{10}e$	X	$\log_{10}(1+X) - X\log_{10}e$
−·95	·888 450	+ ·80	·092 163
−·90	·609 135	+ ·85	·101 979
−·85	·454 759	+ ·90	·112 111
−·80	·351 534	+ ·95	·122 545
−·75	·276 339	+1·00	·133 265
−·70	·218 873	+1·05	·144 255
−·65	·173 641	+1·10	·155 505
−·60	·137 363	+1·15	·167 000
−·55	·107 926	+1·20	·178 731
−·50	·083 883	+1·25	·190 685
−·45	·064 205	+1·30	·202 855
−·40	·048 131	+1·35	·215 230
−·35	·035 084	+1·40	·227 801
−·30	·024 614	+1·45	·240 561
−·25	·016 365	+1·50	·253 502
−·20	·010 051	+1·55	·266 616
−·15	·005 437	+1·60	·279 898
−·10	·002 328	+1·65	·293 340
−·05	·000 562	+1·70	·306 937
·00	·000 000	+1·75	·320 683
+·05	·000 525	+1·80	·334 572
+·10	·002 037	+1·85	·348 600
+·15	·004 446	+1·90	·362 761
+·20	·007 678	+1·95	·377 052
+·25	·011 664	+2·00	·391 468
+·30	·016 345	+2·05	·406 004
+·35	·021 669	+2·10	·420 657
+·40	·027 590	+2·15	·435 422
+·45	·034 065	+2·20	·450 298
+·50	·041 056	+2·25	·465 279
+·55	·048 530	+2·30	·480 363
+·60	·056 457	+2·35	·495 547
+·65	·064 808	+2·40	·510 828
+·70	·073 557	+2·45	·526 202
+·75	·082 683	+2·50	·541 668

TABLE XXVII.

Powers of Natural Numbers.

n	n^2	n^3	n^4	n^5	n^6	n^7	n
1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	2
3	9	27	81	243	729	2187	3
4	16	64	256	1024	4096	16384	4
5	25	125	625	3125	15625	78125	5
6	36	216	1296	7776	46656	279936	6
7	49	343	2401	16807	117649	823543	7
8	64	512	4096	32768	262144	2097152	8
9	81	729	6561	59049	531441	4782969	9
10	100	1000	10000	100000	1000000	10000000	10
11	121	1331	14641	161051	1771561	19487171	11
12	144	1728	20736	248832	2985984	35831808	12
13	169	2197	28561	371293	4826809	62748517	13
14	196	2744	38416	537824	7529536	105413504	14
15	225	3375	50625	759375	11390625	170859375	15
16	256	4096	65536	1048576	16777216	268435456	16
17	289	4913	83521	1419857	24137569	410338673	17
18	324	5832	104976	1889568	34012224	612220032	18
19	361	6859	130321	2476099	47045881	893871739	19
20	400	8000	160000	3200000	64000000	1280000000	20
21	441	9261	194481	4084101	85766121	1801088541	21
22	484	10648	234256	5153632	113379904	2494357888	22
23	529	12167	279841	6436343	148035889	3404825447	23
24	576	13824	331776	7962624	191102976	4586471424	24
25	625	15625	390625	9765625	244140625	6103515625	25
26	676	17576	456976	11881376	308915776	8031810176	26
27	729	19683	531441	14348907	387420489	10460353203	27
28	784	21952	614656	17210368	481890304	13492928512	28
29	841	24389	707281	20511149	594823321	17249876309	29
30	900	27000	810000	24300000	729000000	21870000000	30
31	961	29791	923521	28629151	887503681	27512614111	31
32	1024	32768	1048576	33554432	1073741824	34359738368	32
33	1089	35937	1185921	39135393	1291467969	42618442977	33
34	1156	39304	1336336	45435424	1544804416	52523350144	34
35	1225	42875	1500625	52521875	1838265625	64339296875	35
36	1296	46656	1679616	60466176	2176782336	78364164096	36
37	1369	50653	1874161	69343957	2565726409	94931877133	37
38	1444	54872	2085136	79235168	3010936384	114415582592	38
39	1521	59319	2313441	90224199	3518743761	137231006679	39
40	1600	64000	2560000	102400000	4096000000	163840000000	40
41	1681	68921	2825761	115856201	4750104241	194754273881	41
42	1764	74088	3111696	130691232	5489031744	230539333248	42
43	1849	79507	3418801	147008443	6321363049	271818611107	43
44	1936	85184	3748096	164916224	7256313856	319277809664	44
45	2025	91125	4100625	184528125	8303765625	373669453125	45
46	2116	97336	4477456	205962976	9474296896	435817657216	46
47	2209	103823	4879681	229345007	10779215329	506623120463	47
48	2304	110592	5308416	254803968	12230590464	587068342272	48
49	2401	117649	5764801	282475249	13841287201	678223072849	49
50	2500	125000	6250000	312500000	15625000000	781250000000	50

TABLE XXVII.—(*continued*).

Powers of Natural Numbers.

n	n^2	n^3	n^4	n^5	n^6	n^7	n
51	2601	132651	6765201	345025251	17596287801	897410677851	51
52	2704	140608	7311616	380204032	19770609664	1028071702528	52
53	2809	148877	7890481	418195493	22164361129	1174711139837	53
54	2916	157464	8503056	459165024	24794911296	1338925209984	54
55	3025	166375	9150625	503284375	27680640625	1522435234375	55
56	3136	175616	9834496	550731776	30840979456	1727094849536	56
57	3249	185193	10556001	601692057	34296447249	1954897493193	57
58	3364	195112	11316496	656356768	38068692544	2207984167552	58
59	3481	205379	12117361	714924299	42180533641	2488651484819	59
60	3600	216000	12960000	777600000	46656000000	2799360000000	60
61	3721	226981	13845841	844596301	51520374361	3142742836021	61
62	3844	238328	14776336	916132832	56800235584	3521614606208	62
63	3969	250047	15752961	992436543	62523502209	3938980639167	63
64	4096	262144	16777216	1073741824	68719476736	4398046511104	64
65	4225	274625	17850625	1160290625	75418890625	4902227890625	65
66	4356	287496	18974736	1252332576	82653950016	5455160701056	66
67	4489	300763	20151121	1350125107	90458382169	6060711605323	67
68	4624	314432	21381376	1453933568	98867482624	6722988818432	68
69	4761	328509	22667121	1564031349	107918163081	7446353252589	69
70	4900	343000	24010000	1680700000	117649000000	8235430000000	70
71	5041	357911	25411681	1804229351	128100283921	9095120158391	71
72	5184	373248	26873856	1934917632	139314069504	10030613004288	72
73	5329	389017	28398241	2073071593	151334226289	11047398519097	73
74	5476	405224	29986576	2219006624	164206490176	12151280273024	74
75	5625	421875	31640625	2373046875	177978515625	13348388671875	75
76	5776	438976	33362176	2535525376	192699928576	14645194571776	76
77	5929	456533	35153041	2706784157	208422380089	16048523266853	77
78	6084	474552	37015056	2887174368	225199600704	17565568854912	78
79	6241	493039	38950081	3077056399	243087455521	19203908986159	79
80	6400	512000	40960000	3276800000	262144000000	20971520000000	80
81	6561	531441	43046721	3486784401	282429536481	22876792454961	81
82	6724	551368	45212176	3707398432	304006671424	24928547056768	82
83	6889	571787	47458321	3939040643	326940373369	27136050989627	83
84	7056	592704	49787136	4182119424	351298031616	29509034655744	84
85	7225	614125	52200625	4437053125	377149515625	32057708828125	85
86	7396	636056	54700816	4704270176	404567235136	34792782221696	86
87	7569	658503	57289761	4984209207	433626201009	37725479487783	87
88	7744	681472	59969536	5277319168	464404086784	40867559636992	88
89	7921	704969	62742241	5584059449	496981290961	44231334895529	89
90	8100	729000	65610000	5904900000	531441000000	47829690000000	90
91	8281	753571	68574961	6240321451	567869252041	51676101935731	91
92	8464	778688	71639296	6590815232	606355001344	55784660123648	92
93	8649	804357	74805201	6956883693	646990183449	60170087060757	93
94	8836	830584	78074896	7339040224	689869781056	64847759419264	94
95	9025	857375	81450625	7737809375	735091890625	69833729609375	95
96	9216	884736	84934656	8153726976	782757789696	75144747810816	96
97	9409	912673	88529281	8587340257	832972004929	80798284478113	97
98	9604	941192	92236816	9039207968	885842380864	86812553324672	98
99	9801	970299	96059601	9509900499	941480149401	93206534790699	99
100	10000	1000000	100000000	10000000000	1000000000000	100000000000000	100

TABLE XXVIII.

Sums of Powers of Natural Numbers.

n	$S(n)$	$S(n^2)$	$S(n^3)$	$S(n^4)$	$S(n^5)$	$S(n^6)$	$S(n^7)$	n
1	1	1	1	1	1	1	1	*1*
2	3	5	9	17	33	65	129	*2*
3	6	14	36	98	276	794	2316	*3*
4	10	30	100	354	1300	4890	18700	*4*
5	15	55	225	979	4425	20515	96825	*5*
6	21	91	441	2275	12201	67171	376761	*6*
7	28	140	784	4676	29008	184820	1200304	*7*
8	36	204	1296	8772	61776	446964	3297456	*8*
9	45	285	2025	15333	120825	978405	8080425	*9*
10	55	385	3025	25333	220825	1978405	18080425	*10*
11	66	506	4356	39974	381876	3749966	37567596	*11*
12	78	650	6084	60710	630708	6735950	73399404	*12*
13	91	819	8281	89271	1002001	11562759	136147921	*13*
14	105	1015	11025	127687	1539825	19092295	241561425	*14*
15	120	1240	14400	178312	2299200	30482920	412420800	*15*
16	136	1496	18496	243848	3347776	47260136	680856256	*16*
17	153	1785	23409	327369	4767633	71397705	1091194929	*17*
18	171	2109	29241	432345	6657201	105409929	1703414961	*18*
19	190	2470	36100	562666	9133300	152455810	2597286700	*19*
20	210	2870	44100	722666	12333300	216455810	3877286700	*20*
21	231	3311	53361	917147	16417401	302221931	5678375241	*21*
22	253	3795	64009	1151403	21571033	415601835	8172733129	*22*
23	276	4324	76176	1431244	28007376	563637724	11577558576	*23*
24	300	4900	90000	1763020	35970000	754740700	16164030000	*24*
25	325	5525	105625	2153645	45735625	998881325	22267545625	*25*
26	351	6201	123201	2610621	57617001	1307797101	30299355801	*26*
27	378	6930	142884	3142062	71965908	1695217590	40759709004	*27*
28	406	7714	164836	3756718	89176276	2177107894	54252637516	*28*
29	435	8555	189225	4463999	109687425	2771931215	71502513825	*29*
30	465	9455	216225	5273999	133987425	3500931215	93372513825	*30*
31	496	10416	246016	6197520	162616576	4388434896	120885127936	*31*
32	528	11440	278784	7246096	196171008	5462176720	155244866304	*32*
33	561	12529	314721	8432017	135306401	6753644689	197863309281	*33*
34	595	13685	354025	9768353	180741825	8298449105	250386659425	*34*
35	630	14910	396900	11268978	333263700	10136714730	314725956300	*35*
36	666	16206	443556	12948594	393729876	12313497066	393090120396	*36*
37	703	17575	494209	14822755	463073833	14879223475	488021997529	*37*
38	741	19019	549081	16907891	542309001	17890159859	602437580121	*38*
39	780	20540	608400	19221332	632533200	21408903620	739668586800	*39*
40	820	22140	672400	21781332	734933200	25504903620	903508586800	*40*
41	861	23821	741321	24607093	850789401	30255007861	1098262860681	*41*
42	903	25585	815409	27718789	981480633	35744039605	1328802193929	*42*
43	946	27434	894916	31137590	1128489076	42065402654	1600620805036	*43*
44	990	29370	980100	34885686	1293405300	49321716510	1919898614700	*44*
45	1035	31395	1071225	38986311	1477933425	57625482135	2293568067825	*45*
46	1081	33511	1168561	43463767	1683896401	67099779031	2729385725041	*46*
47	1128	35720	1272384	48343448	1913241408	77878994360	3236008845504	*47*
48	1176	38024	1382976	53651864	2168045376	90109584824	3823077187776	*48*
49	1225	40425	1500625	59416665	2450520625	103950872025	4501300260625	*49*
50	1275	42925	1625625	65666665	2763020625	119575872025	5282550260625	*50*

TABLE XXVIII.—(*continued*).

Sums of Powers of Natural Numbers.

n	$S(n)$	$S(n^2)$	$S(n^3)$	$S(n^4)$	$S(n^5)$	$S(n^6)$	$S(n^7)$	n
51	1326	45526	1758276	72431866	3108045876	137172159826	6179960938476	51
52	1378	48230	1898884	79743482	3488249908	156942769490	7208032641004	52
53	1431	51039	2047761	87633963	3906445401	179107130619	8382743780841	53
54	1485	53955	2205225	96137019	4365610425	203902041915	9721668990825	54
55	1540	56980	2371600	105287644	4868894800	231582682540	11244104225200	55
56	1596	60116	2547216	115122140	5419626576	262423661996	12971199074736	56
57	1653	63365	2732409	125678141	6021318633	296720109245	14926096567929	57
58	1711	66729	2927521	136994637	6677675401	334788801789	17134080735481	58
59	1770	70210	3132900	149111998	7392599700	376969335430	19622732220300	59
60	1830	73810	3348900	162071998	8170199700	423625335430	22422092220300	60
61	1891	77531	3575881	175917839	9014796001	475145709791	25564835056321	61
62	1953	81375	3814209	190694175	9930928833	531945945375	29086449662529	62
63	2016	85344	4064256	206447136	10923365376	594469447584	33025430301696	63
64	2080	89440	4326400	223224352	11997107200	663188924320	37423476812800	64
65	2145	93665	4601025	241074977	13157397825	738607814945	42325704703425	65
66	2211	98021	4888521	260049713	14409730401	821261764961	47780865404481	66
67	2278	102510	5189284	280200834	15759855508	911720147130	53841577009804	67
68	2346	107134	5503716	301582210	17213789076	1010587629754	60564565828236	68
69	2415	111895	5832225	324249331	18777820425	1118505792835	68010919080825	69
70	2485	116795	6175225	348259331	20458520425	1236154792835	76246349080825	70
71	2556	121836	6533136	373671012	22262749776	1364255076756	85341469239216	71
72	2628	127020	6906384	400544868	24197667408	1503569146260	95372082243504	72
73	2701	132349	7295401	428943109	26270739001	1654903372549	106419480762601	73
74	2775	137825	7700625	458929685	28489745625	1819109862725	118570761035625	74
75	2850	143450	8122500	490570310	30862792500	1997088378350	131919149707500	75
76	2926	149226	8561476	523932486	33398317876	2189788306926	146564344279276	76
77	3003	155155	9018009	559085527	36105102033	2398210687015	162612867546129	77
78	3081	161239	9492561	596100583	38992276401	2623410287719	180178436401041	78
79	3160	167480	9985600	635050664	42069332800	2866497743240	199382345387200	79
80	3240	173880	10497600	676010664	45346132800	3128641743240	220353865387200	80
81	3321	180441	11029041	719057385	48832917201	3411071279721	243230657842161	81
82	3403	187165	11580409	764269561	52540315633	3715077951145	268159204898929	82
83	3486	194054	12152196	811727882	56479356276	4042018324514	295295255888556	83
84	3570	201110	12744900	861515018	60661475700	4393316356130	324804290544300	84
85	3655	208335	13359025	913715643	65098528825	4770465871755	356861999372425	85
86	3741	215731	13995081	968416459	69802799001	5175033106891	391654781594121	86
87	3828	223300	14653584	1025706220	74787008208	5608659307900	429380261081904	87
88	3916	231044	15335056	1085675756	80064327376	6073063394684	470247820718896	88
89	4005	238965	16040025	1148417997	85648386825	6570044685645	514479155614425	89
90	4095	247065	16769025	1214027997	91553286825	7101485685645	562308845614425	90
91	4186	255346	17522596	1282602958	97793608276	7669354937686	613984947550156	91
92	4278	263810	18301284	1354242254	104384423508	8275709939030	669769607673804	92
93	4371	272459	19105641	1429047455	111341307201	8922700122479	729939694734561	93
94	4465	281295	19936225	1507122351	118680347425	9612569903535	794787454153825	94
95	4560	290320	20793600	1588572976	126418156800	10347661794160	864621183763200	95
96	4656	299536	21678336	1673507632	134571883776	11130419583856	939765931574016	96
97	4753	308945	22591009	1762036913	143159224033	11963391588785	1020564216052129	97
98	4851	318549	23532201	1854273729	152198432001	12849233969649	1107376769376801	98
99	4950	328350	24502500	1950333330	161708332500	13790714119050	1200583304167500	99
100	5050	338350	25502500	2050333330	171708332500	14790714119050	1300583304167500	100

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·001	·00337	·00736	+·01175	+·01391	+·01134	+·00415	3·09023
·002	·00634	·01290	+·01885	+·01968	+·01269	+·00053	2·87816
·003	·00915	·01778	+·02446	+·02335	+·01228	−·00328	2·74778
·004	·01185	·02222	+·02918	+·02587	+·01111	−·00687	2·65207
·005	·01446	·02634	+·03326	+·02764	+·00952	−·01017	2·57583
·006	·01700	·03020	+·03686	+·02887	+·00770	−·01318	2·51214
·007	·01949	·03386	+·04008	+·02970	+·00575	−·01592	2·45726
·008	·02192	·03734	+·04298	+·03021	+·00371	−·01841	2·40892
·009	·02431	·04066	+·04561	+·03047	+·00164	−·02067	2·36562
·010	·02665	·04384	+·04800	+·03053	−·00044	−·02271	2·32635
·011	·02896	·04690	+·05020	+·03041	−·00253	−·02457	2·29037
·012	·03123	·04985	+·05221	+·03014	−·00460	−·02625	2·25713
·013	·03348	·05270	+·05406	+·02975	−·00664	−·02777	2·22621
·014	·03569	·05545	+·05577	+·02926	−·00866	−·02914	2·19729
·015	·03787	·05811	+·05735	+·02867	−·01064	−·03037	2·17009
·016	·04003	·06069	+·05880	+·02801	−·01259	−·03147	2·14441
·017	·04216	·06320	+·06015	+·02727	−·01449	−·03246	2·12007
·018	·04427	·06564	+·06139	+·02647	−·01636	−·03334	2·09693
·019	·04635	·06801	+·06254	+·02562	−·01818	−·03411	2·07485
·020	·04842	·07031	+·06361	+·02472	−·01996	−·03479	2·05375
·021	·05046	·07256	+·06459	+·02378	−·02170	−·03538	2·03352
·022	·05249	·07475	+·06549	+·02280	−·02340	−·03589	2·01409
·023	·05449	·07688	+·06633	+·02179	−·02505	−·03632	1·99539
·024	·05648	·07897	+·06709	+·02074	−·02666	−·03667	1·97737
·025	·05845	·08100	+·06780	+·01968	−·02823	−·03696	1·95996
·026	·06040	·08299	+·06844	+·01858	−·02976	−·03718	1·94313
·027	·06233	·08493	+·06903	+·01747	−·03125	−·03734	1·92684
·028	·06425	·08682	+·06956	+·01634	−·03270	−·03744	1·91104
·029	·06615	·08868	+·07005	+·01520	−·03411	−·03749	1·89570
·030	·06804	·09049	+·07048	+·01404	−·03547	−·03749	1·88079
·031	·06992	·09227	+·07087	+·01287	−·03680	−·03744	1·86630
·032	·07177	·09400	+·07122	+·01168	−·03810	−·03734	1·85218
·033	·07362	·09570	+·07153	+·01049	−·03935	−·03720	1·83842
·034	·07545	·09737	+·07179	+·00929	−·04057	−·03702	1·82501
·035	·07727	·09900	+·07202	+·00809	−·04176	−·03680	1·81191
·036	·07908	·10060	+·07221	+·00688	−·04291	−·03654	1·79912
·037	·08087	·10216	+·07237	+·00566	−·04402	−·03624	1·78661
·038	·08265	·10370	+·07249	+·00444	−·04510	−·03592	1·77438
·039	·08442	·10520	+·07258	+·00322	−·04615	−·03556	1·76241
·040	·08617	·10668	+·07264	+·00200	−·04717	−·03517	1·75069
·041	·08792	·10812	+·07268	+·00077	−·04815	−·03475	1·73920
·042	·08965	·10954	+·07268	−·00045	−·04910	−·03431	1·72793
·043	·09137	·11093	+·07266	−·00167	−·05003	−·03384	1·71689
·044	·09309	·11229	+·07261	−·00290	−·05092	−·03335	1·70604
·045	·09479	·11363	+·07253	−·00412	−·05178	−·03283	1·69540
·046	·09648	·11495	+·07243	−·00534	−·05261	−·03229	1·68494
·047	·09816	·11623	+·07231	−·00656	−·05342	−·03173	1·67466
·048	·09983	·11750	+·07217	−·00778	−·05420	−·03115	1·66456
·049	·10149	·11874	+·07200	−·00899	−·05495	−·03055	1·65463
·050	·10311	·11996	+·07181	−·01020	−·05567	−·02994	1·64485

τ_1, τ_2 and h are essentially positive.

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·051	·10478	·12115	+·07160	−·01140	−·05637	−·02931	1·63523
·052	·10641	·12232	+·07138	−·01260	−·05704	−·02866	1·62576
·053	·10803	·12347	+·07113	−·01380	−·05769	−·02799	1·61644
·054	·10964	·12460	+·07087	−·01499	−·05831	−·02732	1·60725
·055	·11124	·12571	+·07058	−·01618	−·05891	−·02662	1·59819
·056	·11284	·12680	+·07028	−·01736	−·05949	−·02592	1·58927
·057	·11442	·12787	+·06997	−·01854	−·06004	−·02520	1·58047
·058	·11600	·12892	+·06964	−·01971	−·06057	−·02447	1·57179
·059	·11756	·12995	+·06929	−·02087	−·06107	−·02373	1·56322
·060	·11912	·13096	+·06893	−·02203	−·06155	−·02298	1·55477
·061	·12067	·13196	+·06855	−·02318	−·06202	−·02222	1·54643
·062	·12222	·13293	+·06816	−·02433	−·06246	−·02145	1·53820
·063	·12375	·13389	+·06775	−·02547	−·06288	−·02068	1·53007
·064	·12528	·13483	+·06734	−·02660	−·06328	−·01989	1·52204
·065	·12679	·13575	+·06690	−·02773	−·06365	−·01910	1·51410
·066	·12830	·13666	+·06646	−·02884	−·06401	−·01830	1·50626
·067	·12981	·13754	+·06601	−·02996	−·06435	−·01749	1·49851
·068	·13130	·13842	+·06554	−·03106	−·06467	−·01668	1·49085
·069	·13279	·13927	+·06506	−·03216	−·06498	−·01586	1·48328
·070	·13427	·14011	+·06457	−·03325	−·06526	−·01504	1·47579
·071	·13574	·14094	+·06407	−·03433	−·06552	−·01421	1·46838
·072	·13720	·14175	+·06356	−·03541	−·06577	−·01337	1·46106
·073	·13866	·14254	+·06304	−·03648	−·06600	−·01253	1·45381
·074	·14011	·14332	+·06251	−·03754	−·06621	−·01169	1·44663
·075	·14156	·14409	+·06197	−·03859	−·06641	−·01085	1·43953
·076	·14299	·14484	+·06142	−·03963	−·06659	−·01000	1·43250
·077	·14442	·14558	+·06086	−·04067	−·06675	−·00915	1·42554
·078	·14584	·14630	+·06029	−·04170	−·06690	−·00829	1·41865
·079	·14726	·14701	+·05971	−·04272	−·06703	−·00743	1·41183
·080	·14867	·14771	+·05913	−·04374	−·06715	−·00658	1·40507
·081	·15007	·14839	+·05854	−·04474	−·06725	−·00572	1·39838
·082	·15146	·14906	+·05794	−·04574	−·06733	−·00485	1·39174
·083	·15285	·14971	+·05733	−·04673	−·06741	−·00399	1·38517
·084	·15423	·15036	+·05671	−·04771	−·06746	−·00312	1·37866
·085	·15561	·15099	+·05609	−·04869	−·06751	−·00226	1·37220
·086	·15698	·15160	+·05546	−·04965	−·06753	−·00139	1·36581
·087	·15834	·15221	+·05483	−·05061	−·06755	−·00053	1·35946
·088	·15970	·15280	+·05418	−·05156	−·06755	+·00034	1·35317
·089	·16105	·15339	+·05353	−·05250	−·06754	+·00120	1·34694
·090	·16239	·15396	+·05288	−·05344	−·06751	+·00207	1·34076
·091	·16373	·15451	+·05222	−·05436	−·06748	+·00294	1·33462
·092	·16506	·15506	+·05155	−·05528	−·06743	+·00380	1·32854
·093	·16639	·15560	+·05088	−·05619	−·06736	+·00467	1·32251
·094	·16770	·15612	+·05020	−·05709	−·06729	+·00553	1·31652
·095	·16902	·15663	+·04952	−·05798	−·06720	+·00639	1·31058
·096	·17033	·15713	+·04883	−·05887	−·06710	+·00725	1·30469
·097	·17163	·15763	+·04813	−·05975	−·06699	+·00811	1·29884
·098	·17292	·15811	+·04744	−·06061	−·06687	+·00897	1·29303
·099	·17421	·15858	+·04673	−·06148	−·06674	+·00982	1·28727
·100	·17550	·15904	+·04602	−·06233	−·06660	+·01068	1·28155

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-\alpha)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·101	·17678	·15948	+ ·04531	− ·06317	− ·06644	+ ·01153	1·27587
·102	·17805	·15992	+ ·04459	− ·06401	− ·06628	+ ·01238	1·27024
·103	·17932	·16035	+ ·04387	− ·06484	− ·06610	+ ·01322	1·26464
·104	·18058	·16077	+ ·04315	− ·06566	− ·06592	+ ·01407	1·25908
·105	·18184	·16118	+ ·04242	− ·06647	− ·06572	+ ·01491	1·25357
·106	·18309	·16158	+ ·04169	− ·06727	− ·06551	+ ·01575	1·24808
·107	·18433	·16197	+ ·04095	− ·06807	− ·06530	+ ·01659	1·24264
·108	·18557	·16235	+ ·04021	− ·06886	− ·06507	+ ·01742	1·23723
·109	·18681	·16272	+ ·03947	− ·06964	− ·06484	+ ·01825	1·23186
·110	·18804	·16308	+ ·03872	− ·07041	− ·06459	+ ·01908	1·22653
·111	·18926	·16343	+ ·03797	− ·07117	− ·06434	+ ·01990	1·22123
·112	·19048	·16378	+ ·03721	− ·07193	− ·06408	+ ·02072	1·21596
·113	·19169	·16411	+ ·03646	− ·07268	− ·06381	+ ·02154	1·21073
·114	·19290	·16443	+ ·03570	− ·07342	− ·06353	+ ·02235	1·20553
·115	·19410	·16475	+ ·03493	− ·07415	− ·06324	+ ·02316	1·20036
·116	·19530	·16506	+ ·03417	− ·07488	− ·06294	+ ·02397	1·19522
·117	·19649	·16536	+ ·03340	− ·07559	− ·06264	+ ·02477	1·19012
·118	·19768	·16565	+ ·03263	− ·07630	− ·06233	+ ·02557	1·18504
·119	·19886	·16593	+ ·03186	− ·07700	− ·06201	+ ·02636	1·18000
·120	·20004	·16620	+ ·03108	− ·07770	− ·06168	+ ·02716	1·17499
·121	·20121	·16647	+ ·03030	− ·07838	− ·06134	+ ·02794	1·17000
·122	·20238	·16672	+ ·02952	− ·07906	− ·06100	+ ·02873	1·16505
·123	·20354	·16697	+ ·02874	− ·07973	− ·06065	+ ·02950	1·16012
·124	·20470	·16721	+ ·02796	− ·08039	− ·06029	+ ·03028	1·15522
·125	·20585	·16745	+ ·02717	− ·08105	− ·05992	+ ·03105	1·15035
·126	·20700	·16767	+ ·02638	− ·08169	− ·05955	+ ·03181	1·14551
·127	·20814	·16789	+ ·02559	− ·08233	− ·05917	+ ·03257	1·14069
·128	·20928	·16810	+ ·02480	− ·08297	− ·05878	+ ·03333	1·13590
·129	·21042	·16830	+ ·02401	− ·08359	− ·05839	+ ·03408	1·13113
·130	·21155	·16849	+ ·02321	− ·08421	− ·05799	+ ·03483	1·12639
·131	·21267	·16868	+ ·02241	− ·08482	− ·05758	+ ·03557	1·12168
·132	·21379	·16886	+ ·02162	− ·08542	− ·05717	+ ·03631	1·11699
·133	·21490	·16903	+ ·02082	− ·08601	− ·05675	+ ·03704	1·11232
·134	·21601	·16919	+ ·02001	− ·08660	− ·05632	+ ·03777	1·10768
·135	·21712	·16935	+ ·01921	− ·08718	− ·05589	+ ·03850	1·10306
·136	·21822	·16950	+ ·01841	− ·08775	− ·05546	+ ·03921	1·09847
·137	·21932	·16964	+ ·01760	− ·08831	− ·05501	+ ·03993	1·09390
·138	·22041	·16978	+ ·01680	− ·08887	− ·05456	+ ·04064	1·08935
·139	·22149	·16990	+ ·01599	− ·08942	− ·05411	+ ·04134	1·08482
·140	·22258	·17003	+ ·01518	− ·08996	− ·05365	+ ·04204	1·08032
·141	·22365	·17014	+ ·01437	− ·09050	− ·05318	+ ·04273	1·07584
·142	·22473	·17025	+ ·01356	− ·09103	− ·05271	+ ·04342	1·07138
·143	·22580	·17035	+ ·01275	− ·09155	− ·05224	+ ·04410	1·06694
·144	·22686	·17044	+ ·01194	− ·09206	− ·05176	+ ·04478	1·06252
·145	·22792	·17053	+ ·01113	− ·09257	− ·05127	+ ·04545	1·05812
·146	·22898	·17061	+ ·01032	− ·09307	− ·05078	+ ·04612	1·05374
·147	·23003	·17069	+ ·00950	− ·09356	− ·05028	+ ·04678	1·04939
·148	·23108	·17076	+ ·00869	− ·09405	− ·04978	+ ·04744	1·04505
·149	·23212	·17082	+ ·00788	− ·09452	− ·04928	+ ·04809	1·04073
·150	·23316	·17087	+ ·00706	− ·09499	− ·04877	+ ·04874	1·03643

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·151	·23419	·17092	+ ·00625	− ·09546	− ·04825	+ ·04938	1·03215
·152	·23522	·17097	+ ·00543	− ·09592	− ·04774	+ ·05002	1·02789
·153	·23625	·17100	+ ·00462	− ·09637	− ·04721	+ ·05065	1·02365
·154	·23727	·17103	+ ·00380	− ·09681	− ·04669	+ ·05127	1·01943
·155	·23829	·17106	+ ·00298	− ·09725	− ·04615	+ ·05189	1·01522
·156	·23930	·17108	+ ·00217	− ·09768	− ·04562	+ ·05250	1·01103
·157	·24031	·17109	+ ·00135	− ·09810	− ·04508	+ ·05311	1·00686
·158	·24131	·17110	+ ·00053	− ·09852	− ·04454	+ ·05371	1·00271
·159	·24232	·17110	− ·00028	− ·09892	− ·04399	+ ·05431	·99858
·160	·24331	·17109	− ·00110	− ·09933	− ·04344	+ ·05490	·99446
·161	·24430	·17108	− ·00191	− ·09972	− ·04288	+ ·05549	·99036
·162	·24529	·17107	− ·00273	− ·10011	− ·04232	+ ·05607	·98627
·163	·24628	·17104	− ·00355	− ·10049	− ·04176	+ ·05664	·98220
·164	·24726	·17102	− ·00436	− ·10087	− ·04120	+ ·05721	·97815
·165	·24823	·17098	− ·00518	− ·10124	− ·04063	+ ·05778	·97411
·166	·24921	·17094	− ·00599	− ·10160	− ·04006	+ ·05834	·97009
·167	·25017	·17090	− ·00681	− ·10196	− ·03948	+ ·05889	·96609
·168	·25114	·17085	− ·00762	− ·10231	− ·03890	+ ·05943	·96210
·169	·25210	·17080	− ·00844	− ·10265	− ·03832	+ ·05998	·95812
·170	·25305	·17073	− ·00925	− ·10299	− ·03774	+ ·06051	·95417
·171	·25401	·17067	− ·01007	− ·10332	− ·03715	+ ·06104	·95022
·172	·25495	·17060	− ·01088	− ·10364	− ·03656	+ ·06156	·94629
·173	·25590	·17052	− ·01169	− ·10396	− ·03597	+ ·06208	·94238
·174	·25684	·17044	− ·01251	− ·10427	− ·03537	+ ·06260	·93848
·175	·25778	·17035	− ·01332	− ·10458	− ·03478	+ ·06310	·93459
·176	·25871	·17026	− ·01413	− ·10487	− ·03417	+ ·06360	·93072
·177	·25964	·17016	− ·01494	− ·10517	− ·03357	+ ·06410	·92686
·178	·26056	·17006	− ·01575	− ·10545	− ·03296	+ ·06459	·92301
·179	·26148	·16995	− ·01656	− ·10573	− ·03236	+ ·06507	·91918
·180	·26240	·16984	− ·01737	− ·10601	− ·03175	+ ·06555	·91537
·181	·26331	·16972	− ·01817	− ·10627	− ·03113	+ ·06603	·91156
·182	·26422	·16960	− ·01898	− ·10653	− ·03052	+ ·06649	·90777
·183	·26513	·16948	− ·01979	− ·10679	− ·02990	+ ·06695	·90399
·184	·26603	·16934	− ·02059	− ·10704	− ·02928	+ ·06741	·90023
·185	·26693	·16921	− ·02140	− ·10728	− ·02866	+ ·06786	·89647
·186	·26782	·16907	− ·02220	− ·10752	− ·02803	+ ·06830	·89273
·187	·26871	·16892	− ·02300	− ·10775	− ·02741	+ ·06874	·88901
·188	·26960	·16877	− ·02380	− ·10798	− ·02678	+ ·06917	·88529
·189	·27049	·16861	− ·02460	− ·10819	− ·02615	+ ·06960	·88159
·190	·27137	·16845	− ·02540	− ·10841	− ·02552	+ ·07002	·87790
·191	·27224	·16829	− ·02620	− ·10861	− ·02489	+ ·07044	·87422
·192	·27311	·16812	− ·02700	− ·10882	− ·02425	+ ·07085	·87055
·193	·27398	·16795	− ·02779	− ·10901	− ·02362	+ ·07125	·86689
·194	·27485	·16777	− ·02859	− ·10920	− ·02298	+ ·07165	·86325
·195	·27571	·16759	− ·02938	− ·10939	− ·02234	+ ·07204	·85962
·196	·27657	·16740	− ·03018	− ·10956	− ·02170	+ ·07243	·85600
·197	·27742	·16721	− ·03097	− ·10974	− ·02106	+ ·07281	·85239
·198	·27827	·16701	− ·03176	− ·10990	− ·02041	+ ·07319	·84879
·199	·27912	·16681	− ·03255	− ·11007	− ·01977	+ ·07356	·84520
·200	·27996	·16661	− ·03334	− ·11022	− ·01912	+ ·07392	·84162

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·201	·28080	·16640	− ·03412	− ·11037	− ·01848	+ ·07428	·83805
·202	·28164	·16619	− ·03491	− ·11051	− ·01783	+ ·07464	·83450
·203	·28247	·16597	− ·03569	− ·11065	− ·01718	+ ·07498	·83095
·204	·28330	·16575	− ·03648	− ·11079	− ·01653	+ ·07532	·82742
·205	·28413	·16553	− ·03726	− ·11091	− ·01587	+ ·07566	·82389
·206	·28495	·16530	− ·03804	− ·11104	− ·01522	+ ·07599	·82038
·207	·28577	·16506	− ·03882	− ·11115	− ·01457	+ ·07632	·81687
·208	·28658	·16483	− ·03959	− ·11126	− ·01391	+ ·07664	·81338
·209	·28739	·16459	− ·04037	− ·11137	− ·01326	+ ·07695	·80990
·210	·28820	·16434	− ·04114	− ·11147	− ·01260	+ ·07726	·80642
·211	·28901	·16409	− ·04192	− ·11157	− ·01194	+ ·07756	·80296
·212	·28981	·16384	− ·04269	− ·11166	− ·01129	+ ·07786	·79950
·213	·29060	·16358	− ·04346	− ·11174	− ·01063	+ ·07815	·79606
·214	·29140	·16332	− ·04423	− ·11182	− ·00997	+ ·07844	·79262
·215	·29219	·16305	− ·04499	− ·11189	− ·00931	+ ·07872	·78919
·216	·29298	·16279	− ·04576	− ·11196	− ·00865	+ ·07899	·78577
·217	·29376	·16251	− ·04652	− ·11203	− ·00799	+ ·07926	·78237
·218	·29454	·16224	− ·04728	− ·11208	− ·00733	+ ·07952	·77897
·219	·29532	·16196	− ·04804	− ·11214	− ·00667	+ ·07978	·77557
·220	·29609	·16167	− ·04880	− ·11218	− ·00600	+ ·08004	·77219
·221	·29686	·16139	− ·04956	− ·11223	− ·00534	+ ·08028	·76882
·222	·29763	·16110	− ·05031	− ·11226	− ·00468	+ ·08052	·76546
·223	·29840	·16080	− ·05107	− ·11230	− ·00402	+ ·08076	·76210
·224	·29916	·16050	− ·05182	− ·11233	− ·00335	+ ·08099	·75875
·225	·29991	·16020	− ·05257	− ·11235	− ·00269	+ ·08122	·75541
·226	·30067	·15990	− ·05332	− ·11237	− ·00203	+ ·08144	·75208
·227	·30142	·15959	− ·05406	− ·11238	− ·00136	+ ·08165	·74876
·228	·30216	·15927	− ·05481	− ·11239	− ·00070	+ ·08186	·74545
·229	·30291	·15896	− ·05555	− ·11239	− ·00004	+ ·08207	·74214
·230	·30365	·15864	− ·05629	− ·11239	+ ·00063	+ ·08226	·73885
·231	·30439	·15832	− ·05703	− ·11238	+ ·00129	+ ·08246	·73556
·232	·30512	·15799	− ·05777	− ·11237	+ ·00195	+ ·08265	·73228
·233	·30585	·15766	− ·05851	− ·11235	+ ·00262	+ ·08283	·72900
·234	·30658	·15733	− ·05924	− ·11233	+ ·00328	+ ·08301	·72574
·235	·30730	·15699	− ·05997	− ·11230	+ ·00394	+ ·08318	·72248
·236	·30802	·15665	− ·06070	− ·11227	+ ·00461	+ ·08334	·71923
·237	·30874	·15631	− ·06143	− ·11224	+ ·00527	+ ·08351	·71599
·238	·30945	·15596	− ·06215	− ·11220	+ ·00593	+ ·08366	·71275
·239	·31017	·15561	− ·06288	− ·11215	+ ·00659	+ ·08381	·70952
·240	·31087	·15526	− ·06360	− ·11210	+ ·00726	+ ·08396	·70630
·241	·31158	·15490	− ·06432	− ·11205	+ ·00792	+ ·08410	·70309
·242	·31228	·15454	− ·06504	− ·11199	+ ·00858	+ ·08423	·69988
·243	·31298	·15418	− ·06576	− ·11192	+ ·00924	+ ·08436	·69668
·244	·31367	·15382	− ·06647	− ·11185	+ ·00990	+ ·08449	·69349
·245	·31436	·15345	− ·06718	− ·11178	+ ·01056	+ ·08461	·69031
·246	·31505	·15308	− ·06789	− ·11170	+ ·01122	+ ·08472	·68713
·247	·31574	·15270	− ·06860	− ·11162	+ ·01188	+ ·08483	·68396
·248	·31642	·15232	− ·06931	− ·11154	+ ·01253	+ ·08494	·68080
·249	·31710	·15194	− ·07001	− ·11145	+ ·01319	+ ·08504	·67764
·250	·31778	·15156	− ·07071	− ·11135	+ ·01385	+ ·08513	·67449

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·251	·31845	·15117	− ·07141	− ·11125	+ ·01450	+ ·08522	·67135
·252	·31912	·15078	− ·07211	− ·11115	+ ·01516	+ ·08530	·66821
·253	·31979	·15039	− ·07280	− ·11104	+ ·01581	+ ·08538	·66508
·254	·32045	·14999	− ·07350	− ·11093	+ ·01647	+ ·08546	·66196
·255	·32111	·14959	− ·07419	− ·11081	+ ·01712	+ ·08553	·65884
·256	·32177	·14919	− ·07488	− ·11069	+ ·01777	+ ·08559	·65573
·257	·32242	·14879	− ·07557	− ·11056	+ ·01842	+ ·08565	·65262
·258	·32307	·14838	− ·07625	− ·11043	+ ·01907	+ ·08571	·64952
·259	·32372	·14797	− ·07693	− ·11030	+ ·01972	+ ·08575	·64643
·260	·32437	·14756	− ·07761	− ·11016	+ ·02037	+ ·08580	·64335
·261	·32501	·14714	− ·07829	− ·11002	+ ·02102	+ ·08584	·64027
·262	·32565	·14672	− ·07897	− ·10987	+ ·02166	+ ·08587	·63719
·263	·32628	·14630	− ·07964	− ·10972	+ ·02231	+ ·08590	·63412
·264	·32691	·14588	− ·08031	− ·10956	+ ·02295	+ ·08593	·63106
·265	·32754	·14545	− ·08098	− ·10940	+ ·02360	+ ·08595	·62801
·266	·32817	·14502	− ·08165	− ·10924	+ ·02424	+ ·08596	·62496
·267	·32879	·14459	− ·08231	− ·10907	+ ·02488	+ ·08597	·62191
·268	·32941	·14415	− ·08298	− ·10890	+ ·02552	+ ·08598	·61887
·269	·33003	·14372	− ·08364	− ·10873	+ ·02616	+ ·08598	·61584
·270	·33065	·14328	− ·08429	− ·10855	+ ·02680	+ ·08598	·61281
·271	·33126	·14283	− ·08495	− ·10837	+ ·02743	+ ·08597	·60979
·272	·33187	·14239	− ·08560	− ·10818	+ ·02807	+ ·08596	·60678
·273	·33247	·14194	− ·08625	− ·10799	+ ·02870	+ ·08594	·60376
·274	·33307	·14149	− ·08690	− ·10779	+ ·02933	+ ·08591	·60076
·275	·33367	·14104	− ·08755	− ·10759	+ ·02997	+ ·08589	·59776
·276	·33427	·14058	− ·08819	− ·10739	+ ·03060	+ ·08586	·59477
·277	·33486	·14012	− ·08883	− ·10718	+ ·03122	+ ·08582	·59178
·278	·33545	·13966	− ·08947	− ·10697	+ ·03185	+ ·08578	·58879
·279	·33604	·13920	− ·09011	− ·10676	+ ·03248	+ ·08573	·58581
·280	·33662	·13873	− ·09074	− ·10654	+ ·03310	+ ·08568	·58284
·281	·33720	·13826	− ·09137	− ·10632	+ ·03372	+ ·08563	·57987
·282	·33778	·13779	− ·09200	− ·10609	+ ·03434	+ ·08557	·57691
·283	·33836	·13732	− ·09263	− ·10587	+ ·03496	+ ·08551	·57395
·284	·33893	·13685	− ·09325	− ·10563	+ ·03558	+ ·08544	·57100
·285	·33950	·13637	− ·09388	− ·10540	+ ·03620	+ ·08536	·56805
·286	·34007	·13589	− ·09450	− ·10516	+ ·03681	+ ·08529	·56511
·287	·34063	·13541	− ·09511	− ·10491	+ ·03743	+ ·08521	·56217
·288	·34119	·13492	− ·09573	− ·10466	+ ·03804	+ ·08512	·55924
·289	·34175	·13443	− ·09634	− ·10441	+ ·03865	+ ·08503	·55631
·290	·34230	·13394	− ·09695	− ·10416	+ ·03926	+ ·08494	·55338
·291	·34286	·13345	− ·09756	− ·10390	+ ·03987	+ ·08484	·55047
·292	·34341	·13296	− ·09816	− ·10364	+ ·04047	+ ·08473	·54755
·293	·34395	·13246	− ·09876	− ·10337	+ ·04107	+ ·08463	·54464
·294	·34449	·13196	− ·09936	− ·10310	+ ·04168	+ ·08451	·54174
·295	·34503	·13146	− ·09996	− ·10283	+ ·04228	+ ·08440	·53884
·296	·34557	·13096	− ·10056	− ·10256	+ ·04287	+ ·08428	·53594
·297	·34611	·13046	− ·10115	− ·10228	+ ·04347	+ ·08415	·53305
·298	·34664	·12995	− ·10174	− ·10199	+ ·04407	+ ·08402	·53016
·299	·34717	·12944	− ·10233	− ·10171	+ ·04466	+ ·08389	·52728
·300	·34769	·12893	− ·10291	− ·10142	+ ·04525	+ ·08375	·52440

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·301	·34822	·12841	− ·10349	− ·10113	+ ·04584	+ ·08361	·52153
·302	·34874	·12790	− ·10407	− ·10083	+ ·04643	+ ·08347	·51866
·303	·34925	·12738	− ·10465	− ·10053	+ ·04701	+ ·08332	·51579
·304	·34977	·12686	− ·10522	− ·10023	+ ·04759	+ ·08316	·51293
·305	·35028	·12634	− ·10580	− ·09992	+ ·04818	+ ·08301	·51007
·306	·35079	·12581	− ·10636	− ·09961	+ ·04876	+ ·08284	·50722
·307	·35129	·12529	− ·10693	− ·09930	+ ·04933	+ ·08268	·50437
·308	·35180	·12476	− ·10750	− ·09899	+ ·04991	+ ·08251	·50153
·309	·35230	·12423	− ·10806	− ·09867	+ ·05048	+ ·08233	·49869
·310	·35279	·12370	− ·10862	− ·09834	+ ·05105	+ ·08216	·49585
·311	·35329	·12316	− ·10917	− ·09802	+ ·05162	+ ·08197	·49302
·312	·35378	·12263	− ·10973	− ·09769	+ ·05219	+ ·08179	·49019
·313	·35427	·12209	− ·11028	− ·09736	+ ·05276	+ ·08160	·48736
·314	·35475	·12155	− ·11082	− ·09703	+ ·05332	+ ·08140	·48454
·315	·35524	·12101	− ·11137	− ·09669	+ ·05388	+ ·08121	·48173
·316	·35572	·12046	− ·11191	− ·09635	+ ·05444	+ ·08101	·47891
·317	·35620	·11992	− ·11245	− ·09600	+ ·05500	+ ·08080	·47610
·318	·35667	·11937	− ·11299	− ·09566	+ ·05555	+ ·08059	·47330
·319	·35714	·11882	− ·11353	− ·09531	+ ·05610	+ ·08038	·47050
·320	·35761	·11827	− ·11406	− ·09495	+ ·05665	+ ·08016	·46770
·321	·35808	·11771	− ·11459	− ·09460	+ ·05720	+ ·07994	·46490
·322	·35854	·11716	− ·11512	− ·09424	+ ·05775	+ ·07972	·46211
·323	·35900	·11660	− ·11564	− ·09388	+ ·05829	+ ·07949	·45933
·324	·35946	·11604	− ·11616	− ·09351	+ ·05883	+ ·07926	·45654
·325	·35991	·11548	− ·11668	− ·09315	+ ·05937	+ ·07902	·45376
·326	·36037	·11492	− ·11720	− ·09278	+ ·05991	+ ·07878	·45099
·327	·36082	·11436	− ·11771	− ·09240	+ ·06044	+ ·07854	·44821
·328	·36126	·11379	− ·11822	− ·09203	+ ·06097	+ ·07829	·44544
·329	·36171	·11322	− ·11873	− ·09165	+ ·06150	+ ·07804	·44268
·330	·36215	·11265	− ·11923	− ·09127	+ ·06203	+ ·07779	·43991
·331	·36259	·11208	− ·11974	− ·09088	+ ·06255	+ ·07753	·43715
·332	·36302	·11151	− ·12024	− ·09049	+ ·06308	+ ·07727	·43440
·333	·36346	·11093	− ·12073	− ·09010	+ ·06360	+ ·07701	·43164
·334	·36389	·11036	− ·12123	− ·08971	+ ·06412	+ ·07674	·42889
·335	·36431	·10978	− ·12172	− ·08932	+ ·06463	+ ·07647	·42615
·336	·36474	·10920	− ·12221	− ·08892	+ ·06514	+ ·07620	·42340
·337	·36516	·10862	− ·12270	− ·08852	+ ·06565	+ ·07592	·42066
·338	·36558	·10804	− ·12318	− ·08811	+ ·06616	+ ·07564	·41793
·339	·36600	·10745	− ·12366	− ·08771	+ ·06667	+ ·07535	·41519
·340	·36641	·10687	− ·12414	− ·08730	+ ·06717	+ ·07507	·41246
·341	·36682	·10628	− ·12461	− ·08689	+ ·06767	+ ·07477	·40974
·342	·36723	·10569	− ·12509	− ·08647	+ ·06817	+ ·07448	·40701
·343	·36764	·10510	− ·12555	− ·08606	+ ·06867	+ ·07418	·40429
·344	·36804	·10451	− ·12602	− ·08564	+ ·06916	+ ·07388	·40157
·345	·36844	·10391	− ·12649	− ·08522	+ ·06965	+ ·07358	·39886
·346	·36884	·10332	− ·12695	− ·08479	+ ·07014	+ ·07327	·39614
·347	·36923	·10272	− ·12741	− ·08437	+ ·07062	+ ·07296	·39343
·348	·36962	·10212	− ·12786	− ·08394	+ ·07110	+ ·07264	·39073
·349	·37001	·10152	− ·12831	− ·08351	+ ·07158	+ ·07232	·38802
·350	·37010	·10092	− ·12876	− ·08307	+ ·07206	+ ·07200	·38532

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·351	·37078	·10032	− ·12921	− ·08264	+ ·07254	+ ·07168	·38262
·352	·37116	·09971	− ·12966	− ·08220	+ ·07301	+ ·07135	·37993
·353	·37154	·09911	− ·13010	− ·08176	+ ·07348	+ ·07102	·37723
·354	·37192	·09850	− ·13054	− ·08131	+ ·07395	+ ·07069	·37454
·355	·37229	·09789	− ·13097	− ·08087	+ ·07441	+ ·07035	·37186
·356	·37266	·09728	− ·13140	− ·08042	+ ·07487	+ ·07002	·36917
·357	·37303	·09667	− ·13183	− ·07997	+ ·07533	+ ·06967	·36649
·358	·37340	·09606	− ·13226	− ·07952	+ ·07579	+ ·06933	·36381
·359	·37376	·09544	− ·13269	− ·07906	+ ·07624	+ ·06898	·36113
·360	·37412	·09483	− ·13311	− ·07861	+ ·07669	+ ·06863	·35846
·361	·37447	·09421	− ·13353	− ·07815	+ ·07714	+ ·06827	·35579
·362	·37483	·09359	− ·13394	− ·07768	+ ·07758	+ ·06792	·35312
·363	·37518	·09297	− ·13436	− ·07722	+ ·07803	+ ·06756	·35045
·364	·37553	·09235	− ·13477	− ·07675	+ ·07847	+ ·06719	·34779
·365	·37588	·09173	− ·13517	− ·07629	+ ·07890	+ ·06683	·34513
·366	·37622	·09111	− ·13558	− ·07582	+ ·07934	+ ·06646	·34247
·367	·37656	·09048	− ·13598	− ·07534	+ ·07977	+ ·06609	·33981
·368	·37690	·08985	− ·13638	− ·07487	+ ·08020	+ ·06571	·33715
·369	·37724	·08923	− ·13677	− ·07439	+ ·08062	+ ·06534	·33450
·370	·37757	·08860	− ·13717	− ·07391	+ ·08105	+ ·06496	·33185
·371	·37790	·08797	− ·13756	− ·07343	+ ·08147	+ ·06458	·32921
·372	·37823	·08734	− ·13794	− ·07295	+ ·08188	+ ·06419	·32656
·373	·37855	·08671	− ·13833	− ·07246	+ ·08230	+ ·06380	·32392
·374	·37888	·08607	− ·13871	− ·07198	+ ·08271	+ ·06341	·32128
·375	·37920	·08544	− ·13909	− ·07149	+ ·08312	+ ·06302	·31864
·376	·37951	·08480	− ·13946	− ·07100	+ ·08352	+ ·06262	·31600
·377	·37983	·08416	− ·13984	− ·07050	+ ·08392	+ ·06222	·31337
·378	·38014	·08353	− ·14021	− ·07001	+ ·08432	+ ·06182	·31074
·379	·38045	·08289	− ·14057	− ·06951	+ ·08472	+ ·06142	·30811
·380	·38076	·08225	− ·14094	− ·06901	+ ·08512	+ ·06101	·30548
·381	·38106	·08160	− ·14130	− ·06851	+ ·08551	+ ·06061	·30286
·382	·38136	·08096	− ·14166	− ·06801	+ ·08589	+ ·06019	·30023
·383	·38166	·08032	− ·14201	− ·06750	+ ·08628	+ ·05978	·29761
·384	·38196	·07967	− ·14236	− ·06700	+ ·08666	+ ·05936	·29499
·385	·38225	·07903	− ·14271	− ·06649	+ ·08704	+ ·05895	·29237
·386	·38254	·07838	− ·14306	− ·06598	+ ·08742	+ ·05853	·28976
·387	·38283	·07773	− ·14340	− ·06547	+ ·08779	+ ·05810	·28715
·388	·38312	·07708	− ·14374	− ·06495	+ ·08816	+ ·05766	·28454
·389	·38340	·07643	− ·14408	− ·06444	+ ·08853	+ ·05725	·28193
·390	·38368	·07578	− ·14442	− ·06392	+ ·08889	+ ·05682	·27932
·391	·38396	·07513	− ·14475	− ·06340	+ ·08925	+ ·05638	·27671
·392	·38423	·07447	− ·14508	− ·06288	+ ·08961	+ ·05595	·27411
·393	·38451	·07382	− ·14540	− ·06236	+ ·08997	+ ·05551	·27151
·394	·38478	·07316	− ·14573	− ·06183	+ ·09032	+ ·05507	·26891
·395	·38504	·07251	− ·14604	− ·06131	+ ·09067	+ ·05463	·26631
·396	·38531	·07185	− ·14636	− ·06078	+ ·09101	+ ·05419	·26371
·397	·38557	·07119	− ·14668	− ·06025	+ ·09136	+ ·05374	·26112
·398	·38583	·07053	− ·14699	− ·05972	+ ·09170	+ ·05329	·25853
·399	·38609	·06987	− ·14730	− ·05919	+ ·09203	+ ·05284	·25594
·400	·38634	·06921	− ·14760	− ·05866	+ ·09237	+ ·05239	·25335

TABLE XXIX. *Tetrachoric Functions for Fourfold Correlation Tables.*

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·401	·38659	·06855	− ·14790	− ·05812	+ ·09270	+ ·05193	·25076
·402	·38684	·06789	− ·14820	− ·05758	+ ·09303	+ ·05148	·24817
·403	·38709	·06722	− ·14850	− ·05705	+ ·09335	+ ·05102	·24559
·404	·38734	·06656	− ·14879	− ·05651	+ ·09367	+ ·05056	·24301
·405	·38758	·06589	− ·14908	− ·05596	+ ·09399	+ ·05010	·24043
·406	·38782	·06522	− ·14937	− ·05542	+ ·09430	+ ·04963	·23785
·407	·38805	·06456	− ·14965	− ·05488	+ ·09462	+ ·04916	·23527
·408	·38829	·06389	− ·14993	− ·05433	+ ·09493	+ ·04869	·23269
·409	·38852	·06322	− ·15021	− ·05378	+ ·09523	+ ·04822	·23012
·410	·38875	·06255	− ·15049	− ·05323	+ ·09553	+ ·04775	·22754
·411	·38897	·06188	− ·15076	− ·05268	+ ·09583	+ ·04728	·22497
·412	·38920	·06121	− ·15103	− ·05213	+ ·09613	+ ·04680	·22240
·413	·38942	·06053	− ·15130	− ·05158	+ ·09642	+ ·04632	·21983
·414	·38964	·05986	− ·15156	− ·05102	+ ·09671	+ ·04584	·21727
·415	·38985	·05919	− ·15182	− ·05047	+ ·09700	+ ·04536	·21470
·416	·39007	·05851	− ·15208	− ·04991	+ ·09728	+ ·04488	·21214
·417	·39028	·05784	− ·15233	− ·04935	+ ·09756	+ ·04439	·20957
·418	·39049	·05716	− ·15258	− ·04879	+ ·09784	+ ·04390	·20701
·419	·39069	·05648	− ·15283	− ·04823	+ ·09811	+ ·04341	·20445
·420	·39089	·05580	− ·15308	− ·04767	+ ·09838	+ ·04292	·20189
·421	·39109	·05513	− ·15332	− ·04711	+ ·09865	+ ·04243	·19934
·422	·39129	·05445	− ·15356	− ·04654	+ ·09891	+ ·04194	·19678
·423	·39149	·05377	− ·15380	− ·04598	+ ·09918	+ ·04144	·19422
·424	·39168	·05309	− ·15403	− ·04541	+ ·09943	+ ·04094	·19167
·425	·39187	·05240	− ·15426	− ·04484	+ ·09969	+ ·04044	·18912
·426	·39206	·05172	− ·15449	− ·04427	+ ·09994	+ ·03994	·18657
·427	·39224	·05104	− ·15471	− ·04370	+ ·10019	+ ·03944	·18402
·428	·39243	·05036	− ·15493	− ·04313	+ ·10043	+ ·03894	·18147
·429	·39261	·04967	− ·15515	− ·04256	+ ·10067	+ ·03843	·17892
·430	·39279	·04899	− ·15537	− ·04198	+ ·10091	+ ·03793	·17637
·431	·39296	·04830	− ·15558	− ·04141	+ ·10115	+ ·03742	·17383
·432	·39313	·04761	− ·15579	− ·04083	+ ·10138	+ ·03691	·17128
·433	·39330	·04693	− ·15599	− ·04026	+ ·10161	+ ·03640	·16874
·434	·39347	·04624	− ·15620	− ·03968	+ ·10183	+ ·03589	·16620
·435	·39364	·04555	− ·15640	− ·03910	+ ·10205	+ ·03537	·16366
·436	·39380	·04486	− ·15659	− ·03852	+ ·10227	+ ·03486	·16112
·437	·39396	·04418	− ·15679	− ·03794	+ ·10249	+ ·03434	·15858
·438	·39411	·04349	− ·15698	− ·03735	+ ·10270	+ ·03382	·15604
·439	·39427	·04280	− ·15717	− ·03677	+ ·10291	+ ·03330	·15351
·440	·39442	·04211	− ·15735	− ·03619	+ ·10311	+ ·03278	·15097
·441	·39457	·04141	− ·15753	− ·03560	+ ·10331	+ ·03226	·14843
·442	·39472	·04072	− ·15771	− ·03502	+ ·10351	+ ·03174	·14590
·443	·39486	·04003	− ·15789	− ·03443	+ ·10371	+ ·03121	·14337
·444	·39501	·03934	− ·15806	− ·03384	+ ·10390	+ ·03069	·14084
·445	·39514	·03864	− ·15823	− ·03325	+ ·10409	+ ·03016	·13830
·446	·39528	·03795	− ·15840	− ·03266	+ ·10427	+ ·02963	·13577
·447	·39542	·03726	− ·15856	− ·03207	+ ·10446	+ ·02910	·13324
·448	·39555	·03656	− ·15872	− ·03148	+ ·10463	+ ·02858	·13072
·449	·39568	·03587	− ·15888	− ·03089	+ ·10481	+ ·02804	·12819
·450	·39580	·03517	− ·15904	− ·03030	+ ·10498	+ ·02751	·12566

TABLE XXIX.—(*continued*).

$\frac{1}{2}(1-a)$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	h
·451	·39593	·03447	− ·15919	− ·02970	+ ·10515	+ ·02698	·12314
·452	·39605	·03378	− ·15934	− ·02911	+ ·10532	+ ·02644	·12061
·453	·39617	·03308	− ·15948	− ·02851	+ ·10548	+ ·02591	·11809
·454	·39629	·03238	− ·15962	− ·02792	+ ·10564	+ ·02537	·11556
·455	·39640	·03168	− ·15976	− ·02732	+ ·10579	+ ·02484	·11304
·456	·39651	·03099	− ·15990	− ·02673	+ ·10594	+ ·02430	·11052
·457	·39662	·03029	− ·16003	− ·02613	+ ·10609	+ ·02376	·10799
·458	·39673	·02959	− ·16016	− ·02553	+ ·10624	+ ·02322	·10547
·459	·39683	·02889	− ·16029	− ·02493	+ ·10638	+ ·02268	·10295
·460	·39694	·02819	− ·16041	− ·02433	+ ·10652	+ ·02214	·10043
·461	·39703	·02749	− ·16053	− ·02373	+ ·10665	+ ·02159	·09791
·462	·39713	·02679	− ·16065	− ·02313	+ ·10678	+ ·02105	·09540
·463	·39723	·02609	− ·16077	− ·02253	+ ·10691	+ ·02051	·09288
·464	·39732	·02539	− ·16088	− ·02193	+ ·10704	+ ·01996	·09036
·465	·39741	·02469	− ·16099	− ·02132	+ ·10716	+ ·01941	·08784
·466	·39749	·02398	− ·16109	− ·02072	+ ·10727	+ ·01887	·08533
·467	·39758	·02328	− ·16120	− ·02012	+ ·10739	+ ·01832	·08281
·468	·39766	·02258	− ·16130	− ·01951	+ ·10750	+ ·01777	·08030
·469	·39774	·02188	− ·16139	− ·01891	+ ·10761	+ ·01722	·07778
·470	·39781	·02117	− ·16149	− ·01830	+ ·10771	+ ·01668	·07527
·471	·39789	·02047	− ·16158	− ·01770	+ ·10781	+ ·01613	·07276
·472	·39796	·01977	− ·16166	− ·01709	+ ·10791	+ ·01558	·07024
·473	·39803	·01906	− ·16175	− ·01648	+ ·10801	+ ·01502	·06773
·474	·39809	·01836	− ·16183	− ·01588	+ ·10810	+ ·01447	·06522
·475	·39816	·01765	− ·16191	− ·01527	+ ·10818	+ ·01392	·06271
·476	·39822	·01695	− ·16198	− ·01466	+ ·10827	+ ·01337	·06020
·477	·39828	·01625	− ·16206	− ·01405	+ ·10835	+ ·01281	·05768
·478	·39834	·01554	− ·16212	− ·01344	+ ·10842	+ ·01226	·05517
·479	·39839	·01484	− ·16219	− ·01284	+ ·10850	+ ·01171	·05266
·480	·39844	·01413	− ·16225	− ·01223	+ ·10857	+ ·01115	·05015
·481	·39849	·01342	− ·16231	− ·01162	+ ·10864	+ ·01060	·04764
·482	·39854	·01272	− ·16237	− ·01101	+ ·10870	+ ·01004	·04513
·483	·39858	·01201	− ·16242	− ·01040	+ ·10876	+ ·00949	·04263
·484	·39862	·01131	− ·16247	− ·00979	+ ·10882	+ ·00893	·04012
·485	·39866	·01060	− ·16252	− ·00918	+ ·10887	+ ·00837	·03761
·486	·39870	·00990	− ·16257	− ·00857	+ ·10892	+ ·00782	·03510
·487	·39873	·00919	− ·16261	− ·00796	+ ·10896	+ ·00726	·03259
·488	·39876	·00848	− ·16265	− ·00734	+ ·10901	+ ·00670	·03008
·489	·39879	·00778	− ·16268	− ·00673	+ ·10905	+ ·00614	·02758
·490	·39882	·00707	− ·16271	− ·00612	+ ·10908	+ ·00559	·02507
·491	·39884	·00636	− ·16274	− ·00551	+ ·10912	+ ·00503	·02256
·492	·39886	·00566	− ·16277	− ·00490	+ ·10914	+ ·00447	·02005
·493	·39888	·00495	− ·16279	− ·00429	+ ·10917	+ ·00391	·01755
·494	·39890	·00424	− ·16281	− ·00367	+ ·10919	+ ·00335	·01504
·495	·39891	·00354	− ·16283	− ·00306	+ ·10921	+ ·00279	·01253
·496	·39892	·00283	− ·16284	− ·00245	+ ·10923	+ ·00224	·01003
·497	·39893	·00212	− ·16285	− ·00184	+ ·10924	+ ·00168	·00752
·498	·39894	·00141	− ·16286	− ·00122	+ ·10925	+ ·00112	·00501
·499	·39894	·00071	− ·16287	− ·00061	+ ·10925	+ ·00056	·00251
·500	·39894	·00000	− ·16287	·00000	+ ·10925	·00000	·00000

TABLE XXX. *Supplementary Tables for determining High*

$r = {\cdot}80$.

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·3976	·3766	·3538	·3294	·3039	·2778	·2515	·2254	·2001	·1759	·1531	·1320	·1127
0·1	·3766	·3583	·3380	·3162	·2930	·2689	·2445	·2200	·1960	·1728	·1509	·1304	·1116
0·2	·3538	·3380	·3204	·3011	·2804	·2586	·2361	·2134	·1909	·1690	·1481	·1284	·1102
0·3	·3294	·3162	·3011	·2843	·2661	·2466	·2263	·2056	·1848	·1643	·1446	·1258	·1083
0·4	·3039	·2930	·2804	·2661	·2503	·2332	·2152	·1965	·1775	·1587	·1402	·1226	·1060
0·5	·2778	·2689	·2586	·2466	·2332	·2186	·2028	·1862	·1692	·1520	·1351	·1187	·1031
0·6	·2515	·2445	·2361	·2263	·2152	·2028	·1893	·1748	·1598	·1444	·1291	·1140	·0995
0·7	·2254	·2200	·2134	·2056	·1965	·1862	·1748	·1625	·1494	·1359	·1222	·1086	·0954
0·8	·2001	·1960	·1909	·1848	·1775	·1692	·1598	·1494	·1383	·1266	·1146	·1025	·0906
0·9	·1759	·1728	·1690	·1643	·1587	·1520	·1444	·1359	·1266	·1167	·1064	·0958	·0852
1·0	·1531	·1509	·1481	·1446	·1402	·1351	·1291	·1222	·1146	·1064	·0976	·0886	·0794
1·1	·1320	·1304	·1284	·1258	·1226	·1187	·1140	·1086	·1025	·0958	·0886	·0809	·0731
1·2	·1127	·1116	·1102	·1083	·1060	·1031	·0995	·0954	·0906	·0852	·0794	·0731	·0665
1·3	·0953	·0946	·0936	·0923	·0906	·0885	·0859	·0828	·0791	·0749	·0702	·0652	·0597
1·4	·0798	·0793	·0787	·0778	·0766	·0751	·0733	·0710	·0682	·0650	·0614	·0574	·0530
1·5	·0662	·0659	·0655	·0649	·0641	·0631	·0618	·0601	·0581	·0557	·0529	·0498	·0464
1·6	·0545	·0543	·0540	·0536	·0531	·0524	·0515	·0503	·0489	·0471	·0451	·0427	·0401
1·7	·0444	·0443	·0441	·0438	·0435	·0430	·0424	·0416	·0406	·0394	·0379	·0362	·0342
1·8	·0358	·0357	·0357	·0355	·0353	·0350	·0346	·0341	·0334	·0325	·0315	·0302	·0287
1·9	·0287	·0286	·0286	·0285	·0283	·0281	·0279	·0275	·0271	·0265	·0258	·0249	·0238
2·0	·0227	·0227	·0227	·0226	·0225	·0224	·0223	·0220	·0217	·0213	·0209	·0202	·0195
2·1	·0178	·0178	·0178	·0178	·0177	·0177	·0176	·0174	·0172	·0170	·0167	·0163	·0158
2·2	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0137	·0135	·0134	·0132	·0129	·0126
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104	·0103	·0101	·0099
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0080	·0079	·0078	·0077
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061	·0061	·0060	·0059
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046	·0046	·0045	·0045

$r = {\cdot}85$.

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·4117	·3905	·3670	·3417	·3149	·2873	·2595	·2319	·2052	·1798	·1560	·1341	·1141
0·1	·3905	·3723	·3518	·3292	·3050	·2796	·2537	·2277	·2022	·1777	·1546	·1332	·1136
0·2	·3670	·3518	·3342	·3145	·2930	·2702	·2464	·2222	·1983	·1749	·1527	·1319	·1127
0·3	·3417	·3292	·3145	·2978	·2791	·2588	·2374	·2154	·1931	·1712	·1501	·1301	·1116
0·4	·3149	·3050	·2930	·2791	·2632	·2457	·2268	·2070	·1867	·1665	·1467	·1277	·1099
0·5	·2873	·2796	·2702	·2588	·2457	·2309	·2146	·1972	·1790	·1606	·1423	·1246	·1078
0·6	·2595	·2537	·2464	·2374	·2268	·2146	·2008	·1859	·1700	·1535	·1370	·1206	·1049
0·7	·2319	·2277	·2222	·2154	·2070	·1972	·1859	·1733	·1597	·1453	·1306	·1158	·1014
0·8	·2052	·2022	·1983	·1931	·1867	·1790	·1700	·1597	·1483	·1360	·1232	·1101	·0971
0·9	·1798	·1777	·1749	·1712	·1665	·1606	·1535	·1453	·1360	·1258	·1149	·1035	·0920
1·0	·1560	·1546	·1527	·1501	·1467	·1423	·1370	·1306	·1232	·1149	·1058	·0962	·0862
1·1	·1341	·1332	·1319	·1301	·1277	·1246	·1206	·1158	·1101	·1035	·0962	·0882	·0798
1·2	·1141	·1136	·1127	·1116	·1099	·1078	·1049	·1014	·0971	·0920	·0862	·0798	·0729
1·3	·0963	·0959	·0954	·0947	·0936	·0921	·0901	·0876	·0845	·0807	·0763	·0712	·0656
1·4	·0805	·0803	·0800	·0795	·0788	·0778	·0765	·0748	·0725	·0698	·0665	·0626	·0583
1·5	·0666	·0665	·0664	·0661	·0656	·0650	·0642	·0630	·0615	·0595	·0571	·0543	·0510
1·6	·0547	·0547	·0546	·0544	·0541	·0538	·0532	·0525	·0514	·0501	·0484	·0464	·0439
1·7	·0445	·0445	·0444	·0443	·0442	·0440	·0436	·0432	·0425	·0416	·0405	·0390	·0373
1·8	·0359	·0359	·0359	·0358	·0357	·0356	·0354	·0351	·0347	·0341	·0334	·0324	·0312
1·9	·0287	·0287	·0287	·0287	·0286	·0285	·0284	·0283	·0280	·0276	·0272	·0265	·0257
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0224	·0221	·0218	·0214	·0209
2·1	·0179	·0179	·0179	·0178	·0178	·0178	·0178	·0177	·0176	·0175	·0173	·0171	·0167
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137	·0136	·0135	·0133
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106	·0106	·0105	·0104
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0081	·0081	·0081	·0080
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0061
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0046

Correlations from Tetrachoric Groupings.

$r = \cdot 80.$

h=	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
k=0·0	·0953	·0798	·0662	·0545	·0444	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·1	·0946	·0793	·0659	·0543	·0443	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·2	·0936	·0787	·0655	·0540	·0441	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·3	·0923	·0778	·0649	·0536	·0438	·0355	·0285	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0906	·0766	·0641	·0531	·0435	·0353	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·5	·0885	·0751	·0631	·0524	·0430	·0350	·0281	·0224	·0177	·0138	·0107	·0082	·0062	·0047
0·6	·0859	·0733	·0618	·0515	·0424	·0346	·0279	·0223	·0176	·0137	·0106	·0082	·0062	·0046
0·7	·0828	·0710	·0601	·0503	·0416	·0341	·0275	·0220	·0174	·0137	·0106	·0081	·0062	·0046
0·8	·0791	·0682	·0581	·0489	·0406	·0334	·0271	·0217	·0172	·0135	·0105	·0081	·0061	·0046
0·9	·0749	·0650	·0557	·0471	·0394	·0325	·0265	·0213	·0170	·0134	·0104	·0080	·0061	·0046
1·0	·0702	·0614	·0529	·0451	·0379	·0315	·0258	·0209	·0167	·0132	·0103	·0079	·0061	·0046
1·1	·0652	·0574	·0498	·0427	·0362	·0302	·0249	·0202	·0163	·0129	·0101	·0078	·0060	·0045
1·2	·0597	·0530	·0464	·0401	·0342	·0287	·0238	·0195	·0158	·0126	·0099	·0077	·0059	·0045
1·3	·0541	·0484	·0427	·0372	·0319	·0271	·0226	·0186	·0151	·0121	·0096	·0075	·0058	·0044
1·4	·0484	·0436	·0388	·0341	·0295	·0252	·0212	·0176	·0144	·0117	·0093	·0073	·0057	·0043
1·5	·0427	·0388	·0348	·0309	·0270	·0232	·0197	·0165	·0136	·0111	·0089	·0070	·0055	·0042
1·6	·0372	·0341	·0309	·0276	·0243	·0211	·0181	·0153	·0127	·0104	·0084	·0067	·0053	·0041
1·7	·0319	·0295	·0270	·0243	·0216	·0190	·0164	·0140	·0117	·0097	·0079	·0063	·0050	·0039
1·8	·0271	·0252	·0232	·0211	·0190	·0168	·0146	·0126	·0107	·0089	·0073	·0059	·0047	·0037
1·9	·0226	·0212	·0197	·0181	·0164	·0146	·0129	·0112	·0096	·0081	·0067	·0055	·0044	·0035
2·0	·0186	·0176	·0165	·0153	·0140	·0126	·0112	·0098	·0085	·0072	·0060	·0050	·0040	·0032
2·1	·0151	·0144	·0136	·0127	·0117	·0107	·0096	·0085	·0074	·0064	·0054	·0045	·0037	·0030
2·2	·0121	·0117	·0111	·0104	·0097	·0089	·0081	·0072	·0064	·0055	·0047	·0040	·0033	·0027
2·3	·0096	·0093	·0089	·0084	·0079	·0073	·0067	·0060	·0054	·0047	·0041	·0035	·0029	·0024
2·4	·0075	·0073	·0070	·0067	·0063	·0059	·0055	·0050	·0045	·0040	·0035	·0030	·0025	·0021
2·5	·0058	·0057	·0055	·0053	·0050	·0047	·0044	·0040	·0037	·0033	·0029	·0025	·0022	·0018
2·6	·0044	·0043	·0042	·0041	·0039	·0037	·0035	·0032	·0030	·0027	·0024	·0021	·0018	·0016

$r = \cdot 85.$

h=	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6
k=0·0	·0963	·0805	·0666	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0959	·0803	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0954	·0800	·0664	·0546	·0444	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0947	·0795	·0661	·0544	·0443	·0358	·0287	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·4	·0936	·0788	·0656	·0541	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·5	·0921	·0778	·0650	·0538	·0440	·0356	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·6	·0901	·0765	·0642	·0532	·0436	·0354	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
0·7	·0876	·0748	·0630	·0525	·0432	·0351	·0283	·0225	·0177	·0138	·0107	·0082	·0062	·0047
0·8	·0845	·0725	·0615	·0514	·0425	·0347	·0280	·0224	·0176	·0138	·0107	·0082	·0062	·0047
0·9	·0807	·0698	·0595	·0501	·0416	·0341	·0276	·0221	·0175	·0137	·0106	·0081	·0062	·0046
1·0	·0763	·0665	·0571	·0484	·0405	·0334	·0272	·0218	·0173	·0136	·0106	·0081	·0062	·0046
1·1	·0712	·0626	·0543	·0464	·0390	·0324	·0265	·0214	·0171	·0135	·0105	·0081	·0061	·0046
1·2	·0656	·0583	·0510	·0439	·0373	·0312	·0257	·0209	·0167	·0133	·0104	·0080	·0061	·0046
1·3	·0597	·0535	·0473	·0411	·0352	·0297	·0247	·0202	·0163	·0130	·0102	·0079	·0060	·0046
1·4	·0535	·0485	·0432	·0380	·0329	·0280	·0234	·0194	·0157	·0126	·0100	·0078	·0060	·0045
1·5	·0473	·0432	·0390	·0346	·0302	·0260	·0220	·0183	·0150	·0121	·0097	·0076	·0058	·0045
1·6	·0411	·0380	·0346	·0311	·0274	·0239	·0204	·0172	·0142	·0116	·0093	·0073	·0057	·0044
1·7	·0352	·0329	·0302	·0274	·0245	·0216	·0186	·0159	·0133	·0109	·0088	·0070	·0055	·0043
1·8	·0297	·0280	·0260	·0239	·0216	·0192	·0168	·0144	·0122	·0102	·0083	·0067	·0053	·0041
1·9	·0247	·0234	·0220	·0204	·0186	·0168	·0149	·0129	·0111	·0093	·0077	·0063	·0050	·0039
2·0	·0202	·0194	·0183	·0172	·0159	·0144	·0129	·0114	·0099	·0084	·0070	·0058	·0047	·0037
2·1	·0163	·0157	·0150	·0142	·0133	·0122	·0111	·0099	·0087	·0075	·0063	·0053	·0043	·0034
2·2	·0130	·0126	·0121	·0116	·0109	·0102	·0093	·0084	·0075	·0065	·0056	·0047	·0039	·0032
2·3	·0102	·0100	·0097	·0093	·0088	·0083	·0077	·0070	·0063	·0056	·0049	·0042	·0035	·0029
2·4	·0079	·0078	·0076	·0073	·0070	·0067	·0063	·0058	·0053	·0047	·0042	·0036	·0031	·0025
2·5	·0060	·0060	·0058	·0057	·0055	·0053	·0050	·0047	·0043	·0039	·0035	·0031	·0026	·0022
2·6	·0046	·0045	·0045	·0044	·0043	·0041	·0039	·0037	·0034	·0032	·0029	·0025	·0022	·0019

TABLE XXX. *Supplementary Tables for determining High*

$r = \cdot 90$.

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·4282	·4067	·3822	·3552	·3266	·2969	·2670	·2377	·2094	·1827	·1579	·1353	·1149
0·1	·4067	·3887	·3678	·3441	·3183	·2910	·2630	·2350	·2077	·1817	·1574	·1350	·1147
0·2	·3822	·3678	·3504	·3302	·3076	·2830	·2573	·2311	·2052	·1801	·1564	·1345	·1144
0·3	·3552	·3441	·3302	·3135	·2943	·2728	·2498	·2258	·2016	·1778	·1550	·1336	·1140
0·4	·3266	·3183	·3076	·2943	·2784	·2602	·2401	·2187	·1966	·1744	·1528	·1322	·1132
0·5	·2969	·2910	·2830	·2728	·2602	·2453	·2284	·2097	·1900	·1698	·1497	·1302	·1119
0·6	·2670	·2630	·2573	·2498	·2401	·2284	·2145	·1988	·1817	·1637	·1454	·1274	·1101
0·7	·2377	·2350	·2311	·2258	·2187	·2097	·1988	·1860	·1717	·1561	·1399	·1236	·1075
0·8	·2094	·2077	·2052	·2016	·1966	·1900	·1817	·1717	·1600	·1470	·1331	·1186	·1041
0·9	·1827	·1817	·1801	·1778	·1744	·1698	·1637	·1561	·1470	·1365	·1249	·1124	·0997
1·0	·1579	·1574	·1564	·1550	·1528	·1497	·1454	·1399	·1331	·1249	·1155	·1052	·0942
1·1	·1353	·1350	·1345	·1336	·1322	·1302	·1274	·1236	·1186	·1124	·1052	·0969	·0878
1·2	·1149	·1147	·1144	·1140	·1132	·1119	·1101	·1075	·1041	·0997	·0942	·0878	·0806
1·3	·0967	·0966	·0965	·0962	·0958	·0950	·0939	·0923	·0900	·0869	·0830	·0783	·0727
1·4	·0807	·0807	·0806	·0805	·0802	·0798	·0792	·0782	·0767	·0747	·0720	·0686	·0645
1·5	·0668	·0668	·0667	·0667	·0665	·0663	·0660	·0654	·0645	·0632	·0614	·0591	·0562
1·6	·0548	·0548	·0548	·0547	·0547	·0546	·0544	·0540	·0535	·0528	·0516	·0501	·0481
1·7	·0446	·0446	·0446	·0445	·0445	·0445	·0444	·0442	·0439	·0435	·0428	·0418	·0405
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0356	·0353	·0350	·0344	·0338
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0286	·0284	·0282	·0279	·0274
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0226	·0225	·0223	·0220
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178	·0178	·0178	·0177	·0176	·0175
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0138	·0138	·0137
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0106
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

$r = \cdot 95$.

h=	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k=*0·0*	·4495	·4271	·4005	·3705	·3385	·3055	·2729	·2414	·2116	·1840	·1586	·1357	·1151
0·1	·4271	·4099	·3880	·3622	·3333	·3026	·2713	·2407	·2113	·1839	·1586	·1356	·1151
0·2	·4005	·3880	·3712	·3500	·3252	·2976	·2685	·2392	·2106	·1835	·1585	·1356	·1150
0·3	·3705	·3622	·3500	·3338	·3135	·2898	·2637	·2365	·2092	·1829	·1582	·1355	·1150
0·4	·3385	·3333	·3252	·3135	·2980	·2787	·2564	·2320	·2067	·1816	·1576	·1352	·1149
0·5	·3055	·3026	·2976	·2898	·2787	·2640	·2459	·2250	·2024	·1792	·1563	·1346	·1147
0·6	·2729	·2713	·2685	·2637	·2564	·2459	·2321	·2153	·1960	·1753	·1542	·1335	·1141
0·7	·2414	·2407	·2392	·2365	·2320	·2250	·2153	·2025	·1870	·1694	·1506	·1315	·1131
0·8	·2116	·2113	·2106	·2092	·2067	·2024	·1960	·1870	·1753	·1611	·1452	·1283	·1113
0·9	·1840	·1839	·1835	·1829	·1816	·1792	·1753	·1694	·1611	·1505	·1377	·1234	·1084
1·0	·1586	·1586	·1585	·1582	·1576	·1563	·1542	·1506	·1452	·1377	·1281	·1167	·1041
1·1	·1357	·1356	·1356	·1355	·1352	·1346	·1335	·1315	·1283	·1234	·1167	·1082	·0981
1·2	·1151	·1151	·1150	·1150	·1149	·1147	·1141	·1131	·1113	·1084	·1041	·0981	·0906
1·3	·0968	·0968	·0968	·0968	·0967	·0966	·0964	·0959	·0950	·0934	·0908	·0870	·0818
1·4	·0808	·0808	·0808	·0808	·0807	·0807	·0806	·0804	·0800	·0792	·0778	·0755	·0721
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0667	·0665	·0661	·0654	·0642	·0622
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0547	·0545	·0542	·0536	·0525
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0445	·0445	·0445	·0443	·0440	·0435
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0358	·0357	·0355
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0286	·0285
2·0	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227	·0227
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0178
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

r = ·90.

h=	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k=*0·0*	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0965	·0806	·0667	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0962	·0805	·0667	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0958	·0802	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0950	·0798	·0663	·0546	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0939	·0792	·0660	·0544	·0444	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0923	·0782	·0654	·0540	·0442	·0357	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·8	·0900	·0767	·0645	·0535	·0439	·0356	·0286	·0227	·0178	·0139	·0107	·0082	·0062	·0047
0·9	·0869	·0747	·0632	·0528	·0435	·0353	·0284	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·0	·0830	·0720	·0614	·0516	·0428	·0350	·0282	·0225	·0177	·0138	·0107	·0082	·0062	·0047
1·1	·0783	·0686	·0591	·0501	·0418	·0344	·0279	·0223	·0176	·0138	·0107	·0082	·0062	·0047
1·2	·0727	·0645	·0562	·0481	·0405	·0338	·0274	·0220	·0175	·0137	·0106	·0082	·0062	·0047
1·3	·0664	·0596	·0526	·0456	·0388	·0325	·0267	·0216	·0173	·0136	·0106	·0081	·0062	·0046
1·4	·0596	·0543	·0485	·0426	·0367	·0310	·0258	·0211	·0169	·0134	·0105	·0081	·0062	·0046
1·5	·0526	·0485	·0439	·0391	·0341	·0292	·0246	·0203	·0164	·0131	·0103	·0080	·0061	·0046
1·6	·0456	·0426	·0391	·0353	·0312	·0271	·0231	·0193	·0158	·0127	·0101	·0079	·0060	·0046
1·7	·0388	·0367	·0341	·0312	·0281	·0247	·0214	·0181	·0150	·0122	·0098	·0077	·0059	·0045
1·8	·0325	·0310	·0292	·0271	·0247	·0221	·0194	·0167	·0140	·0116	·0094	·0074	·0058	·0044
1·9	·0267	·0258	·0246	·0231	·0214	·0194	·0173	·0151	·0129	·0108	·0088	·0071	·0056	·0043
2·0	·0216	·0211	·0203	·0193	·0181	·0167	·0151	·0134	·0116	·0099	·0082	·0067	·0053	·0042
2·1	·0173	·0169	·0164	·0158	·0150	·0140	·0129	·0116	·0102	·0088	·0075	·0062	·0050	·0040
2·2	·0136	·0134	·0131	·0127	·0122	·0116	·0108	·0099	·0088	·0078	·0067	·0056	·0046	·0037
2·3	·0106	·0105	·0103	·0101	·0098	·0094	·0088	·0082	·0075	·0067	·0058	·0050	·0042	·0034
2·4	·0081	·0081	·0080	·0079	·0077	·0074	·0071	·0067	·0062	·0056	·0050	·0044	·0037	·0031
2·5	·0062	·0062	·0061	·0060	·0059	·0058	·0056	·0053	·0050	·0046	·0042	·0037	·0032	·0027
2·6	·0046	·0046	·0046	·0046	·0045	·0044	·0043	·0042	·0040	·0037	·0034	·0031	·0027	·0024

r = ·95.

h=	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k=*0·0*	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0967	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0966	·0807	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0964	·0806	·0668	·0548	·0446	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0959	·0804	·0667	·0548	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0950	·0800	·0665	·0547	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0934	·0792	·0661	·0545	·0445	·0359	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0908	·0778	·0654	·0542	·0443	·0358	·0287	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0870	·0755	·0642	·0536	·0440	·0357	·0286	·0227	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0818	·0721	·0622	·0525	·0435	·0355	·0285	·0227	·0178	·0139	·0107	·0082	·0062	·0047
1·3	·0752	·0676	·0593	·0508	·0426	·0350	·0283	·0226	·0178	·0139	·0107	·0082	·0062	·0047
1·4	·0676	·0619	·0554	·0483	·0411	·0342	·0279	·0224	·0177	·0139	·0107	·0082	·0062	·0047
1·5	·0593	·0554	·0505	·0450	·0390	·0330	·0273	·0221	·0176	·0138	·0107	·0082	·0062	·0047
1·6	·0508	·0483	·0450	·0409	·0362	·0312	·0263	·0215	·0173	·0137	·0106	·0082	·0062	·0047
1·7	·0426	·0411	·0390	·0362	·0328	·0289	·0248	·0207	·0169	·0135	·0105	·0081	·0062	·0047
1·8	·0350	·0342	·0330	·0312	·0289	·0261	·0229	·0195	·0162	·0131	·0104	·0080	·0062	·0046
1·9	·0283	·0279	·0273	·0263	·0248	·0229	·0205	·0179	·0152	·0125	·0101	·0079	·0061	·0046
2·0	·0226	·0224	·0221	·0215	·0207	·0195	·0179	·0160	·0139	·0117	·0096	·0077	·0060	·0046
2·1	·0178	·0177	·0176	·0173	·0169	·0162	·0152	·0139	·0124	·0107	·0090	·0073	·0058	·0045
2·2	·0139	·0139	·0138	·0137	·0135	·0131	·0125	·0117	·0107	·0095	·0082	·0068	·0055	·0043
2·3	·0107	·0107	·0107	·0106	·0105	·0104	·0101	·0096	·0090	·0082	·0072	·0062	·0051	·0041
2·4	·0082	·0082	·0082	·0082	·0081	·0080	·0079	·0077	·0073	·0068	·0062	·0054	·0046	·0038
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0061	·0060	·0058	·0055	·0051	·0046	·0040	·0034
2·6	·0047	·0047	·0047	·0047	·0047	·0046	·0046	·0046	·0045	·0043	·0041	·0038	·0034	·0030

TABLE XXX. *Supplementary Tables for determining High*

$r = 1{\cdot}00.$

h =	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*	*1·0*	*1·1*	*1·2*
k = *0·0*	·5000	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·1	·4602	·4602	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·2	·4207	·4207	·4207	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·3	·3821	·3821	·3821	·3821	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·4	·3446	·3446	·3446	·3446	·3446	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·5	·3085	·3085	·3085	·3085	·3085	·3085	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·6	·2743	·2743	·2743	·2743	·2743	·2743	·2743	·2420	·2119	·1841	·1587	·1357	·1151
0·7	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2420	·2119	·1841	·1587	·1357	·1151
0·8	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·2119	·1841	·1587	·1357	·1151
0·9	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1841	·1587	·1357	·1151
1·0	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1587	·1357	·1151
1·1	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1357	·1151
1·2	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151	·1151
1·3	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968	·0968
1·4	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808	·0808
1·5	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668	·0668
1·6	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548	·0548
1·7	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446	·0446
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359	·0359
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0287
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

Correlations from Tetrachoric Groupings.

$r = 1{\cdot}00.$

h =	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*
k = *0·0*	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·4	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·5	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·6	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·7	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·8	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
0·9	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·0	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·1	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·2	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·3	·0968	·0808	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·4	·0808	·0808	·0668	·0548	·0446	·0359	·0287	·0228	0179	·0139	·0107	·0082	·0062	·0047
1·5	·0668	·0668	·0668	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·6	·0548	·0548	·0548	·0548	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·7	·0446	·0446	·0446	·0446	·0446	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·8	·0359	·0359	·0359	·0359	·0359	·0359	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
1·9	·0287	·0287	·0287	·0287	·0287	·0287	·0287	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·0	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0228	·0179	·0139	·0107	·0082	·0062	·0047
2·1	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0179	·0139	·0107	·0082	·0062	·0047
2·2	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0139	·0107	·0082	·0062	·0047
2·3	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0107	·0082	·0062	·0047
2·4	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0082	·0062	·0047
2·5	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0062	·0047
2·6	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047	·0047

TABLE XXXI. *The Γ-Function.*

p	Log Γ(*p*), Negative Characteristic, $\bar{1}$									
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1·00	·999,999$\dot{9}$	7497	5001	2512	0030	·998,7555	5087	2627	0173	$-\overline{7727}$
1·01	·997,5287	2855	0430	$-\overline{8011}$	$-\overline{5600}$	·996,3196	0798	$-\overline{8408}$	$-\overline{6025}$	$\overline{3648}$
1·02	·995,1279	$-\overline{8916}$	$-\overline{6561}$	$-\overline{4212}$	$-\overline{1870}$	·993,9535	7207	4886	2572	0265
1·03	·992,7964	5671	3384	1104	$-\overline{8831}$	·991,6564	4305	2052	$-\overline{9806}$	$-\overline{7567}$
1·04	·990,5334	3108	0889	$-\overline{8677}$	$-\overline{6471}$	·989,4273	2080	$-\overline{9895}$	$-\overline{7716}$	$-\overline{5544}$
1·05	·988,3379	1220	$-\overline{9068}$	$-\overline{6922}$	$-\overline{4783}$	·987,2651	0525	$-\overline{8406}$	$-\overline{6294}$	$-\overline{4188}$
1·06	·986,2089	$-\overline{9996}$	$-\overline{7910}$	$-\overline{5830}$	$-\overline{3757}$	·985,1690	$-\overline{9630}$	$-\overline{7577}$	$-\overline{5530}$	$-\overline{3489}$
1·07	·984,1455	$-\overline{9428}$	$-\overline{7407}$	$-\overline{5392}$	$-\overline{3384}$	·983,1382	$-\overline{9387}$	$-\overline{7398}$	$-\overline{5415}$	$-\overline{3439}$
1·08	·982,1469	$-\overline{9506}$	$-\overline{7549}$	$-\overline{5599}$	$-\overline{3655}$	·981,1717	$-\overline{9785}$	$-\overline{7860}$	$-\overline{5941}$	$-\overline{4029}$
1·09	·980,2123	0223	$-\overline{8329}$	$-\overline{6442}$	$-\overline{4561}$	·979,2686	0818	$-\overline{8956}$	$-\overline{7100}$	$-\overline{5250}$
1·10	·978,3407	1570	$-\overline{9738}$	$-\overline{7914}$	$-\overline{6095}$	·977,4283	2476	0676	$-\overline{8882}$	$-\overline{7095}$
1·11	·976,5313	3538	1768	0005	$-\overline{8248}$	·975,6497	4753	3014	1281	$-\overline{9555}$
1·12	·974,7834	6120	4411	2709	1013	·973,9323	7638	5960	4288	2622
1·13	·973,0962	$-\overline{9308}$	$-\overline{7659}$	$-\overline{6017}$	$-\overline{4381}$	·972,2751	1126	$-\overline{9508}$	$-\overline{7896}$	$-\overline{6289}$
1·14	·971,4689	3094	1505	$-\overline{9922}$	$-\overline{8345}$	·970,6774	5209	3650	2096	0549
1·15	·969,9007	7471	5941	4417	2898	·969,1386	$-\overline{9879}$	$-\overline{8378}$	$-\overline{6883}$	$-\overline{5393}$
1·16	·968,3910	2432	0960	$-\overline{9493}$	$-\overline{8033}$	·967,6578	5129	3686	2248	0816
1·17	·966,9390	7969	6554	5145	3742	·966,2344	0952	$-\overline{9566}$	$-\overline{8185}$	$-\overline{6810}$
1·18	·965,5440	4076	2718	1366	0019	·964,8677	7341	6011	4687	3368
1·19	·964,2054	0746	$-\overline{9444}$	$-\overline{8147}$	$-\overline{6856}$	·963,5570	4290	3016	1747	0483
1·20	·962,9225	7973	6725	5484	4248	·962,3017	1792	0573	$-\overline{9358}$	$-\overline{8150}$
1·21	·961,6946	5748	4556	3369	2188	·961,1011	$-\overline{9841}$	$-\overline{8675}$	$-\overline{7515}$	$-\overline{6361}$
1·22	·960,5212	4068	2930	1796	0669	·959,9546	8430	7318	6212	5111
1·23	·959,4015	2925	1840	0760	$-\overline{9685}$	·958,8616	7553	6494	5441	4393
1·24	·958,3350	2313	1280	0253	$-\overline{9232}$	·957,8215	7204	6198	5197	4201
1·25	·957,3211	2226	1246	0271	$-\overline{9301}$	·956,8337	7377	6423	5474	4530
1·26	·956,3592	2658	1730	0806	$-\overline{9888}$	·955,8975	8067	7165	6267	5374
1·27	·955,4487	3604	2727	1855	0988	·955,0126	$-\overline{9268}$	$-\overline{8416}$	$-\overline{7570}$	$-\overline{6728}$
1·28	·954,5891	5059	4232	3410	2593	·954,1782	0975	0173	$-\overline{9376}$	$-\overline{8585}$
1·29	·953,7798	7016	6239	5467	4700	·953,3938	3181	2429	1682	0940
1·30	·953,0203	$-\overline{9470}$	$-\overline{8743}$	$-\overline{8021}$	$-\overline{7303}$	·952,6590	5883	5180	4482	3789
1·31	·952,3100	2417	1739	1065	0396	·951,9732	9073	8419	7770	7125
1·32	·951,6485	5850	5220	4595	3975	·951,3359	2748	2142	1541	0944
1·33	·951,0353	$-\overline{9766}$	$-\overline{9184}$	$-\overline{8606}$	$-\overline{8034}$	·950,7466	6903	6344	5791	5242
1·34	·950,4698	4158	3624	3094	2568	·950,2048	1532	1021	0514	0012
1·35	·949,9515	9023	8535	8052	7573	·949,7100	6630	6166	5706	5251
1·36	·949,4800	4355	3913	3477	3044	·949,2617	2194	1776	1362	0953
1·37	·949,0549	0149	$-\overline{9754}$	$-\overline{9363}$	$-\overline{8977}$	·948,8595	8218	7846	7478	7115
1·38	·948,6756	6402	6052	5707	5366	·948,5030	4698	4371	4049	3731
1·39	·948,3417	3108	2803	2503	2208	·948,1916	1630	1348	1070	0797
1·40	·948,0528	0263	0003	$-\overline{9748}$	$-\overline{9497}$	·947,9250	9008	8770	8537	8308
1·41	·947,8084	7864	7648	7437	7230	·947,7027	6829	6636	6446	6261
1·42	·947,6081	5905	5733	5565	5402	·947,5243	5089	4939	4793	4652
1·43	·947,4515	4382	4254	4130	4010	·947,3894	3783	3676	3574	3476
1·44	·947,3382	3292	3207	3125	3049	·947,2976	2908	2844	2784	2728
1·45	·947,2677	2630	2587	2549	2514	·947,2484	2459	2437	2419	2406
1·46	·947,2397	2393	2392	2396	2404	·947,2416	2432	2452	2477	2506
1·47	·947,2539	2576	2617	2662	2712	·947,2766	2824	2886	2952	3022
1·48	·947,3097	3175	3258	3345	3436	·947,3531	3630	3734	3841	3953
1.49	·947,4068	4188	4312	4440	4572	947,4708	4848	4992	5141	5293
1.50	·947,5449	5610	5774	5943	6116	·947,6292	6473	6658	6847	7040

A horizontal bar means that the third figure of the mantissa has changed, a negative sign that it must be *lowered* one unit.

DIFFERENCES :—NEGATIVE down to rule										
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*p*
2503	2496	2489	2482	2475	2468	2460	2454	2446	2440	*1·00*
2432	2425	2419	2411	2404	2398	2390	2383	2377	2369	*1·01*
2363	2355	2349	2342	2335	2328	2321	2314	2307	2301	*1·02*
2293	2287	2280	2273	2267	2259	2253	2246	2239	2233	*1·03*
2226	2219	2212	2206	2198	2193	2185	2179	2172	2165	*1·04*
2159	2152	2146	2139	2132	2126	2119	2112	2106	2099	*1·05*
2093	2086	2080	2073	2067	2060	2053	2047	2041	2034	*1·06*
2027	2021	2015	2008	2002	1995	1989	1983	1976	1970	*1·07*
1963	1957	1950	1944	1938	1932	1925	1919	1912	1906	*1·08*
1900	1894	1887	1881	1875	1868	1862	1856	1850	1843	*1·09*
1837	1832	1824	1819	1812	1807	1800	1794	1787	1782	*1·10*
1775	1770	1763	1757	1751	1744	1739	1733	1726	1721	*1·11*
1714	1709	1702	1696	1690	1685	1678	1672	1666	1660	*1·12*
1654	1649	1642	1636	1630	1625	1618	1612	1607	1600	*1·13*
1595	1589	1583	1577	1571	1565	1559	1554	1547	1542	*1·14*
1536	1530	1524	1519	1512	1507	1501	1495	1490	1483	*1·15*
1478	1472	1467	1460	1455	1449	1443	1438	1432	1426	*1·16*
1421	1415	1409	1403	1398	1392	1386	1381	1375	1370	*1·17*
1364	1358	1352	1347	1342	1336	1330	1324	1319	1314	*1·18*
1308	1302	1297	1291	1286	1280	1274	1269	1264	1258	*1·19*
1252	1248	1241	1236	1231	1225	1219	1215	1208	1204	*1·20*
1198	1192	1187	1181	1177	1170	1166	1160	1154	1149	*1·21*
1144	1138	1134	1127	1123	1116	1112	1106	1101	1096	*1·22*
1090	1085	1080	1075	1069	1063	1059	1053	1048	1043	*1·23*
1037	1033	1027	1021	1017	1011	1006	1001	996	990	*1·24*
985	980	975	970	964	960	954	949	944	938	*1·25*
934	928	924	918	913	908	902	898	893	887	*1·26*
883	877	872	867	862	858	852	846	842	837	*1·27*
832	827	822	817	811	807	802	797	791	787	*1·28*
782	777	772	767	762	757	752	747	742	737	*1·29*
733	727	722	718	713	707	703	698	693	689	*1·30*
683	678	674	669	664	659	654	649	645	640	*1·31*
635	630	625	620	616	611	606	601	597	591	*1·32*
587	582	578	572	568	563	559	553	549	544	*1·33*
540	534	530	526	520	516	511	507	502	497	*1·34*
492	488	483	479	473	470	464	460	455	451	*1·35*
445	442	436	433	427	423	418	414	409	404	*1·36*
400	395	391	386	382	377	372	368	363	359	*1·37*
354	350	345	341	336	332	327	322	318	314	*1·38*
309	305	300	295	292	286	282	278	273	269	*1·39*
265	260	255	251	247	242	238	233	229	224	*1·40*
220	216	211	207	203	198	193	190	185	180	*1·41*
176	172	168	163	159	154	150	146	141	137	*1·42*
133	128	124	120	116	111	107	102	98	94	*1·43*
90	85	82	76	73	68	64	60	56	51	*1·44*
47	43	−38	−35	−30	−25	−22	−18	−13	− 9	*1·45*
− 4	− 1	+ 4	+ 8	+12	+16	+20	+25	+29	+33	*1·46*
+37	+41	45	50	54	58	62	66	70	75	*1·47*
78	83	87	91	95	99	104	107	112	115	*1·48*
120	124	128	132	136	140	144	149	152	156	*1·49*
161	164	169	173	176	181	185	189	193	197	*1·50*

* Differences change sign at horizontal rule.

TABLE XXXI. *The Γ-Function.*

p	Log $\Gamma(p)$, Negative Characteristic, $\bar{1}$									
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1·51	·947,7237	7437	7642	7851	8064	·947,8281	8502	8727	8956	9189
1·52	·947,9426	9667	9912	+$\overline{0161}$	+$\overline{0414}$	·948,0671	0932	1196	1465	1738
1·53	·948,2015	2295	2580	2868	3161	·948,3457	3758	4062	4370	4682
1·54	·948,4998	5318	5642	5970	6302	·948,6638	6977	7321	7668	8019
1·55	·948,8374	8733	9096	9463	9834	·949,0208	0587	0969	1355	1745
1·56	·949,2139	2537	2938	3344	3753	·949,4166	4583	5004	5429	5857
1·57	·949,6289	6725	7165	7609	8056	·949,8508	8963	9422	9885	+$\overline{0351}$
1·58	·950,0822	1296	1774	2255	2741	·950,3230	3723	4220	4720	5225
1·59	·950,5733	6245	6760	7280	7803	·950,8330	8860	9395	9933	+$\overline{0475}$
1·60	·951,1020	1569	2122	2679	3240	·951,3804	4372	4943	5519	6098
1·61	·951,6680	7267	7857	8451	9048	·951,9649	+$\overline{0254}$	+$\overline{0862}$	+$\overline{1475}$	+$\overline{2091}$
1·62	·952,2710	3333	3960	4591	5225	·952,5863	6504	7149	7798	8451
1·63	·952,9107	9766	+$\overline{0430}$	+$\overline{1097}$	+$\overline{1767}$	·953,2442	3120	3801	4486	5175
1·64	·953,5867	6563	7263	7966	8673	·953,9383	+$\overline{0097}$	+$\overline{0815}$	+$\overline{1536}$	+$\overline{2260}$
1·65	·954,2989	3721	4456	5195	5938	·954,6684	7434	8187	8944	9704
1·66	·955,0468	1236	2007	2782	3560	·955,4342	5127	5916	6708	7504
1·67	·955,8303	9106	9913	+$\overline{0723}$	+$\overline{1536}$	·956,2353	3174	3998	4825	5656
1·68	·956,6491	7329	8170	9015	9864	·957,0716	1571	2430	3293	4159
1·69	·957,5028	5901	6777	7657	8540	·957,9427	+$\overline{0317}$	+$\overline{1211}$	+$\overline{2108}$	+$\overline{3008}$
1·70	·958,3912	4820	5731	6645	7563	·958,8484	9409	+$\overline{0337}$	+$\overline{1268}$	+$\overline{2203}$
1·71	·959,3141	4083	5028	5977	6929	·959,7884	8843	9805	+$\overline{0771}$	+$\overline{1740}$
1·72	·960,2712	3688	4667	5650	6636	·960,7625	8618	9614	+$\overline{0613}$	+$\overline{1616}$
1·73	·961,2622	3632	4645	5661	6681	·961,7704	8730	9760	+$\overline{0793}$	+$\overline{1830}$
1·74	·962,2869	3912	4959	6009	7062	·962,8118	9178	+$\overline{0241}$	+$\overline{1308}$	+$\overline{2378}$
1·75	·963,3451	4527	5607	6690	7776	·963,8866	9959	+$\overline{1055}$	+$\overline{2155}$	+$\overline{3258}$
1·76	·964,4364	5473	6586	7702	8821	·964,9944	+$\overline{1070}$	+$\overline{2199}$	+$\overline{3331}$	+$\overline{4467}$
1·77	·965,5606	6749	7894	9043	+$\overline{0195}$	·966,1350	2509	3671	4836	6004
1·78	·966,7176	8351	9529	+$\overline{0710}$	+$\overline{1895}$	·967,3082	4274	5468	6665	7866
1·79	·967,9070	+$\overline{0277}$	+$\overline{1488}$	+$\overline{2701}$	+$\overline{3918}$	·968,5138	6361	7588	8818	+$\overline{0051}$
1·80	·969,1287	2526	3768	5014	6263	·969,7515	8770	+$\overline{0029}$	+$\overline{1291}$	+$\overline{2555}$
1·81	·970,3823	5095	6369	7646	8927	·971,0211	1498	2788	4082	5378
1·82	·971,6678	7981	9287	+$\overline{0596}$	+$\overline{1908}$	·972,3224	4542	5864	7189	8517
1·83	·972,9848	+$\overline{1182}$	+$\overline{2520}$	+$\overline{3860}$	+$\overline{5204}$	·973,6551	7900	9254	+$\overline{0610}$	+$\overline{1969}$
1·84	·974,3331	4697	6065	7437	8812	·975,0190	1571	2955	4342	5733
1·85	·975,7126	8522	9922	+$\overline{1325}$	+$\overline{2730}$	·976,4139	5551	6966	8384	9805
1·86	·977,1230	2657	4087	5521	6957	·977,8397	9839	+$\overline{1285}$	+$\overline{2734}$	+$\overline{4186}$
1·87	·978,5640	7098	8559	+$\overline{0023}$	+$\overline{1490}$	·979,2960	4433	5909	7389	8871
1·88	·980,0356	1844	3335	4830	6327	·980,7827	9331	+$\overline{0837}$	+$\overline{2346}$	+$\overline{3859}$
1·89	·981,5374	6893	8414	9939	+$\overline{1466}$	·982,2996	4530	6066	7606	9148
1·90	·983,0693	2242	3793	5348	6905	·983,8465	+$\overline{0028}$	+$\overline{1595}$	+$\overline{3164}$	+$\overline{4736}$
1·91	·984,6311	7890	9471	+$\overline{1055}$	+$\overline{2642}$	·985,4232	5825	7421	9020	+$\overline{0621}$
1·92	·986,2226	3834	5445	7058	8675	·987,0294	1917	3542	5170	6802
1·93	·987,8436	+$\overline{0073}$	+$\overline{1713}$	+$\overline{3356}$	+$\overline{5002}$	·988,6651	8302	9957	+$\overline{1614}$	+$\overline{3275}$
1.94	·989,4938	6605	8274	9946	+$\overline{1621}$	·990,3299	4980	6663	8350	+$\overline{0039}$
1·95	·991,1732	3427	5125	6826	8530	·992,0237	1947	3659	5375	7093
1·96	·992,8815	+$\overline{0539}$	+$\overline{2266}$	+$\overline{3995}$	+$\overline{5728}$	·993,7464	9202	+$\overline{0943}$	+$\overline{2688}$	+$\overline{4435}$
1·97	·994,6185	7937	9693	+$\overline{1451}$	+$\overline{3213}$	·995,4977	6744	8513	+$\overline{0286}$	+$\overline{2062}$
1·98	·996,3840	5621	7405	9192	+$\overline{0982}$	·997,2774	4569	6368	8169	9972
1·99	·998,1779	3588	5401	7216	9034	·999,0854	2678	4504	6333	8165

A horizontal bar means that the third figure of the mantissa has changed, a positive sign that it must be *raised* one unit.

Differences :—on this page, Positive										
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*p*
200	205	209	213	217	221	225	229	233	237	*1·51*
241	245	249	253	257	261	264	269	273	277	*1·52*
280	285	288	293	296	301	304	308	312	316	*1·53*
320	324	328	332	336	339	344	347	351	355	*1·54*
359	363	367	371	374	379	382	386	390	394	*1·55*
398	401	406	409	413	417	421	425	428	432	*1·56*
436	440	444	447	452	455	459	463	466	471	*1·57*
474	478	481	486	489	493	497	500	505	508	*1·58*
512	515	520	523	527	530	535	538	542	545	*1·59*
549	553	557	561	564	568	571	576	579	582	*1·60*
587	590	594	597	601	605	608	613	616	619	*1·61*
623	627	631	634	638	641	645	649	653	656	*1·62*
659	664	667	670	675	678	681	685	689	692	*1·63*
696	700	703	707	710	714	718	721	724	729	*1·64*
732	735	739	743	746	750	753	757	760	764	*1·65*
768	771	775	778	782	785	789	792	796	799	*1·66*
803	807	810	813	817	821	824	827	831	835	*1·67*
838	841	845	849	852	855	859	863	866	869	*1·68*
873	876	880	883	887	890	894	897	900	904	*1·69*
908	911	914	918	921	925	928	931	935	938	*1·70*
942	945	949	952	955	959	962	966	969	972	*1·71*
976	979	983	986	989	993	996	999	1003	1006	*1·72*
1010	1013	1016	1020	1023	1026	1030	1033	1037	1039	*1·73*
1043	1047	1050	1053	1056	1060	1063	1067	1070	1073	*1·74*
1076	1080	1083	1086	1090	1093	1096	1100	1103	1106	*1·75*
1109	1113	1116	1119	1123	1126	1129	1132	1136	1139	*1·76*
1143	1145	1149	1152	1155	1159	1162	1165	1168	1172	*1·77*
1175	1178	1181	1185	1187	1192	1194	1197	1201	1204	*1·78*
1207	1211	1213	1217	1220	1223	1227	1230	1233	1236	*1·79*
1239	1242	1246	1249	1252	1255	1259	1262	1264	1268	*1·80*
1272	1274	1277	1281	1284	1287	1290	1294	1296	1300	*1·81*
1303	1306	1309	1312	1316	1318	1322	1325	1328	1331	*1·82*
1334	1338	1340	1344	1347	1349	1354	1356	1359	1362	*1·83*
1366	1368	1372	1375	1378	1381	1384	1387	1391	1393	*1·84*
1396	1400	1403	1405	1409	1412	1415	1418	1421	1425	*1·85*
1427	1430	1434	1436	1440	1442	1446	1449	1452	1454	*1·86*
1458	1461	1464	1467	1470	1473	1476	1480	1482	1485	*1·87*
1488	1491	1495	1497	1500	1504	1506	1509	1513	1515	*1·88*
1519	1521	1525	1527	1530	1534	1536	1540	1542	1545	*1·89*
1549	1551	1555	1557	1560	1563	1567	1569	1572	1575	*1·90*
1579	1581	1584	1587	1590	1593	1596	1599	1601	1605	*1·91*
1608	1611	1613	1617	1619	1623	1625	1628	1632	1634	*1·92*
1637	1640	1643	1646	1649	1651	1655	1657	1661	1663	*1·93*
1667	1669	1672	1675	1678	1681	1683	1687	1689	1693	*1·94*
1695	1698	1701	1704	1707	1710	1712	1716	1718	1722	*1·95*
1724	1727	1729	1733	1736	1738	1741	1745	1747	1750	*1·96*
1752	1756	1758	1762	1764	1767	1769	1773	1776	1778	*1·97*
1781	1784	1787	1790	1792	1795	1799	1801	1803	1807	*1·98*
1809	1813	1815	1818	1820	1824	1826	1829	1832	1835	*1·99*

TABLE XXXII. *Subtense from Arc and Chord*

Table to pass from measured index $\beta = 100$ *(arc − chord)/chord of a curve to the index and may be closely represented by a common catenary. Suggested use: to pass*

Values of α for given values of β as argument.

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
13	23·1	23·2	23·2	23·3	23·4	23·5	23·6	23·7	23·8	23·9
14	24·0	24·1	24·2	24·3	24·4	24·5	24·6	24·7	24·7	24·8
15	24·9	25·0	25·1	25·2	25·3	25·4	25·5	25·6	25·6	25·7
16	25·8	25·9	26·0	26·1	26·2	26·3	26·4	26·4	26·5	26·6
17	26·7	26·8	26·9	27·0	27·0	27·1	27·2	27·3	27·4	27·5
18	27·6	27·7	27·7	27·8	27·9	28·0	28·1	28·2	28·3	28·3
19	28·4	28·5	28·6	28·7	28·7	28·8	28·9	29·0	29·1	29·2
20	29·2	29·3	29·4	29·5	29·6	29·6	29·7	29·8	29·9	30·0
21	30·0	30·1	30·2	30·3	30·4	30·4	30·5	30·6	30·7	30·8
22	30·8	30·9	31·0	31·1	31·2	31·2	31·3	31·4	31·5	31·6
23	31·6	31·7	31·8	31·9	31·9	32·0	32·1	32·2	32·3	32·3
24	32·4	32·5	32·6	32·6	32·7	32·8	32·9	32·9	33·0	33·1
25	33·2	33·3	33·3	33·4	33·5	33·6	33·6	33·7	33·8	33·9
26	33·9	34·0	34·1	34·2	34·2	34·3	34·4	34·5	34·5	34·6
27	34·7	34·8	34·8	34·9	35·0	35·1	35·1	35·2	35·3	35·3
28	35·4	35·5	35·6	35·6	35·7	35·8	35·9	35·9	36·0	36·1
29	36·2	36·2	36·3	36·4	36·4	36·5	36·6	36·7	36·7	36·8
30	36·9	36·9	37·0	37·1	37·2	37·2	37·3	37·4	37·5	37·5
31	37·6	37·7	37·7	37·8	37·9	38·0	38·0	38·1	38·2	38·2
32	38·3	38·4	38·4	38·5	38·6	38·7	38·7	38·8	38·9	38·9
33	39·0	39·1	39·2	39·2	39·3	39·4	39·4	39·5	39·6	39·6
34	39·7	39·8	39·8	39·9	40·0	40·1	40·1	40·2	40·3	40·3
35	40·4	40·5	40·5	40·6	40·7	40·7	40·8	40·9	41·0	41·0
36	41·1	41·2	41·2	41·3	41·4	41·4	41·5	41·6	41·6	41·7
37	41·8	41·8	41·9	42·0	42·0	42·1	42·2	42·2	42·3	42·4
38	42·4	42·5	42·6	42·6	42·7	42·8	42·9	42·9	43·0	43·1
39	43·1	43·2	43·3	43·3	43·4	43·5	43·5	43·6	43·7	43·7
40	43·8	43·9	43·9	44·0	44·1	44·1	44·2	44·3	44·3	44·4
41	44·5	44·5	44·6	44·6	44·7	44·8	44·8	44·9	45·0	45·0
42	45·1	45·2	45·2	45·3	45·4	45·4	45·5	45·6	45·6	45·7
43	45·8	45·8	45·9	46·0	46·0	46·1	46·2	46·2	46·3	46·4
44	46·4	46·5	46·5	46·6	46·7	46·7	46·8	46·9	46·9	47·0
45	47·1	47·1	47·2	47·3	47·3	47·4	47·5	47·5	47·6	47·6
46	47·7	47·8	47·8	47·9	48·0	48·0	48·1	48·2	48·2	48·3
47	48·4	48·4	48·5	48·5	48·6	48·7	48·7	48·8	48·9	48·9
48	49·0	49·1	49·1	49·2	49·2	49·3	49·4	49·4	49·5	49·6
49	49·6	49·7	49·8	49·8	49·9	49·9	50·0	50·1	50·1	50·2
50	50·3	50·3	50·4	50·5	50·5	50·6	50·6	50·7	50·8	50·8
51	50·9	51·0	51·0	51·1	51·1	51·2	51·3	51·3	51·4	51·5
52	51·5	51·6	51·6	51·7	51·8	51·8	51·9	52·0	52·0	52·1
53	52·1	52·2	52·3	52·3	52·4	52·5	52·5	52·6	52·6	52·7
54	52·8	52·8	52·9	53·0	53·0	53·1	53·1	53·2	53·3	53·3
55	53·4	53·4	53·5	53·6	53·6	53·7	53·8	53·8	53·9	53·9
56	54·0	54·1	54·1	54·2	54·3	54·3	54·4	54·4	54·5	54·6
57	54·6	54·7	54·7	54·8	54·9	54·9	55·0	55·0	55·1	55·2
58	55·2	55·3	55·4	55·4	55·5	55·5	55·6	55·7	55·7	55·8
59	55·8	55·9	56·0	56·0	56·1	56·1	56·2	56·3	56·3	56·4
60	56·5	56·5	56·6	56·6	56·7	56·8	56·8	56·9	56·9	57·0
61	57·1	57·1	57 2	57·2	57·3	57·4	57·4	57·5	57·5	57·6
62	57·7	57·7	57·8	57·8	57·9	58·0	58·0	58·1	58·1	58·2
63	58·3	58·3	58·4	58·4	58·5	58·6	58·6	58·7	58·7	58·8
64	58·9	58·9	59·0	59·0	59·1	59·2	59·2	59·3	59·3	59·4

in the case of the Common Catenary.

α = 100 *subtense/chord, on the assumption that the curve is symmetrical about the subtense from callipers and tape measurements of the nasal bridge to the ratio of "rise" to "span."*

Values of α for given values of β as argument.

β	*·0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
65	59·5	59·5	59·6	59·6	59·7	59·8	59·8	59·9	59·9	60·0
66	60·1	60·1	60·2	60·2	60·3	60·4	60·4	60·5	60·5	60·6
67	60·7	60·7	60·8	60·8	60·9	61·0	61·0	61·1	61·1	61·2
68	61·3	61·3	61·4	61·4	61·5	61·6	61·6	61·7	61·7	61·8
69	61·9	61·9	62·0	62·0	62·1	62·1	62·2	62·3	62·3	62·4
70	62·4	62·5	62·6	62·6	62·7	62·7	62·8	62·9	62·9	63·0
71	63·0	63·1	63·1	63·2	63·3	63·3	63·4	63·4	63·5	63·6
72	63·6	63·7	63·7	63·8	63·9	63·9	64·0	64·0	64·1	64·1
73	64·2	64·3	64·3	64·4	64·4	64·5	64·6	64·6	61·7	64·7
74	64·8	64·9	64·9	65·0	65·0	65·1	65·1	65·2	65·3	65·3
75	65·4	65·4	65·5	65·6	65·6	65·7	65·7	65·8	65·8	65·9
76	66·0	66·0	66·1	66·2	66·2	66·3	66·3	66·4	66·4	66·5
77	66·5	66·6	66·7	66·7	66·8	66·8	66·9	66·9	67·0	67·1
78	67·1	67·2	67·2	67·3	67·4	67·4	67·5	67·5	67·6	67·6
79	67·7	67·8	67·8	67·9	67·9	68·0	68·0	68·1	68·2	68·2
80	68·3	68·3	68·4	68·5	68·5	68·6	68·6	68·7	68·7	68·8
81	68·9	68·9	69·0	69·0	69·1	69·1	69·2	69·3	69·3	69·4
82	69·4	69·5	69·5	69·6	69·7	69·7	69·8	69·8	69·9	70·0
83	70·0	70·1	70·1	70·2	70·2	70·3	70·4	70·4	70·5	70·5
84	70·6	70·6	70·7	70·8	70·8	70·9	70·9	71·0	71·0	71·1
85	71·2	71·2	71·3	71·3	71·4	71·4	71·5	71·6	71·6	71·7
86	71·7	71·8	71·8	71·9	72·0	72·0	72·1	72·1	72·2	72·2
87	72·3	72·4	72·4	72·5	72·5	72·6	72·6	72·7	72·8	72·8
88	72·9	72·9	73·0	73·0	73·1	73·2	73·2	73·3	73·3	73·4
89	73·4	73·5	73·6	73·6	73·7	73·7	73·8	73·8	73·9	73·9
90	74·0	74·1	74·1	74·2	74·2	74·3	74·3	74·4	74·5	74·5
91	74·6	74·6	74·7	74·7	74·8	74·9	74·9	75·0	75·0	75·1
92	75·1	75·2	75·3	75·3	75·4	75·4	75·5	75·5	75·6	75·6
93	75·7	75·8	75·8	75·9	75·9	76·0	76·0	76·1	76·2	76·2
94	76·3	76·3	76·4	76·4	76·5	76·6	76·6	76·7	76·7	76·8
95	76·8	76·9	76·9	77·0	77·1	77·1	77·2	77·2	77·3	77·3
96	77·4	77·5	77·5	77·6	77·6	77·7	77·7	77·8	77·8	77·9
97	78·0	78·0	78·1	78·1	78·2	78·2	78·3	78·3	78·4	78·5
98	78·5	78·6	78·6	78·7	78·7	78·8	78·9	78·9	79·0	79·0
99	79·1	79·1	79·2	79·2	79·3	79·4	79·4	79·5	79·5	79·6
100	79·6	79·7	79·8	79·8	79·9	79·9	80·0	80·0	80·1	80·1
101	80·2	80·3	80·3	80·4	80·4	80·5	80·5	80·6	80·6	80·7
102	80·8	80·8	80·9	80·9	81·0	81·0	81·1	81·1	81·2	81·3
103	81·3	81·4	81·4	81·5	81·5	81·6	81·6	81·7	81·8	81·8
104	81·9	81·9	82·0	82·0	82·1	82·1	82·2	82·3	82·3	82·4
105	82·4	82·5	82·5	82·6	82·6	82·7	82·8	82·8	82·9	82·9
106	83·0	83·0	83·1	83·1	83·2	83·3	83·3	83·4	83·4	83·5
107	83·5	83·6	83·6	83·7	83·8	83·8	83·9	83·9	84·0	84·0
108	84·1	84·1	84·2	84·3	84·3	84·4	84·4	84·5	84·5	84·6
109	84·6	84·7	84·8	84·8	84·9	84·9	85·0	85·0	85·1	85·1
110	85·2	85·3	85·3	85·4	85·4	85·5	85·5	85·6	85·6	85·7
111	85·8	85·8	85·9	85·9	86·0	86·0	86·1	86·1	86·2	86·2
112	86·3	86·4	86·4	86·5	86·5	86·6	86·6	86·7	86·7	86·8
113	86·9	86·9	87·0	87·0	87·1	87·1	87·2	87·2	87·3	87·4
114	87·4	87·5	87·5	87·6	87·6	87·7	87·7	87·8	87·8	87·9
115	88·0	88·0	88·1	88·1	88·2	88·2	88·3	88·3	88·4	88·5
116	88·5	88·6	88·6	88·7	88·7	88·8	88·8	88·9	88·9	89·0

TABLE XXXIII, A *and* B. *Supplementary Tables of Subtense from Arc and Chord.*

TABLE XXXIII.

Supplementary Tables for Subtense Index α as calculated from the arcual value β on the Catenary Hypothesis.

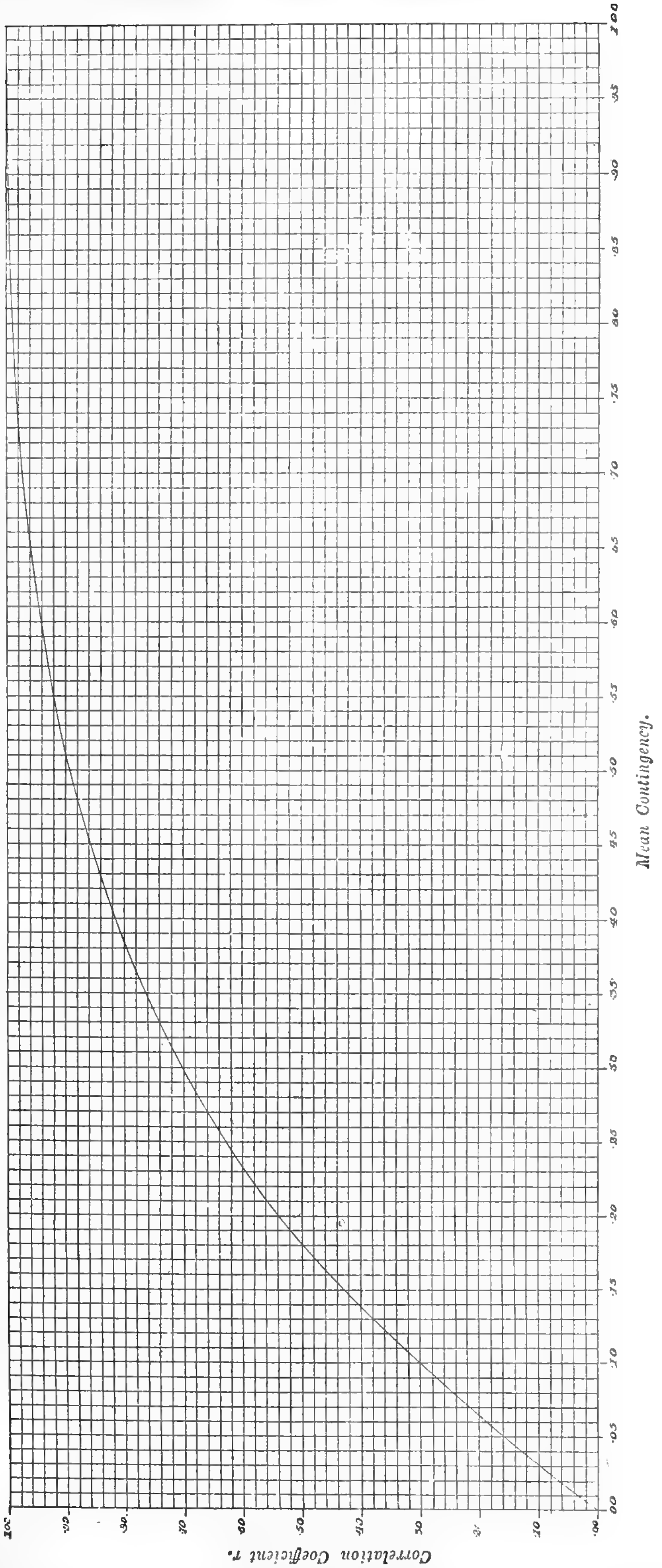
(A) *Values of α for low β.*

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
6	15·3	15·4	15·6	15·7	15·8	16·0	16·1	16·2	16·3	16·5
7	16·6	16·7	16·8	17·0	17·1	17·2	17·3	17·4	17·6	17·7
8	17·8	17·9	18·0	18·1	18·3	18·4	18·5	18·6	18·7	18·8
9	18·9	19·1	19·2	19·3	19·4	19·5	19·6	19·7	19·8	19·9
10	20·0	20·1	20·2	20·3	20·4	20·6	20·7	20·8	20·9	21·0
11	21·1	21·2	21·3	21·4	21·5	21·6	21·7	21·8	21·9	22·0
12	22·1	22·2	22·3	22·4	22·5	22·6	22·7	22·8	22·9	23·0

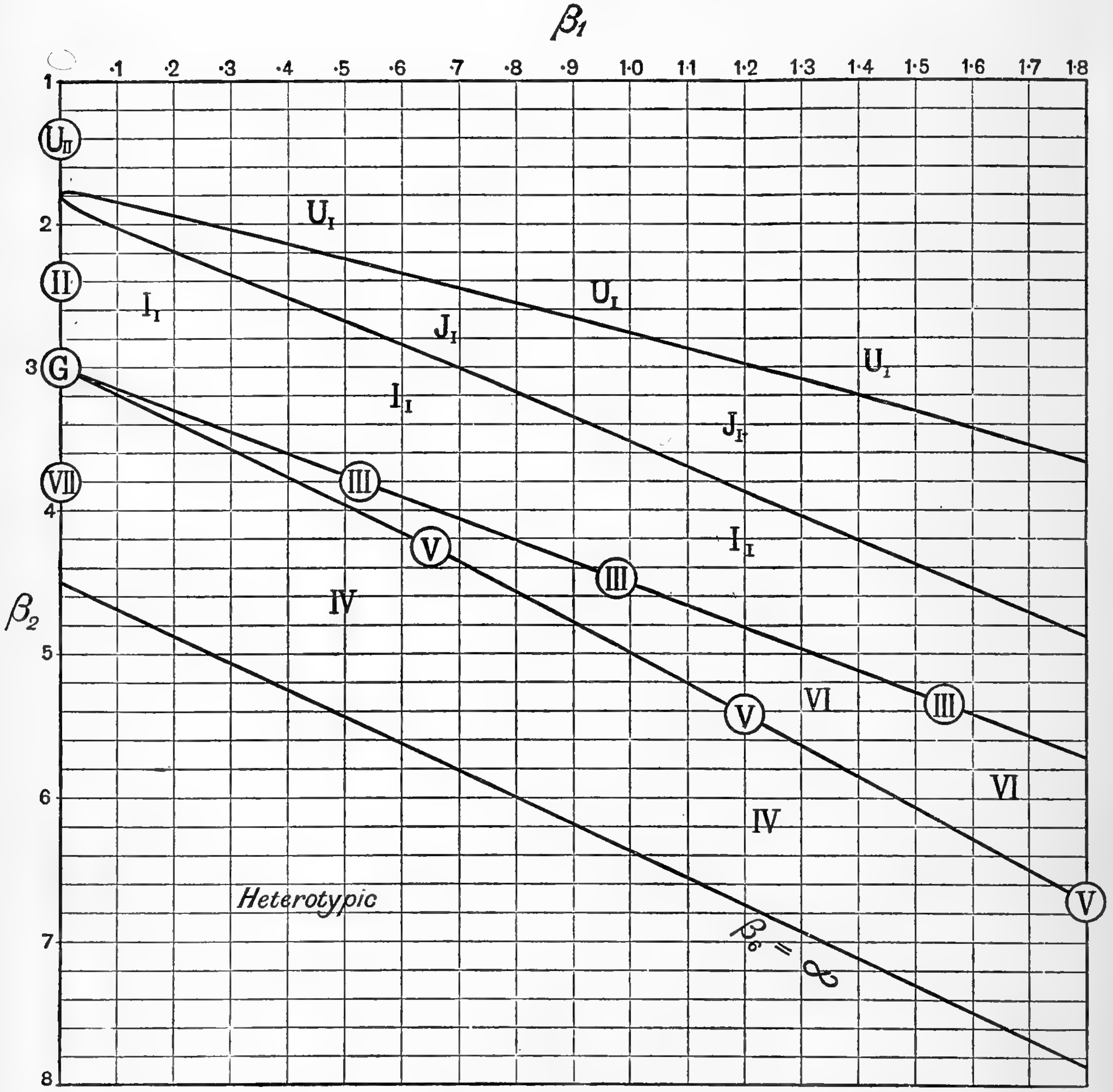
(B) *Values of α for high β.*

β	α	β	α	β	α	β	α	β	α	β	α
101	80·2	126	94·0	151	107·5	176	120·8	201	133·9	226	147·0
102	80·8	127	94·5	152	108·0	177	121·3	202	134·5	227	147·5
103	81·3	128	95·1	153	108·5	178	121·8	203	135·0	228	148·0
104	81·9	129	95·6	154	109·1	179	122·4	204	135·5	229	148·6
105	82·4	130	96·2	155	109·6	180	122·9	205	136·0	230	149·1
106	83·0	131	96·7	156	110·2	181	123·4	206	136·6	231	149·6
107	83·5	132	97·2	157	110·7	182	124·0	207	137·1	232	150·1
108	84·1	133	97·8	158	111·2	183	124·5	208	137·6	233	150·6
109	84·6	134	98·3	159	111·8	184	125·0	209	138·1	234	151·2
110	85·2	135	98·9	160	112·3	185	125·5	210	138·7	235	151·7
111	85·8	136	99·4	161	112·8	186	126·1	211	139·2	236	152·2
112	86·3	137	99·9	162	113·4	187	126·6	212	139·7	237	152·7
113	86·9	138	100·5	163	113·9	188	127·1	213	140·2	238	153·2
114	87·4	139	101·0	164	114·4	189	127·7	214	140·8	239	153·8
115	88·0	140	101·6	165	114·9	190	128·2	215	141·3	240	154·3
116	88·5	141	102·1	166	115·5	191	128·7	216	141·8	241	154·8
117	89·1	142	102·7	167	116·0	192	129·2	217	142·3	242	155·3
118	89·6	143	103·2	168	116·5	193	129·8	218	142·8	243	155·8
119	90·1	144	103·7	169	117·1	194	130·3	219	143·4	244	156·3
120	90·7	145	104·3	170	117·6	195	130·8	220	143·9	245	156·9
121	91·2	146	104·8	171	118·1	196	131·3	221	144·4	246	157·4
122	91·8	147	105·3	172	118·7	197	131·8	222	144·9	247	157·9
123	92·3	148	105·9	173	119·2	198	132·4	223	145·4	248	158·4
124	92·9	149	106·4	174	119·7	199	132·9	224	146·0	249	159·0
125	93·4	150	106·9	175	120·2	200	133·4	225	146·5	250	159·5

XXXIV. *Diagram to find Correlation r from Mean Contingency on the Hypothesis of a Normal Distribution.*



XXXV. *Diagram to determine the type of a Frequency Distribution from a knowledge of the Constants* β_1 *and* β_2. *Customary Values of* β_1 *and* β_2.



XXXVI. *Diagram showing Distribution of Frequency Types for High Values for β_1 and β_2.*

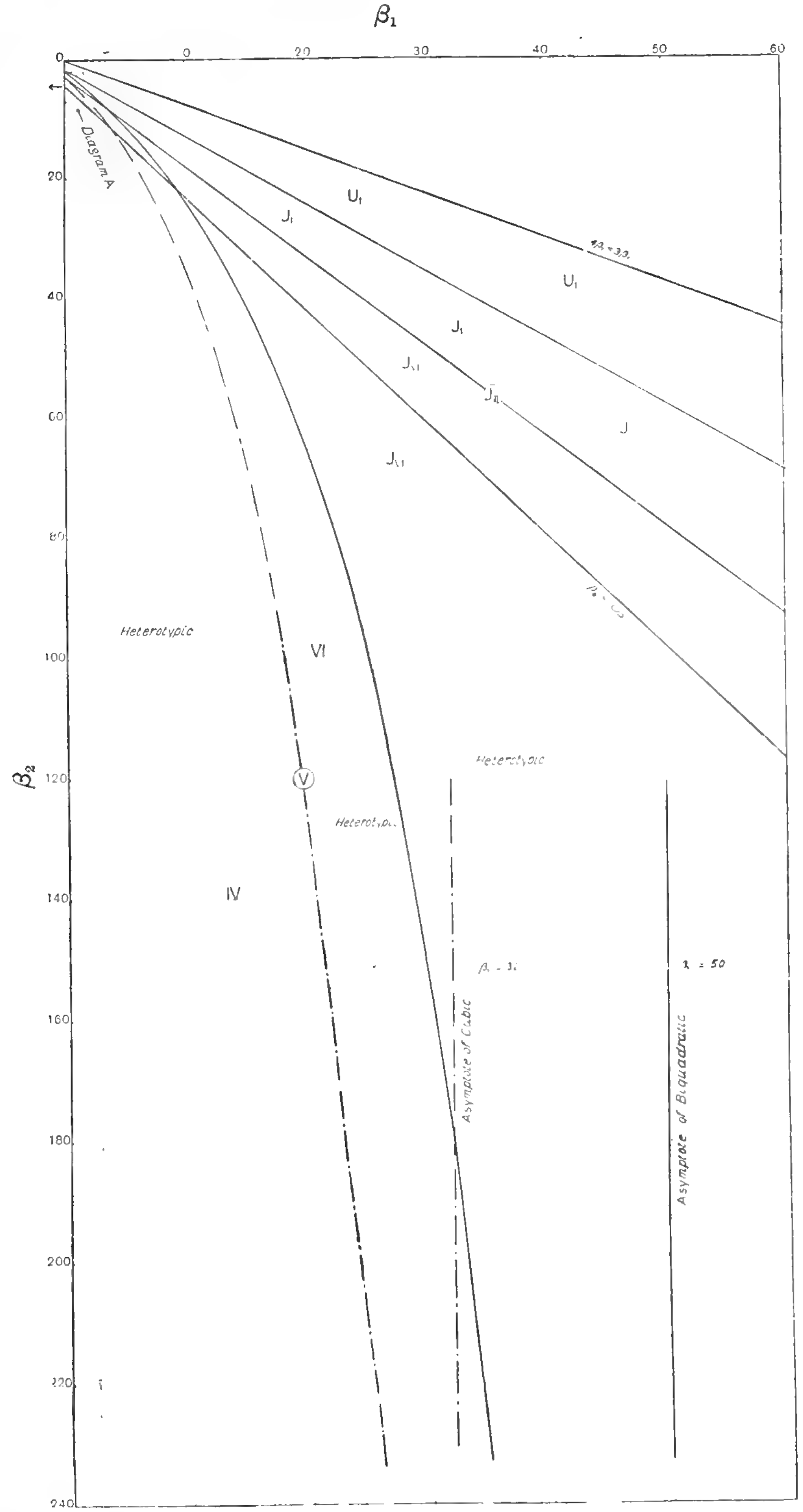


TABLE XXXVII. *To find the Probable Error of β_1.*

Values of $\sqrt{N}\,\Sigma_\beta$.

β_1

β_2	*0·00*	*0·05*	*0·10*	*0·15*	*0·20*	*0·25*	*0·30*	*0·35*	*0·40*	*0·45*	*0·50*	*0·55*	*0·60*	*0·65*	*0·70*	*0·75*
2·0	0·00	0·58	0·93	1·15	1·37	1·57	1·77	1·97	2·17	2·38	2·58	2·80	3·02	3·24	3·46	3·71
2·1	0·00	0·59	0·95	1·12	1·30	1·50	1·70	1·90	2·10	2·29	2·48	2·69	2·91	3·12	3·34	3·57
2·2	0·00	0·60	0·97	1·13	1·30	1·48	1·67	1·86	2·05	2·22	2·41	2·61	2·81	3·01	3·22	3·44
2·3	0·00	0·62	0·99	1·15	1·32	1·49	1·67	1·84	2·02	2·19	2·36	2·55	2·74	2·93	3·12	3·32
2·4	0·00	0·64	1·02	1·20	1·37	1·54	1·70	1·86	2·02	2·18	2·34	2·51	2·68	2·85	3·03	3·22
2·5	0·00	0·66	1·05	1·26	1·45	1·61	1·76	1·91	2·05	2·19	2·33	2·49	2·65	2·81	2·97	3·14
2·6	0·00	0·69	1·10	1·34	1·54	1·72	1·86	2·00	2·13	2·25	2·37	2·50	2·64	2·78	2·92	3·08
2·7	0·00	0·73	1·15	1·42	1·64	1·83	1·96	2·09	2·22	2·32	2·42	2·53	2·65	2·77	2·90	3·05
2·8	0·00	0·77	1·22	1·51	1·75	1·94	2·07	2·20	2·32	2·41	2·50	2·60	2·70	2·80	2·93	3·05
2·9	0·00	0·81	1·30	1·61	1·87	2·06	2·20	2·33	2·44	2·53	2·62	2·70	2·79	2·89	2·99	3·09
3·0	0·00	0·87	1·40	1·73	2·01	2·20	2·34	2·47	2·57	2·67	2·76	2·84	2·92	3·00	3·09	3·18
3·1	0·00	0·94	1·53	1·86	2·17	2·35	2·51	2·64	2·75	2·85	2·94	3·00	3·08	3·15	3·23	3·30
3·2	0·00	1·02	1·67	2·02	2·33	2·52	2·71	2·84	2·95	3·05	3·14	3·22	3·27	3·33	3·40	3·46
3·3	0·00	1·12	1·82	2·20	2·50	2·71	2·92	3·06	3·18	3·28	3·37	3·44	3·50	3·55	3·60	3·65
3·4	0·00	1·24	1·99	2·38	2·68	2·93	3·14	3·30	3·43	3·53	3·63	3·70	3·75	3·79	3·83	3·87
3·5	0·00	1·37	2·16	2·57	2·89	3·17	3·39	3·56	3·69	3·81	3·91	3·98	4·02	4·06	4·10	4·12
3·6	0·00	1·50	2·33	2·78	3·11	3·43	3·65	3·84	3·99	4·12	4·22	4·29	4·33	4·37	4·40	4·41
3·7	0·00	1·64	2·50	2·99	3·36	3·70	3·93	4·14	4·31	4·44	4·54	4·61	4·66	4·70	4·72	4·74
3·8	0·00	1·78	2·67	3·20	3·62	3·97	4·23	4·46	4·64	4·77	4·87	4·95	5·00	5·05	5·07	5·09
3·9	0·00	1·93	2·86	3·43	3·89	4·25	4·54	4·79	4·97	5·11	5·23	5·32	5·38	5·43	5·46	5·48
4·0	0·00	2·10	3·07	3·69	4·17	4·55	4·87	5·13	5·32	5·48	5·62	5·72	5·79	5·84	5·88	5·89
4·1	—	—	3·29	3·87	4·47	4·87	5·21	5·49	5·69	5·87	6·03	6·15	6·23	6·28	6·32	6·33
4·2	—	—	3·53	4·19	4·79	5·21	5·58	5·88	6·10	6·30	6·46	6·60	6·69	6·75	6·80	6·81
4·3	—	—	3·78	4·52	5·13	5·58	5·97	6·29	6·54	6·75	6·93	7·07	7·18	7·25	7·29	7·31
4·4	—	—	4·05	4·85	5·49	5·98	6·40	6·74	7·01	7·24	7·42	7·57	7·68	7·76	7·80	7·83
4·5	—	—	4·33	5·18	5·88	6·42	6·87	7·23	7·52	7·75	7·95	8·10	8·21	8·29	8·34	8·37
4·6	—	—	—	—	—	—	7·37	7·76	8·07	8·30	8·51	8·66	8·76	8·85	8·91	8·95
4·7	—	—	—	—	—	—	7·90	8·31	8·64	8·90	9·11	9·25	9·35	9·44	9·50	9·54
4·8	—	—	—	—	—	—	8·46	8·88	9·24	9·54	9·75	9·89	9·99	10·08	10·14	10·18
4·9	—	—	—	—	—	—	9·05	9·47	9·86	10·21	10·42	10·58	10·69	10·78	10·84	10·80
5·0	—	—	—	—	—	—	9·66	10·08	10·50	10·90	11·19	11·33	11·44	11·53	11·60	11·64
5·1	—	—	—	—	—	—	—	—	—	—	—	—	12·26	12·36	12·42	12·43
5·2	—	—	—	—	—	—	—	—	—	—	—	—	13·10	13·26	13·29	13·29
5·3	—	—	—	—	—	—	—	—	—	—	—	—	13·98	14·15	14·18	14·18
5·4	—	—	—	—	—	—	—	—	—	—	—	—	14·91	15·05	15·10	15·11
5·5	—	—	—	—	—	—	—	—	—	—	—	—	15·90	15·98	16·05	16·07
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XXXVII.—(*continued*).

Values of $\sqrt{N}\Sigma_{\beta_1}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
3·96	4·21	4·47	4·73	5·00	5·27	5·55	5·83	6·12	6·41	6·71	7·01	7·31	7·62	7·94	*2·0*
3·80	4·03	4·27	4·53	4·80	5·07	5·34	5·62	5·90	6·18	6·48	6·77	7·07	7·37	7·69	*2·1*
3·66	3·88	4·11	4·36	4·63	4·88	5·15	5·42	5·69	5·96	6·25	6·54	6·84	7·14	7·45	*2·2*
3·52	3·74	3·96	4·20	4·46	4·71	4·96	5·22	5·48	5·75	6·02	6·31	6·61	6·91	7·21	*2·3*
3·41	3·62	3·83	4·05	4·29	4·54	4·78	5·03	5·28	5·55	5·82	6·10	6·38	6·68	6·97	*2·4*
3·32	3·51	3·71	3·92	4·15	4·38	4·61	4·85	5·10	5·36	5·62	5·89	6·16	6·45	6·74	*2·5*
3·25	3·42	3·60	3·80	4·01	4·23	4·45	4·68	4·92	5·17	5·43	5·68	5·94	6·22	6·51	*2·6*
3·20	3·35	3·51	3·69	3·89	4·10	4·32	4·54	4·76	5·00	5·24	5·48	5·73	6·00	6·28	*2·7*
3·18	3·32	3·47	3·63	3·80	4·00	4·21	4·41	4·62	4·84	5·07	5·30	5·53	5·79	6·06	*2·8*
3·19	3·32	3·45	3·60	3·75	3·92	4·11	4·30	4·49	4·70	4·91	5·12	5·34	5·59	5·85	*2·9*
3·27	3·38	3·49	3·61	3·74	3·87	4·03	4·21	4·39	4·58	4·78	4·98	5·19	5·42	5·68	*3·0*
3·38	3·47	3·57	3·67	3·77	3·89	4·02	4·16	4·31	4·48	4·66	4·85	5·05	5·28	5·53	*3·1*
3·53	3·60	3·68	3·76	3·84	3·93	4·03	4·15	4·28	4·43	4·59	4·75	4·92	5·15	5·40	*3·2*
3·70	3·75	3·81	3·88	3·95	4·02	4·10	4·19	4·28	4·42	4·56	4·69	4·84	5·04	5·28	*3·3*
3·90	3·93	3·97	4·03	4·08	4·14	4·20	4·26	4·34	4·45	4·56	4·66	4·78	4·97	5·18	*3·4*
4·14	4·17	4·19	4·22	4·26	4·30	4·34	4·39	4·45	4·52	4·60	4·68	4·79	4·93	5·12	*3·5*
4·42	4·44	4·45	4·47	4·49	4·51	4·54	4·57	4·62	4·67	4·74	4·78	4·81	4·95	5·09	*3·6*
4·74	4·75	4·76	4·76	4·77	4·78	4·79	4·81	4·84	4·87	4·90	4·92	4·95	5·04	5·13	*3·7*
5·10	5·10	5·09	5·08	5·08	5·07	5·07	5·06	5·06	5·08	5·09	5·11	5·14	5·18	5·22	*3·8*
5·49	5·48	5·46	5·44	5·42	5·40	5·37	5·35	5·33	5·32	5·32	5·33	5·34	5·35	5·37	*3·9*
5·89	5·88	5·86	5·83	5·80	5·76	5·72	5·69	5·65	5·62	5·60	5·58	5·57	5·57	5·59	*4·0*
6·33	6·32	6·30	6·26	6·21	6·16	6·11	6·06	6·02	5·98	5·94	5·91	5·88	5·86	5·86	*4·1*
6·80	6·79	6·76	6·71	6·65	6·60	6·54	6·48	6·42	6·36	6·31	6·27	6·24	6·21	6·18	*4·2*
7·30	7·28	7·25	7·19	7·13	7·07	7·01	6·93	6·87	6·80	6·74	6·67	6·62	6·57	6·53	*4·3*
7·83	7·80	7·76	7·71	7·65	7·58	7·51	7·44	7·37	7·28	7·20	7·12	7·05	6·98	6·92	*4·4*
8·38	8·36	8·32	8·28	8·21	8·14	8·07	7·99	7·90	7·81	7·71	7·61	7·51	7·42	7·34	*4·5*
8·96	8·95	8·91	8·86	8·79	8·72	8·64	8·55	8·45	8·35	8·24	8·13	8·00	7·90	7·80	*4·6*
9·57	9·57	9·53	9·47	9·40	9·33	9·24	9·14	9·04	8·93	8·82	8·69	8·55	8·42	8·31	*4·7*
10·20	10·23	10·16	10·10	10·05	9·97	9·88	9·77	9·67	9·55	9·42	9·28	9·14	9·00	8·87	*4·8*
10·91	10·92	10·87	10·80	10·74	10·66	10·57	10·44	10·32	10·18	10·04	9·90	9·76	9·63	9·50	*4·9*
11·66	11·65	11·61	11·55	11·48	11·39	11·29	11·17	11·04	10·90	10·77	10·62	10·46	10·30	10·14	*5·0*
12·45	12·43	12·38	12·32	12·24	12·14	12·03	11·91	11·78	11·64	11·50	11·34	11·15	10·96	10·82	*5·1*
13·28	13·25	13·20	13·13	13·04	12·92	12·80	12·67	12·54	12·40	12·24	12·07	11·88	11·72	11·54	*5·2*
14·16	14·12	14·07	13·98	13·87	13·76	13·63	13·47	13·35	13·20	13·02	12·84	12·66	12·48	12·30	*5·3*
15·09	15·06	15·00	14·90	14·78	14·65	14·51	14·36	14·22	14·05	13·81	13·67	13·48	13·29	13·11	*5·4*
16·06	16·02	15·96	15·87	15·76	15·63	15·49	15·33	15·17	15·00	14·81	14·61	14·40	14·18	13·97	*5·5*
—	—	17·02	16·91	16·79	16·67	16·51	16·34	16·18	15·95	15·70	15·50	15·30	15·07	14·84	*5·6*
—	—	18·14	17·99	17·88	17·75	17·58	17·40	17·23	16·94	16·70	16·47	16·26	16·04	15·77	*5·7*
—	—	19·34	19·13	19·02	18·87	18·69	18·48	18·26	17·98	17·74	17·50	17·26	17·01	16·76	*5·8*
—	—	20·57	20·36	20·20	20·03	19·84	19·62	19·39	19·11	18·84	18·59	18·32	18·05	17·78	*5·9*
—	—	21·86	21·65	21·45	21·25	21·03	20·79	20·54	20·29	20·02	19·76	19·47	19·18	18·90	*6·0*
—	—	—	—	—	—	22·36	22·18	21·92	21·61	21·31	20·97	20·61	20·30	20·13	*6·1*
—	—	—	—	—	—	23·77	23·61	23·32	23·00	22·63	22·22	21·82	21·50	21·29	*6·2*
—	—	—	—	—	—	25·33	25·09	24·74	24·38	24·00	23·55	23·13	22·78	22·50	*6·3*
—	—	—	—	—	—	26·95	26·64	26·27	25·86	25·43	26·00	24·52	24·12	23·82	*6·4*
—	—	—	—	—	—	28·61	28·18	27·73	27·30	26·89	26·46	26·06	25·65	25·24	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	27·67	27·21	26·75	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	29·40	28·90	28·35	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	31·15	30·61	29·94	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	33·02	32·41	31·72	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	34·89	34·16	33·59	*7·0*

TABLE XXXVIII. *To find Probable Error of* β_2.

Values of $\sqrt{N}\Sigma_{\beta_2}$.

β_1

β_2	*0·00*	*0·05*	*0·10*	*0·15*	*0·20*	*0·25*	*0·30*	*0·35*	*0·40*	*0·45*	*0·50*	*0·55*	*0·60*	*0·65*	*0·70*	*0·75*
2·0	1·41	1·60	1·74	1·93	2·11	2·28	2·44	2·60	2·77	2·94	3·12	3·29	3·46	3·65	3·86	4·05
2·1	1·57	1·76	1·89	2·00	2·10	2·20	2·35	2·51	2·68	2·86	3·05	3·24	3·44	3·64	3·84	4·04
2·2	1·75	1·94	2·07	2·16	2·20	2·28	2·40	2·53	2·68	2·85	3·04	3·23	3·43	3·63	3·83	4·03
2·3	1·95	2·16	2·28	2·35	2·42	2·49	2·58	2·67	2·78	2·92	3·09	3·27	3·45	3·64	3·84	4·03
2·4	2·18	2·39	2·53	2·60	2·72	2·82	2·90	2·96	3·02	3·10	3·22	3·35	3·50	3·68	3·86	4·03
2·5	2·46	2·68	2·83	2·97	3·09	3·19	3·27	3·31	3·32	3·36	3·47	3·53	3·63	3·75	3·88	4·03
2·6	2·78	3·03	3·24	3·38	3·52	3·60	3·66	3·69	3·69	3·70	3·75	3·78	3·83	3·87	3·95	4·07
2·7	3·17	3·48	3·71	3·87	3·98	4·03	4·08	4·12	4·11	4·09	4·07	4·06	4·06	4·06	4·07	4·15
2·8	3·64	4·02	4·26	4·42	4·52	4·58	4·60	4·59	4·57	4·52	4·44	4·39	4·34	4·32	4·31	4·34
2·9	4·22	4·65	4·94	5·11	5·20	5·22	5·18	5·13	5·07	4·99	4·90	4·80	4·70	4·63	4·60	4·61
3·0	4·90	5·48	5·76	5·89	5·95	5·93	5·86	5·76	5·65	5·53	5·41	5·30	5·20	5·12	5·05	5·00
3·1	5·75	6·41	6·72	6·88	6·90	6·82	6·70	6·54	6·38	6·22	6·07	5·92	5·79	5·69	5·60	5·53
3·2	6·77	7·55	7·90	8·00	7·97	7·83	7·63	7·42	7·21	7·03	6·86	6·70	6·53	6·39	6·26	6·14
3·3	8·00	8·83	9·22	9·30	9·22	9·02	8·80	8·53	8·29	8·05	7·83	7·60	7·38	7·20	7·02	6·84
3·4	9·37	10·28	10·68	10·76	10·67	10·46	10·20	9·91	9·62	9·31	9·01	8·73	8·44	8·18	7·92	7·66
3·5	10·85	11·75	12·31	12·52	12·46	12·25	11·95	11·60	11·24	10·86	10·45	10·03	9·63	9·26	8·90	8·54
3·6	12·67	13·74	14·40	14·78	14·53	14·21	13·80	13·38	12·95	12·55	12·10	11·60	11·06	10·54	10·02	9·55
3·7	14·78	15·98	16·78	17·09	16·93	16·53	16·05	15·58	15·09	14·61	14·08	13·49	12·74	12·02	11·36	10·80
3·8	17·50	18·83	19·83	20·03	19·78	19·36	18·76	18·22	17·64	16·98	16·25	15·30	14·42	13·60	12·88	12·27
3·9	20·80	22·50	23·68	23·81	23·34	22·67	21·98	21·14	20·29	19·45	18·58	17·54	16·50	15·54	14·77	14·06
4·0	24·74	26·83	28·47	28·05	27·24	26·29	25·25	24·18	23·03	22·02	21·01	20·01	19·04	18·12	17·23	16·36
4·1	—	—	35·00	34·17	32·88	31·36	29·77	28·13	26·60	25·12	23·82	22·64	21·54	20·53	19·56	18·62
4·2	—	—	43·3	41·4	39·2	37·2	35·2	33·2	31·2	29·2	27·4	26·0	24·7	23·4	22·3	21·3
4·3	—	—	55·3	51·6	48·0	44·6	41·2	38·6	36·2	33·8	31·8	30·1	28·5	26·9	25·6	24·3
4·4	—	—	72·7	66·0	59·	54·1	49·5	45·7	42·1	39·2	36·8	34·8	32·9	31·0	29·2	27·7
4·5	—	—	96·5	82·7	72·7	65·3	59·8	54·7	50·8	47·2	44·0	41·0	38·5	36·2	34·1	32·0
4·6	—	—	—	—	—	—	75·0	68·0	62·2	56·9	52·2	48·2	45·1	42·2	39·6	37·2
4·7	—	—	—	—	—	—	101·3	87·2	76·8	68·3	62·0	56·9	52·7	49·1	45·9	42·8
4·8	—	—	—	—	—	—	140·0	115·2	96·2	82·6	72·7	66·1	60·9	56·7	52·9	49·3
4·9	—	—	—	—	—	—	204·5	150·8	122·3	102·5	89·1	80·2	72·4	66·6	61·4	56·7
5·0	—	—	—	—	—	—	325·7	206·0	154·2	126·8	110·1	96·9	86·6	78·1	71·2	65·6
5·1	—	—	—	—	—	—	—	—	—	—	—	—	103·6	94·4	85·9	78·0
5·2	—	—	—	—	—	—	—	—	—	—	—	—	130·4	116·4	104·0	91·0
5·3	—	—	—	—	—	—	—	—	—	—	—	—	175·2	144·8	124·4	109·6
5·4	—	—	—	—	—	—	—	—	—	—	—	—	224·4	178·0	151·0	132·6
5·5	—	—	—	—	—	—	—	—	—	—	—	—	340·8	246·0	195·3	163·2
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XXXVIII.—(*continued*).

Values of $\sqrt{N}\,\Sigma_{\beta_2}$.

β_1.

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
4·24	4·43	4·62	4·81	5·00	5·19	5·38	5·56	5·75	5·94	6·12	6·30	6·49	6·67	6·84	*2·0*
4·23	4·41	4·59	4·77	4·96	5·15	5·34	5·53	5·72	5·90	6·08	6·27	6·47	6·65	6·83	*2·1*
4·22	4·39	4·56	4·74	4·93	5·12	5·31	5·50	5·69	5·87	6·05	6·24	6·44	6·63	6·82	*2·2*
4·20	4·36	4·53	4·71	4·90	5·08	5·27	5·46	5·65	5·84	6·02	6·21	6·41	6·61	6·80	*2·3*
4·19	4·35	4·51	4·69	4·87	5·05	5·23	5·42	5·61	5·80	5·99	6·18	6·38	6·58	6·78	*2·4*
4·18	4·34	4·50	4·67	4·85	5·03	5·21	5·39	5·58	5·77	5·96	6·15	6·35	6·54	6·74	*2·5*
4·20	4·35	4·50	4·67	4·84	5·01	5·20	5·36	5·54	5·72	5·91	6·11	6·30	6·49	6·68	*2·6*
4·26	4·38	4·52	4·68	4·84	5·01	5·18	5·34	5·51	5·67	5·85	6·05	6·25	6·44	6·62	*2·7*
4·40	4·50	4·60	4·72	4·86	5·03	5·19	5·34	5·49	5·65	5·83	6·02	6·21	6·39	6·58	*2·8*
4·63	4·67	4·73	4·82	4·93	5·05	5·20	5·35	5·50	5·66	5·82	6·00	6·18	6·36	6·54	*2·9*
4·98	4·97	4·99	5·03	5·10	5·18	5·28	5·39	5·52	5·66	5·82	5·98	6·15	6·33	6·51	*3·0*
5·47	5·42	5·38	5·34	5·36	5·37	5·41	5·48	5·58	5·70	5·83	5·97	6·14	6·32	6·52	*3·1*
6·03	5·92	5·83	5·75	5·67	5·62	5·60	5·62	5·68	5·78	5·90	6·03	6·18	6·34	6·52	*3·2*
6·67	6·51	6·35	6·22	6·09	6·00	5·90	5·94	5·92	5·95	6·01	6·12	6·25	6·37	6·53	*3·3*
7·41	7·17	6·95	6·77	6·61	6·48	6·29	6·26	6·24	6·22	6·22	6·26	6·34	6·47	6·61	*3·4*
8·22	7·92	7·64	7·38	7·17	6·99	6·84	6·72	6·63	6·57	6·54	6·53	6·56	6·61	6·71	*3·5*
9·14	8·80	8·51	8·23	7·98	7·70	7·53	7·40	7·22	7·09	6·99	6·98	6·95	6·92	6·93	*3·6*
10·34	9·94	9·58	9·25	8·96	8·66	8·36	8·14	7·90	7·75	7·61	7·51	7·42	7·31	7·23	*3·7*
11·77	11·29	10·82	10·37	9·98	9·62	9·31	9·03	8·73	8·51	8·29	8·11	7·94	7·78	7·60	*3·8*
13·42	12·85	12·31	11·79	11·30	10·86	10·41	10·02	9·64	9·34	9·03	8·77	8·52	8.30	8·10	*3·9*
15·58	14·84	14·10	13·42	12·79	12·20	11·64	11·13	10·65	10·21	9·83	9·51	9·20	8·92	8·67	*4·0*
17·72	16·85	16·01	15·21	14·44	13·70	13·00	12·34	11·73	11·17	10·67	10·24	9·87	9·58	9·32	*4·1*
20·2	19·2	18·3	17·3	16·4	15·5	14·7	14·0	13·3	12·6	12·0	11·5	11·0	10·5	10·3	*4·2*
23·1	22·0	20·9	19·8	18·7	17·6	16·7	15·8	15·0	14·2	13·5	12·8	12·3	11·8	11·3	*4·3*
26·3	25·0	23·8	22·5	21·3	20·1	19·0	18·0	17·1	16·1	15·3	14·6	13 9	13·2	12·6	*4·4*
30·1	28·4	26·8	25·3	23·9	22·6	21·4	20·3	19·3	18·3	17·3	16·4	15·6	14·8	14·1	*4·5*
34·7	32·5	30·5	28·8	27·3	25·6	24·2	22·9	21·7	20·6	19·5	18·4	17·5	16·7	16·1	*4·6*
40·0	37·4	35·0	32·8	30·9	29·2	27·6	26·1	24·7	23·3	22·0	20·9	19·8	18·8	18·1	*4·7*
46·1	43·1	40·3	37·7	35·3	33·2	31·4	29·7	28·0	26·4	25·0	23·6	22·3	21·2	20·3	*4·8*
52·4	48·8	46·8	43·1	40·2	37·8	35·6	33·6	31·6	29·8	28·1	26·6	25·1	23·8	22·7	*4·9*
60·6	56·1	52·3	48·8	45·5	42·6	40·0	37·6	35·4	33·4	31·5	29·8	28·1	26·6	25·2	*5·0*
71·2	65·1	60·6	56·5	52·6	49·1	45·8	43·1	40·5	38·0	35·6	33·6	31·7	30·0	28·4	*5·1*
83·0	76·4	70·5	65·4	60·6	56·3	52·5	49·3	46·3	43·4	40·4	38·0	35·6	33·6	31·7	*5·2*
98·8	89·6	81·9	75·6	70·2	65·0	60·2	56·2	52·5	48·9	45·5	42·6	39·9	37·4	35·2	*5·3*
118·4	105·2	96·0	87·6	80·4	74·0	68·3	63·4	58·8	54·7	51·0	47·7	44·5	41·5	38·9	*5·4*
141·4	124·0	111·2	99·6	91·2	84·0	77·4	71·2	65·7	61·2	56·9	52·9	49·4	46·2	43·5	*5·5*
—	—	131·2	117·4	105·2	96·0	87·3	79·3	72·8	67·8	63·2	58·6	54·8	51·4	48·8	*5·6*
—	—	160·0	142·4	126·4	113·4	102·2	93·0	84·4	77·3	71·1	65·6	60·8	57·2	54·7	*5·7*
—	—	199·2	175·8	154·8	134·2	119·6	107·0	97·2	88·4	80·6	74·4	69·4	64·9	61·5	*5·8*
—	—	266·0	221·6	192·8	163·6	142·8	128·0	114·6	104·0	94·6	86·0	79·6	74·4	70·2	*5·9*
—	—	378·1	284·0	231·5	198·2	171·6	151·5	136·2	123·8	112·8	103·4	94·8	87·5	81·4	*6·0*
—	—	—	—	—	—	206·3	186·3	167·5	150·0	134·2	121·5	111·0	101·8	92·8	*6·1*
—	—	—	—	—	—	264	232	205	280	160	141	128	116	107	*6·2*
—	—	—	—	—	—	350	297	251	216	188	164	148	132	122	*6·3*
—	—	—	—	—	—	510	376	308	263	225	196	172	152	138	*6·4*
—	—	—	—	—	—	889	524	387	313	264	229	200	177	161	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	237	204	184	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	286	249	220	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	363	305	268	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	485	392	333	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	747	510	416	*7·0*

TABLE XXXIX. *To find the correlation in errors of β_1 and β_2.*

Values of $R_{\beta_1\beta_2}$.

β_1

β_2	*0·00*	*0·05*	*0·10*	*0·15*	*0·20*	*0·25*	*0·30*	*0·35*	*0·40*	*0·45*	*0·50*	*0·55*	*0·60*	*0·65*	*0·70*	*0·75*
2·0	0·00	·570	·706	·770	·823	·863	·894	·917	·935	·949	·960	·968	·976	·983	·989	·992
2·1	0·00	·557	·685	·755	·798	·838	·870	·895	·914	·936	·948	·959	·969	·977	·983	·986
2·2	0·00	·551	·672	·728	·771	·814	·847	·874	·896	·919	·934	·948	·960	·968	·975	·980
2·3	0·00	·550	·663	·719	·765	·799	·829	·859	·880	·900	·919	·935	·948	·957	·965	·971
2·4	0·00	·551	·660	·712	·752	·787	·814	·843	·867	·886	·905	·920	·936	·945	·954	·962
2·5	0·00	·554	·659	·706	·745	·776	·805	·834	·858	·878	·893	·908	·924	·933	·941	·949
2·6	0·00	·557	·662	·706	·742	·773	·799	·825	·851	·871	·883	·898	·913	·921	·930	·938
2·7	0·00	·557	·668	·710	·744	·773	·800	·825	·846	·863	·876	·889	·902	·911	·920	·928
2·8	0·00	·556	·674	·716	·750	·779	·803	·826	·842	·858	·871	·883	·893	·901	·910	·919
2·9	0·00	·550	·680	·724	·760	·787	·810	·830	·844	·857	·868	·878	·887	·895	·903	·912
3·0	0·00	·542	·684	·738	·774	·796	·816	·835	·847	·857	·867	·875	·883	·890	·898	·906
3·1	0·00	·534	·687	·744	·781	·808	·825	·840	·850	·858	·867	·874	·882	·889	·897	·903
3·2	0·00	·524	·688	·746	·786	·811	·830	·842	·852	·860	·868	·875	·882	·889	·896	·902
3·3	0·00	·512	·688	·747	·788	·814	·832	·845	·855	·863	·870	·876	·882	·888	·895	·901
3·4	0·00	·501	·686	·748	·790	·816	·833	·848	·858	·865	·872	·878	·883	·889	·895	·900
3·5	0·00	·490	·681	·747	·790	·815	·833	·849	·860	·867	·873	·879	·884	·890	·895	·900
3·6	0·00	·477	·676	·745	·788	·813	·832	·850	·860	·867	·874	·880	·886	·891	·896	·900
3·7	0·00	·462	·670	·741	·784	·810	·831	·848	·859	·867	·874	·881	·887	·892	·897	·901
3·8	0·00	·450	·662	·736	·779	·803	·828	·845	·858	·866	·874	·882	·888	·893	·898	·901
3·9	0·00	·438	·654	·720	·770	·796	·822	·841	·856	·866	·875	·882	·889	·894	·899	·903
4·0	0·00	·422	·645	·713	·760	·788	·816	·837	·853	·865	·873	·881	·888	·894	·899	·903
4·1	—	—	·630	·702	·748	·780	·807	·830	·849	·862	·871	·880	·887	·892	·897	·901
4·2	—	—	·608	·682	·733	·770	·793	·822	·842	·857	·867	·877	·884	·890	·894	·899
4·3	—	—	·580	·658	·712	·753	·784	·811	·832	·848	·860	·871	·878	·885	·890	·897
4·4	—	—	·540	·628	·688	·732	·770	·796	·819	·837	·851	·863	·872	·880	·887	·894
4·5	—	—	·481	·590	·657	·709	·749	·780	·804	·824	·841	·853	·865	·874	·882	·890
4·6	—	—	—	—	—	—	·716	·754	·784	·808	·828	·842	·856	·868	·877	·886
4·7	—	—	—	—	—	—	·674	·723	·759	·788	·812	·830	·846	·860	·870	·879
4·8	—	—	—	—	—	—	·615	·681	·727	·761	·791	·815	·834	·849	·861	·872
4·9	—	—	—	—	—	—	·532	·620	·680	·728	·766	·795	·818	·835	·850	·862
5·0	—	—	—	—	—	—	·362	·534	·628	·687	·731	·767	·798	·822	·837	·851
5·1	—	—	—	—	—	—	—	—	—	—	—	—	·768	·799	·820	·837
5·2	—	—	—	—	—	—	—	—	—	—	—	—	·730	·768	·799	·820
5·3	—	—	—	—	—	—	—	—	—	—	—	—	·679	·729	·769	·799
5·4	—	—	—	—	—	—	—	—	—	—	—	—	·608	·686	·736	·774
5·5	—	—	—	—	—	—	—	—	—	—	—	—	·496	·601	·674	·724
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XXXIX.—(*continued*).

Values of $R_{\beta_1\beta_2}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	
·993	·995	·997	·999	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	*2·0*
·989	·991	·994	·996	·998	·998	·999	·999	·999	1·000	1·000	1·000	1·000	1·000	1·000	*2·1*
·983	·986	·989	·992	·995	·996	·997	·998	·998	·999	1·000	1·000	1·000	1·000	1·000	*2·2*
·976	·980	·984	·988	·992	·993	·994	·995	·997	·998	·999	·999	·999	1·000	1·000	*2·3*
·968	·973	·978	·983	·987	·989	·991	·993	·995	·996	·998	·998	·999	·999	1·000	*2·4*
·958	·965	·972	·977	·982	·985	·988	·990	·992	·994	·996	·997	·998	·999	1·000	*2·5*
·947	·956	·964	·970	·976	·980	·984	·986	·988	·991	·993	·995	·997	·998	·999	*2·6*
·937	·947	·957	·963	·968	·973	·977	·980	·983	·986	·989	·992	·995	·997	·998	*2·7*
·928	·939	·949	·955	·960	·965	·970	·974	·978	·981	·985	·989	·992	·994	·996	*2·8*
·921	·932	·942	·947	·952	·957	·963	·968	·972	·976	·980	·984	·988	·990	·992	*2·9*
·915	·923	·931	·937	·943	·948	·954	·960	·966	·971	·975	·979	·983	·986	·988	*3·0*
·909	·915	·922	·929	·936	·942	·947	·953	·959	·965	·970	·974	·978	·981	·984	*3·1*
·907	·912	·918	·924	·930	·936	·941	·946	·952	·958	·963	·968	·973	·977	·980	*3·2*
·906	·909	·914	·919	·925	·930	·935	·940	·946	·951	·956	·961	·966	·971	·975	*3·3*
·905	·908	·912	·916	·920	·925	·930	·935	·940	·945	·950	·954	·958	·964	·974	*3·4*
·904	·907	·910	·914	·918	·922	·926	·931	·936	·940	·944	·948	·952	·958	·965	*3·5*
·904	·907	·910	·914	·918	·921	·924	·928	·932	·935	·938	·942	·946	·952	·959	*3·6*
·905	·907	·910	·914	·917	·920	·923	·927	·930	·933	·935	·937	·940	·946	·953	*3·7*
·905	·908	·911	·914	·917	·920	·922	·925	·928	·930	·932	·934	·936	·941	·948	*3·8*
·906	·909	·911	·914	·917	·919	·921	·924	·927	·929	·931	·933	·935	·939	·944	*3·9*
·906	·909	·912	·914	·917	·919	·921	·923	·926	·928	·930	·932	·934	·936	·940	*4·0*
·905	·908	·911	·914	·917	·919	·921	·923	·925	·927	·930	·931	·932	·933	·934	*4·1*
·905	·907	·910	·913	·916	·919	·921	·923	·924	·926	·929	·929	·930	·930	·929	*4·2*
·903	·906	·910	·913	·916	·918	·920	·922	·924	·926	·929	·928	·928	·927	·924	*4·3*
·900	·904	·908	·912	·916	·918	·920	·922	·923	·926	·928	·927	·927	·925	·922	*4·4*
·897	·902	·906	·910	·915	·918	·920	·922	·923	·926	·928	·927	·926	·923	·920	*4·5*
·893	·898	·903	·908	·913	·916	·919	·920	·922	·925	·927	·926	·925	·923	·920	*4·6*
·887	·894	·900	·905	·910	·913	·917	·919	·921	·924	·926	·925	·925	·923	·922	*4·7*
·881	·890	·896	·901	·906	·910	·914	·917	·920	·923	·925	·926	·926	·925	·925	*4·8*
·874	·884	·890	·895	·901	·907	·911	·915	·919	·922	·925	·926	·927	·927	·928	*4·9*
·863	·875	·883	·889	·896	·903	·908	·913	·918	·922	·925	·927	·928	·930	·932	*5·0*
·851	·864	·875	·882	·890	·898	·905	·911	·917	·922	·925	·928	·931	·933	·936	*5·1*
·837	·852	·866	·875	·884	·892	·901	·909	·916	·921	·924	·928	·933	·937	·941	*5·2*
·820	·839	·853	·865	·876	·885	·895	·904	·913	·918	·923	·929	·935	·940	·945	*5·3*
·798	·818	·837	·853	·867	·877	·888	·898	·908	·915	·921	·928	·935	·941	·947	*5·4*
·764	·792	·817	·837	·854	·867	·880	·890	·900	·910	·918	·925	·933	·940	·947	*5·5*
—	—	·789	·815	·835	·852	·868	·880	·890	·904	·911	·917	·926	·935	·944	*5·6*
—	—	·750	·786	·811	·835	·854	·869	·880	·892	·901	·909	·917	·927	·938	*5·7*
—	—	·701	·748	·783	·811	·835	·852	·866	·879	·890	·897	·905	·915	·928	*5·8*
—	—	·640	·700	·748	·781	·810	·828	·846	·861	·875	·883	·892	·901	·913	*5·9*
—	—	·544	·639	·703	·746	·778	·802	·825	·842	·857	·867	·879	·886	·893	*6·0*
—	—	—	—	—	—	·741	·769	·796	·820	·837	·852	·866	·872	·873	*6·1*
—	—	—	—	—	—	·691	·727	·762	·792	·815	·836	·852	·858	·856	*6·2*
—	—	—	—	—	—	·628	·678	·724	·761	·790	·818	·838	·845	·842	*6·3*
—	—	—	—	—	—	·526	·606	·675	·724	·763	·793	·818	·831	·834	*6·4*
—	—	—	—	—	—	·354	·526	·619	·680	·726	·761	·791	·814	·831	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	·761	·790	·832	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	·721	·760	·837	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	·670	·727	·845	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	·600	·683	·857	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	·468	·602	·876	*7·0*

β_2

TABLE XL. *To find the Probable Error of the distance from Mean to Mode.*

Values of $\frac{\sqrt{N}}{\sigma}\Sigma_d$.

β_1.

β_2	*0·00*	*0·05*	*0·10*	*0·15*	*0·20*	*0·25*	*0·30*	*0·35*	*0·40*	*0·45*	*0·50*	*0·55*	*0·60*	*0·65*	*0·70*	*0·75*	*0·80*
2·0	3·54	—	—	—	—	—	—	—	—	—	3·03	2·44	2·10	1·80	1·58	1·42	1·30
2·1	2·15	4·36	—	—	—	—	—	—	—	—	—	—	—	3·10	2·53	2·16	1·88
2·2	1·87	2·75	9·65	—	—	—	—	—	—	—	—	—	—	—	3·91	3·17	2·60
2·3	1·64	1·86	3·00	—	—	—	—	—	—	—	—	—	—	—	—	—	3·78
2·4	1·46	1·58	2·07	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2·5	1·35	1·46	1·67	2·08	2·87	4·04	5·21	—	—	—	—	—	—	—	—	—	—
2·6	1·28	1·37	1·58	1·98	2·60	3·42	4·43	6·72	—	—	—	—	—	—	—	—	—
2·7	1·25	1·30	1·50	1·83	2·34	2·98	3·75	5·06	7·48	—	—	—	—	—	—	—	—
2·8	1·23	1·28	1·43	1·71	2·11	2·60	3·17	4·12	5·28	7·45	—	—	—	—	—	—	—
2·9	1·22	1·27	1·38	1·60	1·90	2·27	2·69	3·20	3·84	4·78	6·65	—	—	—	—	—	—
3·0	1·23	1·26	1·34	1·51	1·73	1·98	2·29	2·63	3·06	3·58	4·28	5·18	6·43	8·24	10·89	—	—
3·1	1·25	1·27	1·32	1·44	1·58	1·76	2·00	2·23	2·54	2·94	3·42	3·93	4·52	5·50	6·76	8·66	—
3·2	1·27	1·28	1·30	1·38	1·48	1·60	1·75	1·92	2·13	2·37	2·72	3·12	3·54	4·21	5·07	6·22	8·00
3.3	1·29	1·29	1·28	1·32	1·39	1·47	1·55	1·68	1·83	2·03	2·27	2·57	2·90	3·39	4·04	4·66	5·53
3.4	1·30	1·29	1·28	1·29	1·31	1·37	1·45	1·54	1·63	1·79	2·00	2·24	2·51	2·88	3·36	3·86	4·50
3·5	1·31	1·30	1·29	1·27	1·25	1·30	1·37	1·45	1·54	1·66	1·83	2·03	2·26	2·55	2·89	3·18	3·61
3·6	1·32	1·31	1·30	1·26	1·22	1·26	1·32	1·40	1·50	1·61	1·74	1·89	2·08	2·31	2·56	2·86	3·24
3·7	1·31	1·31	1·31	1·26	1·22	1·25	1·30	1·37	1·46	1·57	1·69	1·82	1·97	2·14	2·34	2·62	2·95
3·8	1·30	1·31	1·32	1·28	1·25	1·27	1·32	1·38	1·46	1·55	1·65	1·76	1·88	2·03	2·20	2·43	2·69
3·9	1·29	1·33	1·35	1·33	1·30	1·32	1·36	1·41	1·48	1·56	1·64	1·73	1·84	1·96	2·11	2·27	2·49
4·0	1·27	1·37	1·40	1·39	1·39	1·40	1·42	1·46	1·51	1·58	1·65	1·73	1·83	1·94	2·06	2·19	2·36
4·1	—	—	1·47	1·48	1·50	1·51	1·53	1·55	1·57	1·61	1·66	1·75	1·85	1·94	2·03	2·13	2·26
4·2	—	—	1·58	1·62	1·64	1·65	1·65	1·65	1·65	1·66	1·70	1·77	1·85	1·94	2·03	2·12	2·22
4·3	—	—	1·75	1·77	1·78	1·79	1·78	1·76	1·75	1·75	1·78	1·83	1·90	1·97	2·05	2·13	2·23
4·4	—	—	1·98	1·97	1·95	1·94	1·93	1·90	1·88	1·89	1·92	1·96	2·01	2·07	2·13	2·20	2·29
4·5	—	—	2·27	2·20	2·15	2·11	2·10	2·09	2·09	2·10	2·12	2·15	2·18	2·22	2·26	2·32	2·38
4·6	—	—	—	—	—	—	2·44	2·40	2·38	2·36	2·34	2·36	2·38	2·40	2·42	2·45	2·48
4·7	—	—	—	—	—	—	2·93	2·85	2·78	2·71	2·67	2·65	2·64	2·63	2·62	2·61	2·61
4·8	—	—	—	—	—	—	3·74	3·52	3·33	3·16	3·07	3·00	2·95	2·90	2·85	2·81	2·80
4·9	—	—	—	—	—	—	5·44	4·64	4·16	3·87	3·63	3·45	3·32	3·24	3·11	3·05	3·03
5·0	—	—	—	—	—	—	10·66	6·83	5·53	4·84	4·37	4·04	3·79	3·62	3·47	3·37	3·31
5·1	—	—	—	—	—	—	—	—	—	—	—	—	4·46	4·21	3·99	3·85	3·74
5·2	—	—	—	—	—	—	—	—	—	—	—	—	5·38	5·05	4·73	4·47	4·24
5·3	—	—	—	—	—	—	—	—	—	—	—	—	6·84	6·19	5·66	5·27	4·92
5·4	—	—	—	—	—	—	—	—	—	—	—	—	9·24	7·96	7·00	6·24	5·74
5·5	—	—	—	—	—	—	—	—	—	—	—	—	14·81	10·89	8·87	7·64	6·81
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XL—(*continued*).

Values of $\frac{\sqrt{N}}{\sigma}\Sigma_d$.

β_1

0·85	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
1·20	1·13	1·07	1·02	·97	·92	·87	·83	·80	·76	·72	·68	·64	·60	*2·0*
1·64	1·49	1·38	1·29	1·21	1·14	1·08	1·03	·99	·94	·90	·86	·82	·78	*2·1*
2·22	1·97	1·80	1·66	1·54	1·45	1·38	1·32	1·26	1·20	1·14	1·08	1·02	·96	*2·2*
3·10	2·63	2·46	2·15	1·98	1·85	1·75	1·67	1·58	1·50	1·41	1·32	1·24	1·16	*2·3*
—	3·72	3·20	2·81	2·56	2·36	2·22	2·09	1·96	1·84	1·73	1·62	1·51	1·39	*2·4*
—	—	—	3·94	3·40	3·08	2·87	2·66	2·47	2·29	2·12	1·96	1·80	1·64	*2·5*
—	—	—	—	—	4·32	3·82	3·48	3·14	2·86	2·60	2·34	2·10	1·91	*2·6*
—	—	—	—	—	—	—	4·82	4·20	3·66	3·18	2·78	2·46	2·19	*2·7*
—	—	—	—	—	—	—	—	5·73	4·82	3·88	3·28	2·87	2·49	*2·8*
—	—	—	—	—	—	—	—	—	6·63	4·81	3·94	3·31	2·80	*2·9*
—	—	—	—	—	—	—	—	—	—	6·15	4·80	3·79	3·12	*3·0*
—	—	—	—	—	—	—	—	—	—	—	—	4·29	3·48	*3·1*
—	—	—	—	—	—	—	—	—	—	—	—	—	3·88	*3·2*
6·80	—	—	—	—	—	—	—	—	—	—	—	—	—	*3·3*
5·38	6·55	—	—	—	—	—	—	—	—	—	—	—	—	*3·4*
4·21	4·95	6·00	7·33	9·36	12·16	—	—	—	—	—	—	—	—	*3·5*
3·74	4·34	5·12	6·03	7·17	8·80	11·52	—	—	—	—	—	—	—	*3·6*
3·34	3·80	4·32	5·01	5·78	6·65	8·28	11·12	—	—	—	—	—	—	*3·7*
3·00	3·35	3·74	4·23	4·72	5·41	6·36	8·00	10·22	—	—	—	—	—	*3·8*
2·74	3·00	3·32	3·66	4·04	4·50	5·18	6·16	7·43	9·32	—	—	—	—	*3·9*
2·55	2·77	3·02	3·29	3·60	3·98	4·50	5·12	5·92	6·91	8·00	9·23	11·08	—	*4·0*
2·42	2·60	2·79	3·00	3·27	3·61	4·06	4·61	5·20	5·90	6·80	7·86	9·48	11·58	*4·1*
2·35	2·50	2·67	2·86	3·09	3·38	3·71	4·13	4·60	5·15	5·86	6·72	7·76	9·10	*4·2*
2·35	2·48	2·62	2·77	2·96	3·21	3·49	3·82	4·18	4·59	5·14	5·82	6·66	7·59	*4·3*
2·39	2·50	2·61	2·73	2·89	3·10	3·32	3·55	3·83	4·16	4·60	5·16	5·82	6·60	*4·4*
2·45	2·53	2·63	2·74	2·87	3·02	3·20	3·39	3·61	3·87	4·21	4·66	5·18	5·86	*4·5*
2·52	2·60	2·69	2·79	2·89	3·01	3·15	3·31	3·48	3·67	3·95	4·31	4·77	5·36	*4·6*
2·64	2·69	2·77	2·85	2·93	3·02	3·13	3·25	3·39	3·55	3·78	4·10	4·50	4·99	*4·7*
2·81	2·82	2·86	2·93	3·00	3·08	3·17	3·27	3·38	3·50	3·68	3·94	4·26	4·66	*4·8*
3·02	3·03	3·04	3·08	3·12	3·16	3·22	3·30	3·39	3·50	3·63	3·81	4·07	4·35	*4·9*
3·28	3·26	3·25	3·26	3·28	3·31	3·34	3·39	3·44	3·51	3·61	3·73	3·90	4·09	*5·0*
3·64	3·55	3·51	3·49	3·47	3·47	3·48	3·49	3·52	3·56	3·61	3·68	3·77	3·90	*5·1*
4·04	3·90	3·81	3·74	3·68	3·65	3·63	3·62	3·61	3·62	3·63	3·65	3·69	3·76	*5·2*
4·60	4·35	4·18	4·05	3·95	3·88	3·81	3·75	3·72	3·70	3·69	3·68	3·69	3·70	*5·3*
5·33	4·98	4·71	4·52	4·37	4·24	4·12	4·01	3·92	3·85	3·79	3·75	3·73	3·72	*5·4*
6·21	5·74	5·36	5·08	4·86	4·66	4·48	4·32	4·20	4·09	3·99	3·92	3·87	3·82	*5·5*
—	6·69	6·27	5·83	5·49	5·19	4·94	4·73	4·56	4·42	4·30	4·18	4·11	4·04	*5·6*
—	8·11	7·48	6·82	6·32	5·90	5·55	5·27	5·03	4·84	4·68	4·54	4·43	4·35	*5·7*
—	10·18	9·11	8·12	7·45	6·85	6·35	5·95	5·62	5·37	5·16	5·00	4·87	4·76	*5·8*
—	13·53	11·44	9·84	8·71	7·94	7·32	6·82	6·45	6·13	5·85	5·61	5·43	5·29	*5·9*
—	19·95	14·26	11·92	10·48	9·38	8·55	7·89	7·38	6·95	6·62	6·33	6·10	5·90	*6·0*
—	—	—	—	—	11·64	10·26	9·31	8·62	8·03	7·53	7·15	6·87	6·60	*6·1*
—	—	—	—	—	14·83	12·55	11·19	10·24	9·40	8·64	8·08	7·68	7·36	*6·2*
—	—	—	—	—	19·65	15·85	13·69	12·21	11·01	10·02	9·19	8·65	8·22	*6·3*
—	—	—	—	—	28·03	20·85	17·09	14·56	12·84	11·45	10·45	9·69	9·11	*6·4*
—	—	—	—	—	47·99	28·04	21·30	17·44	15·07	13·20	11·90	10·83	10·07	*6·5*
—	—	—	—	—	—	—	—	—	—	—	14·2	12·9	12·4	*6·6*
—	—	—	—	—	—	—	—	—	—	—	17·2	15·6	14·7	*6·7*
—	—	—	—	—	—	—	—	—	—	—	21·8	19·6	18·3	*6·8*
—	—	—	—	—	—	—	—	—	—	—	29·4	26·0	24·1	*6·9*
—	—	—	—	—	—	—	—	—	—	—	43·0	37·8	34·9	*7·0*

TABLE XLI. *To find the Probable Error of the Skewness sk.*

Values of $\sqrt{N}\,\Sigma_{sk}$.

β_1

β_2	*0·00*	*0·05*	*0·10*	*0·15*	*0·20*	*0·25*	*0·30*	*0·35*	*0·40*	*0·45*	*0·50*	*0·55*	*0·60*	*0·65*	*0·70*	*0·75*
2·0	3·54	—	—	—	—	—	—	—	—	—	3·41	2·80	2·43	2·12	1·89	1·72
2·1	2·15	4·20	—	—	—	—	—	—	—	—	—	—	—	3·67	3·02	2·58
2·2	1·87	2·63	9·50	—	—	—	—	—	—	—	—	—	—	—	5·57	4·10
2·3	1·64	1·78	2·88	—	—	—	—	—	—	—	—	—	—	—	—	—
2·4	1·46	1·49	2·02	—	—	—	—	—	—	—	—	—	—	—	—	—
2·5	1·35	1·41	1·62	2·02	2·80	4·06	5·08	—	—	—	—	—	—	—	—	—
2·6	1·28	1·30	1·43	1·75	2·18	2·82	3·65	4·96	—	—	—	—	—	—	—	—
2·7	1·25	1·25	1·31	1·52	1·84	2·29	2·85	3·68	4·84	—	—	—	—	—	—	—
2·8	1·23	1·22	1·24	1·36	1·59	1·89	2·21	2·70	3·36	4·36	—	—	—	—	—	—
2·9	1·22	1·20	1·20	1·28	1·43	1·63	1·86	2·19	2·60	3·33	4·30	—	—	—	—	—
3·0	1·23	1·21	1·20	1·25	1·34	1·48	1·62	1·84	2·12	2·49	3·00	3·76	4·62	6·02	8·12	—
3·1	1·25	1·22	1·21	1·23	1·27	1·36	1·50	1·67	1·88	2·16	2·50	3·06	3·76	4·72	6·10	8·08
3·2	1·27	1·23	1·22	1·22	1·23	1·29	1·40	1·53	1·70	1·90	2·17	2·58	3·17	3·97	4·87	5·83
3·3	1·29	1·25	1·23	1·21	1·21	1·24	1·33	1·44	1·58	1·74	1·94	2·27	2·73	3·28	3·88	4·53
3·4	1·30	1·27	1·24	1·21	1·20	1·23	1·29	1·38	1·49	1·61	1·78	2·04	2·36	2·75	3·18	3·63
3·5	1·31	1·29	1·25	1·21	1·20	1·22	1·28	1·35	1·43	1·54	1·68	1·87	2·09	2·37	2·68	2·98
3·6	1·32	1·30	1·26	1·22	1·20	1·22	1·26	1·32	1·40	1·50	1·61	1·75	1·91	2·13	2·39	2·65
3·7	1·31	1·31	1·28	1·24	1·20	1·23	1·27	1·32	1·39	1·47	1·56	1·67	1·80	1·98	2·19	2·40
3·8	1·30	1·32	1·30	1·27	1·23	1·25	1·28	1·33	1·38	1·46	1·54	1·63	1·75	1·88	2·04	2·22
3·9	1·29	1·34	1·33	1·30	1·28	1·28	1·30	1·34	1·39	1·45	1·53	1·61	1·71	1·82	1·96	2·12
4·0	1·27	1·36	1·38	1·36	1·35	1·35	1·36	1·39	1·43	1·48	1·56	1·63	1·72	1·81	1·92	2·04
4·1	—	—	1·46	1·45	1·45	1·44	1·44	1·46	1·50	1·54	1·60	1·67	1·74	1·82	1·91	2·03
4·2	—	—	1·58	1·57	1·57	1·56	1·55	1·56	1·59	1·62	1·67	1·72	1·78	1·85	1·93	2·02
4·3	—	—	1·75	1·74	1·73	1·71	1·70	1·69	1·70	1·72	1·76	1·81	1·86	1·92	1·98	2·06
4·4	—	—	1·95	1·94	1·92	1·90	1·88	1·85	1·86	1·87	1·90	1·93	1·97	2·01	2·06	2·12
4·5	—	—	2·26	2·19	2·13	2·09	2·07	2·06	2·05	2·06	2·07	2·08	2·11	2·15	2·19	2·23
4·6	—	—	—	—	—	—	2·48	2·42	2·37	2·35	2·33	2·31	2·31	2·32	2·33	2·35
4·7	—	—	—	—	—	—	3·11	2·93	2·82	2·73	2·65	2·60	2·57	2·54	2·52	2·50
4·8	—	—	—	—	—	—	3·78	3·53	3·35	3·21	3·08	2·97	2·89	2·82	2·76	2·71
4·9	—	—	—	—	—	—	5·48	4·66	4·17	3·87	3·63	3·44	3·30	3·17	3·07	3·01
5·0	—	—	—	—	—	—	11·12	6·96	5·52	4·82	4·36	4·02	3·77	3·58	3·45	3·37
5·1	—	—	—	—	—	—	—	—	—	—	—	—	4·45	4·16	4·00	3·87
5·2	—	—	—	—	—	—	—	—	—	—	—	—	5·38	5·02	4·72	4·49
5·3	—	—	—	—	—	—	—	—	—	—	—	—	6·84	6·18	5·66	5·26
5·4	—	—	—	—	—	—	—	—	—	—	—	—	9·24	7·76	6·80	6·22
5·5	—	—	—	—	—	—	—	—	—	—	—	—	14·80	10·67	8·87	7·71
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLI—(*continued*).

Values of $\sqrt{N}\Sigma_{sk}$.

β_1

0·80	*0·85*	*0·90*	*0·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
1·59	1·48	1·39	1·30	1·24	1·19	1·14	1·10	1·06	1·02	·99	·95	·91	·87	·83	*2·0*
2·20	1·95	1·80	1·68	1·58	1·52	1·47	1·42	1·37	1·32	1·26	1·20	1·15	1·10	1·05	*2·1*
3·22	2·65	2·29	2·08	1·98	1·91	1·84	1·78	1·72	1·66	1·59	1·52	1·45	1·38	1·31	*2·2*
5·23	3·80	3·04	2·75	2·53	2·40	2·30	2·21	2·12	2·03	1·94	1·85	1·76	1·67	1·58	*2·3*
—	—	4·29	3·64	3·31	3·12	2·94	2·78	2·63	2·49	2·36	2·23	2·10	1·98	1·86	*2·4*
—	—	—	—	4·77	4·27	3·84	3·55	3·29	3·06	2·85	2·66	2·49	2·32	2·15	*2·5*
—	—	—	—	—	—	5·72	5·00	4·39	3·94	3·54	3·20	2·93	2·67	2·44	*2·6*
—	—	—	—	—	—	—	—	6·25	5·20	4·46	3·88	3·42	3·08	2·74	*2·7*
—	—	—	—	—	—	—	—	—	7·05	5·68	4·78	4·03	3·54	3·05	*2·8*
—	—	—	—	—	—	—	—	—	—	7·45	6·00	4·85	4·11	3·37	*2·9*
—	—	—	—	—	—	—	—	—	—	—	7·80	6·05	4·77	3·70	*3·0*
—	—	—	—	—	—	—	—	—	—	—	—	—	5·57	4·10	*3·1*
7·00	—	—	—	—	—	—	—	—	—	—	—	—	—	4·58	*3·2*
5·30	6·24	—	—	—	—	—	—	—	—	—	—	—	—	—	*3·3*
4·16	4·71	5·60	—	—	—	—	—	—	—	—	—	—	—	—	*3·4*
3·38	3·89	4·59	5·58	6·85	8·38	11·48	—	—	—	—	—	—	—	—	*3·5*
2·95	3·38	3·99	4·66	5·50	6·48	7·55	9·92	—	—	—	—	—	—	—	*3·6*
2·67	3·05	3·49	3·97	4·52	5·22	6·00	7·36	9·42	—	—	—	—	—	—	*3·7*
2·44	2·75	3·08	3·43	3·80	4·25	4·78	5·64	7·08	9·02	—	—	—	—	—	*3·8*
2·29	2·53	2·79	3·07	3·38	3·70	4·15	4·77	5·62	6·89	8·76	—	—	—	—	*3·9*
2·20	2·38	2·58	2·82	3·06	3·33	3·69	4·14	4·77	5·50	6·42	7·40	8·57	10·12	—	*4·0*
2·16	2·31	2·47	2·65	2·85	3·09	3·34	3·72	4·15	4·61	5·14	5·84	6·80	8·44	11·00	*4·1*
2·14	2·26	2·40	2·55	2·71	2·88	3·10	3·37	3·67	4·03	4·44	5·04	5·94	7·12	8·67	*4·2*
2·16	2·27	2·38	2·50	2·65	2·80	2·96	3·14	3·37	3·65	4·01	4·55	5·28	6·18	7·21	*4·3*
2·20	2·30	2·41	2·52	2·64	2·76	2·89	3·04	3·23	3·48	3·78	4·22	4·78	5·44	6·26	*4·4*
2·29	2·35	2·44	2·53	2·63	2·75	2·88	3·02	3·20	3·40	3·65	3·97	4·40	4·91	5·56	*4·5*
2·39	2·43	2·48	2·56	2·66	2·77	2·89	3·01	3·16	3·34	3·55	3·79	4·16	4·55	5·10	*4·6*
2·52	2·54	2·58	2·63	2·71	2·80	2·90	3·01	3·14	3·29	3·47	3·67	3·96	4·28	4·72	*4·7*
2·70	2·72	2·74	2·78	2·83	2·89	2·97	3·06	3·16	3·28	3·41	3·58	3·80	4·06	4·42	*4·8*
2·98	2·97	2·96	2·97	3·00	3·04	3·08	3·14	3·21	3·30	3·40	3·53	3·68	3·88	4·16	*4·9*
3·31	3·25	3·21	3·20	3·21	3·22	3·24	3·26	3·31	3·36	3·43	3·51	3·60	3·73	3·96	*5·0*
3·75	3·64	3·55	3·49	3·45	3·42	3·41	3·41	3·43	3·44	3·46	3·51	3·57	3·65	3·78	*5·1*
4·28	4·10	3·96	3·85	3·76	3·68	3·63	3·60	3·57	3·55	3·53	3·53	3·55	3·60	3·67	*5·2*
4·93	4·69	4·48	4·29	4·13	4·02	3·92	3·84	3·76	3·68	3·63	3·60	3·58	3·59	3·62	*5·3*
5·78	5·42	5·09	4·80	4·56	4·40	4·26	4·13	4·00	3·89	3·80	3·73	3·68	3·65	3·63	*5·4*
6·94	6·32	5·82	5·40	5·07	4·84	4·64	4·46	4·30	4·17	4·06	3·95	3·87	3·80	3·75	*5·5*
—	—	6·75	6·22	5·79	5·46	5·19	4·97	4·77	4·60	4·44	4·30	4·19	4·10	4·01	*5·6*
—	—	8·15	7·30	6·73	6·26	5·91	5·61	5·34	5·10	4·90	4·73	4·59	4·47	4·38	*5·7*
—	—	10·20	8·76	7·98	7·26	6·76	6·34	5·99	5·68	5·44	5·24	5·06	4·91	4·78	*5·8*
—	—	13·53	10·83	9·66	8·71	7·90	7·28	6·78	6·40	6·10	5·84	5·62	5·44	5·28	*5·9*
—	—	19·96	14·30	12·02	10·51	9·39	8·56	7·90	7·39	6·96	6·61	6·33	6·10	5·89	*6·0*
—	—	—	—	—	—	11·64	10·26	9·31	8·56	8·03	7·53	7·15	6·82	6·54	*6·1*
—	—	—	—	—	—	14·83	12·55	11·19	10·13	9·30	8·64	8·08	7·63	7·24	*6·2*
—	—	—	—	—	—	19·65	15·85	13·69	12·21	11·01	10·02	9·19	8·55	8·01	*6·3*
—	—	—	—	—	—	28·03	20·85	17·09	14·56	12·84	11·45	10·45	9·60	8·92	*6·4*
—	—	—	—	—	—	47·99	28·04	21·30	17·44	15·07	13·20	11·90	10·83	10·07	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	14·2	12·9	12·4	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	17·2	15·6	14·7	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	21·8	19·6	18·3	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	29·4	26·0	24·1	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	43·0	37·8	34·9	*7·0*

TABLE XLII. *To give values of β_3, β_4, β_5 and β_6 in terms*

TABLE XLII (*a*).

Values of β_3.

β_2

β_1	*2·0*	*2·5*	*3·0*	*3·5*	*4·0*	*4·5*	*5·0*	*5·5*	*6·0*	*6·5*	*7·0*
0·0	0	0	0	0	0						
0·1	0·48493	0·68971	0·94286	1·25688	1·64906	2·14375					
0·2	0·91585	1·32958	1·78182	2·37049	3·10000	4·01176					
0·3	1·29873	1·85270	2·53043	3·36094	4·38305	5·65000	7·2368				
0·4	1·63902	2·34286	3·20000	4·24478	5·52277	7·09474	9·0462				
0·5	1·94118	2·78126	3·80000	5·03582	6·53846	8·37500	10·6364				
0·6	2·20909	3·17350	4·33846	5·80178	7·44706	9·51429	12·0414	15·1585			
0·7	2·44615	3·52441	4·82222	6·38287	8·26202	10·53182	13·2885	16·6624			
0·8	2·65532	3·83820	5·25714	6·95698	8·99462	11·44375	14·4000	17·9932			
0·9	2·83917	4·12064	5·64828	7·47438	9·65454	12·26250	15·3940	19·1758	23·7791		
1·0	3·00000	4·36842	6·00000	7·94121	10·24999	13·00000	16·2857	20·2308	25·0000		
1·1	3·13980	4·59081	6·31613	8·36246	10·78796	13·66538	17·0877	21·1750	26·0857	32·0328	
1·2	3·26038	4·78812	6·60000	8·74286	11·27443	14·26667	17·8105	22·0225	27·0546	33·1082	
1·3	3·36330	4·96250	6·85454	9·08619	11·71461	14·81071	18·4633	22·7851	27·9217	34·0635	
1·4	3·45000	5·11589	7·08235	9·39582	12·11304	15·30345	19·0536	23·4727	28·7000	34·9164	42·3613
1·5	3·52174	5·25000	7·28571	9·67501	12·47368	15·75000	19·5864	24·0937	29·4000	35·6786	43·1538

TABLE XLII (*b*).

Values of β_4.

β_2

β_1	*2·0*	*2·5*	*3·0*	*3·5*	*4·0*	*4·5*	*5·0*	*5·5*	*6·0*	*6·5*	*7·0*
0·0	5·00000	8·92856	15·0000	23·7288	31·0000						
0·1	5·27356	9·41054	15·7973	25·7430	41·7660	69·3682					
0·2	5·44361	9·75086	16·2648	26·4018	42·5000	69·4796					
0·3	5·53293	9·86224	16·4907	26·6520	42·5613	68·5776	114·4732				
0·4	5·55998	9·91072	16·5385	26·6144	42·1807	67·0888	109·4534				
0·5	5·53802	9·87751	16·4545	26·3742	41·5076	65·2679	104·4652				
0·6	5·47791	9·81734	16·2732	26·1077	40·6453	63·2707	99·6442	162·125			
0·7	5·38824	9·63895	16·0200	25·5026	39·6623	61·1946	95·0525	151·253			
0·8	5·27513	9·45991	15·7143	24·9478	38·6061	59·1016	90·7143	141·707			
0·9	5·14437	9·25645	15·3706	24·3462	37·5099	57·0279	86·6331	133·240	210·995		
1·0	5·00000	9·02746	15·0000	23·7495	36·3971	55·0000	82·8022	125·664	195·000		
1·1	4·84537	8·78075	14·6111	23·0744	35·2835	53·0316	79·2091	118·839	181·299	286·374	
1·2	4·68319	8·53522	14·2105	22·4107	34·1811	51·1309	75·8392	112·653	169·394	261·436	
1·3	4·51562	8·27700	13·8032	21·7535	33·0971	49·3447	72·6772	107·016	158·930	240·845	
1·4	4·34440	8·01454	13·3931	21·1002	32·0367	47·5471	69·7076	101·850	149·643	223·304	343·147
1·5	4·17097	7·75000	12·9832	20·4546	31·0037	45·9038	66·9117	97·112	141·333	208·129	313·704

of β_1 and β_2 on the assumption that the Frequency falls into one or other of Pearson's Types.

TABLE XLII (*c*).

Values of β_5.

	β_2										
β_1	*2·0*	*2·5*	*3·0*	*3·5*	*4·0*	*4·5*	*5·0*	*5·5*	*6·0*	*6·5*	*7·0*
0·0	0	0	0	0	0						
0·1	1·99086	4·39480	9·3207	19·9714	45·9387	128·529					
0·2	3·59438	8·03374	16·5960	34·7825	76·6000	193·361					
0·3	4·86677	10·68765	22·2196	45·7142	97·2263	228·104	668·284				
0·4	5·85929	12·85477	26·5187	53·7090	111·0237	246·506	650·398				
0·5	6·51704	14·51540	29·7545	59·4655	120·0543	255·295	614·633				
0·6	7·17383	15·7892	32·1362	63·9266	125·6629	258·147	581·205	1618·635			
0·7	7·56616	16·6546	33·8306	66·2045	128·8283	257·225	550·107	1368·373			
0·8	7·81963	17·2668	34·9714	67·8804	130·2010	253·872	521·257	1196·612			
0·9	7·95777	17·6667	35·6658	68·7533	130·2587	248·937	495·375	1068·877	2769·42		
1·0	8·00000	17·8291	36·0000	69·0644	129·3434	243·000	469·637	968·318	2280·00		
1·1	7·96281	17·8472	36·0437	68·7730	127·7158	236·441	446·547	886·541	1945·69	5313·80	
1·2	7·86015	17·7503	35·8535	68·1357	125·5684	229·524	425·062	818·040	1700·98	4135·56	
1·3	7·70375	17·5396	35·4754	67·2181	123·0362	222·562	405·663	759·486	1512·94	3388·18	
1·4	7·50358	17·2423	34·9467	66·0678	120·5142	214·828	386·347	708·620	1363·20	2870·08	7265·31
1·5	7·26808	16·8768	34·2983	64·7210	117·2460	207·227	368·843	663·926	1240·65	2488·62	5719·68

TABLE XLII (*d*).

Values of β_6.

	β_2										
β_1	*2·0*	*2·5*	*3·0*	*3·5*	*4·0*	*4·5*	*5·0*	*5·5*	*6·0*	*6·5*	*7·0*
0·0	14·0000	39·0649	105·000	290·678	868·015						
0·1	16·4616	45·7741	124·835	355·508	1243·832	10228·33					
0·2	17·7296	50·2472	132·998	369·894	1190·700	6204·69					
0·3	18·1764	51·0927	134·215	361·909	1089·739	4485·38	107697·95				
0·4	18·0667	50·2458	131·337	344·886	977·506	3471·87	25413·18				
0·5	17·5474	48·7896	126·107	323·447	877·884	2792·19	13737·63				
0·6	16·8560	46·8558	119·601	303·252	784·431	2303·07	9048·43	119230·33			
0·7	15·9787	44·3106	112·492	277·658	701·500	1934·79	6534·78	40994·77			
0·8	15·0148	41·7081	105·200	255·716	628·450	1648·52	5045·80	22660·09			
0·9	14·0113	39·0906	97·984	235·072	564·277	1420·51	4024·45	14836·90	137288·7		
1·0	13·0000	36·4119	91·000	216·137	507·894	1235·50	3286·65	10612·25	57584·9		
1·1	12·0030	33·7916	84·339	198·263	456·575	1083·04	2741·39	8135·91	33078·5	797653·2	
1·2	11·0354	31·3418	78·047	181·987	414·455	955·78	2322·13	6314·06	21891·8	155693·9	
1·3	10·1070	28·9775	72·146	167·142	375·834	848·97	1994·05	5108·55	15690·2	75009·7	
1·4	9·2240	26·7355	66·637	153·582	342·057	755·79	1726·18	4219·50	11846·9	44891·9	565740
1·5	8·3899	24·6268	61·512	141·477	310·976	676·32	1508·92	3544·82	9281·2	30280·3	180793

TABLE XLIII.

Probable Error of Criterion κ_2. Values of $\sqrt{N}\Sigma_{\kappa_2}$ for values of β_1, β_2.

β_1

β_2	·00	·05	·10	·15	·20	·25	·30	·35	·40	·45	·50	·55	·60	·65	·70
2·0	·000	·242	·332	·399	·454	·498	·545	·582	·631	·671	·716	·758	·806	·854	·899
2·1	·000	·271	·367	·430	·483	·521	·557	·600	·639	·678	·717	·760	·800	·843	·890
2·2	·000	·310	·415	·480	·527	·565	·597	·626	·656	·691	·728	·767	·809	·845	·890
2·3	·000	·355	·477	·550	·596	·635	·660	·678	·700	·725	·753	·790	·826	·858	·895
2·4	·000	·417	·560	·642	·691	·722	·748	·759	·770	·787	·806	·830	·857	·884	·914
2·5	·000	·500	·697	·771	·816	·841	·855	·860	·867	·873	·881	·892	·909	·928	·947
2·6	·000	·660	·840	·946	1·01	1·03	1·04	1·02	1·01	1·00	1·00	1·00	1·00	1·00	1·00
2·7	·000	1·04	1·30	1·35	1·34	1·33	1·30	1·27	1·24	1·19	1·15	1·12	1·10	1·09	1·08
2·8	·000	1·83	1·97	1·98	1·93	1·84	1·74	1·64	1·55	1·47	1·40	1·33	1·27	1·23	1·20
2·9	·000	3·51	3·71	3·42	3·00	2·66	2·42	2·22	2·05	1·89	1·73	1·61	1·51	1·45	1·39
3·0	—	18·8	9·89	6·94	5·30	4·30	3·62	3·10	2·73	2·43	2·18	2·00	1·84	1·71	1·61
3·1	·000	—	62·0	20·5	8·47	6·36	5·22	4·42	3·78	3·34	2·96	2·66	2·41	2·20	2·01
3·2	·000	7·82	91·7	—	49·7	20·2	12·2	8·48	6·44	5·09	4·28	3·71	3·21	2·86	2·58
3·3	·000	2·99	11·7	70·2	—	142	32·4	15·4	10·8	8·56	6·88	5·62	4·66	3·98	3·46
3·4	·000	1·82	4·80	13·8	55·5	—	344	182	28·9	16·8	11·8	8·69	6·88	5·60	4·76
3·5	·000	1·43	2·89	6·43	15·8	50·6	380	—	127	46·5	25·1	16·1	11·5	8·55	6·91
3·6	·000	1·17	2·18	4·08	8·00	17·2	46·5	215	—	277	76·4	35·4	22·4	15·8	11·5
3·7	·000	1·04	1·79	3·08	5·18	9·36	24·6	44·3	155	—	767	133	54·6	28·5	19·2
3·8	·000	·979	1·54	2·54	4·09	6·33	14·3	26·2	44·0	126	855	—	230	80·0	40·8
3·9	·000	·920	1·41	2·20	3·32	4·91	8·84	13·1	20·5	38·2	105	448	—	—	125
4·0	·000	·869	1·35	2·00	2·83	4·08	5·98	9·08	13·7	22·3	41·4	98·0	296	—	—
4·1	—	—	1·30	1·94	2·60	3·61	5·08	7·25	10·4	15·8	24·4	40·5	78·7	216	—
4·2	—	—	1·33	1·94	2·58	3·43	4·54	6·09	8·49	11·8	16·8	25·3	39·4	67·2	169
4·3	—	—	1·44	2·01	2·63	3·37	4·20	5·41	7·19	9·52	12·8	17·8	26·2	45·2	72·2
4·4	—	—	1·58	2·15	2·74	3·39	4·20	5·18	6·58	8·56	10·9	14·6	20·2	29·0	40·4
4·5	—	—	1·81	2·32	2·94	3·59	4·36	5·29	6·45	7·76	9·85	12·4	16·1	21·1	29·9
4·6	—	—	—	—	—	—	4·90	5·63	6·60	7·84	9·20	11·1	13·6	17·2	24·2
4·7	—	—	—	—	—	—	5·87	6·46	7·11	7·97	8·96	10·2	12·2	15·3	20·0
4·8	—	—	—	—	—	—	7·43	7·61	7·94	8·41	9·16	10·1	12·0	14·3	17·5
4·9	—	—	—	—	—	—	10·1	9·45	9·08	9·36	9·80	10·6	12·0	13·7	15·9
5·0	—	—	—	—	—	—	15·1	11·3	10·4	10·4	10·8	11·5	12·4	13·6	15·4
5·1	—	—	—	—	—	—	—	—	—	—	—	—	13·5	14·0	15·3
5·2	—	—	—	—	—	—	—	—	—	—	—	—	15·3	15·5	16·0
5·3	—	—	—	—	—	—	—	—	—	—	—	—	18·3	17·8	17·7
5·4	—	—	—	—	—	—	—	—	—	—	—	—	22·8	20·6	20·0
5·5	—	—	—	—	—	—	—	—	—	—	—	—	30·6	26·0	23·3
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLIII—(*continued*).

Probable Error of Criterion κ_2. *Values of* $\sqrt{N}\,\Sigma_{\kappa_2}$ *for values of* β_1, β_2.

β_1

·75	*·80*	*·85*	*·90*	*·95*	*1·00*	*1·05*	*1·10*	*1·15*	*1·20*	*1·25*	*1·30*	*1·35*	*1·40*	*1·45*	*1·50*	β_2
·949	1·00	1·06	1·12	1·18	1·25	1·31	1·39	1·46	1·56	1·64	1·75	1·86	1·97	2·11	2·25	*2·0*
·937	·992	1·05	1·10	1·16	1·22	1·28	1·35	1·42	1·49	1·58	1·67	1·77	1·88	2·00	2·12	*2·1*
·936	·987	1·04	1·09	1·14	1·19	1·25	1·31	1·37	1·43	1·51	1·59	1·69	1·79	1·89	1·99	*2·2*
·939	·982	1·04	1·08	1·12	1·17	1·22	1·27	1·32	1·38	1·45	1·53	1·62	1·70	1·79	1·87	*2·3*
·950	·990	1·03	1·07	1·11	1·15	1·19	1·24	1·29	1·34	1·40	1·47	1·55	1·62	1·69	1·77	*2·4*
·972	·998	1·03	1·07	1·10	1·14	1·18	1·22	1·27	1·32	1·37	1·43	1·49	1·55	1·61	1·68	*2·5*
1·01	1·03	1·06	1·09	1·12	1·15	1·18	1·21	1·26	1·31	1·36	1·41	1·46	1·51	1·57	1·63	*2·6*
1·07	1·08	1·09	1·11	1·14	1·17	1·19	1·21	1·25	1·30	1·35	1·39	1·43	1·48	1·53	1·59	*2·7*
1·17	1·15	1·16	1·17	1·18	1·19	1·21	1·23	1·27	1·31	1·35	1·39	1·43	1·47	1·52	1·58	*2·8*
1·33	1·28	1·26	1·26	1·25	1·25	1·25	1·27	1·29	1·32	1·35	1·39	1·43	1·47	1·52	1·57	*2·9*
1·53	1·47	1·42	1·38	1·36	1·34	1·34	1·34	1·35	1·36	1·38	1·40	1·43	1·46	1·51	1·56	*3·0*
1·85	1·72	1·62	1·56	1·52	1·49	1·46	1·44	1·43	1·43	1·43	1·44	1·45	1·47	1·51	1·57	*3·1*
2·34	2·13	1·96	1·83	1·75	1·68	1·62	1·57	1·54	1·52	1·51	1·50	1·50	1·51	1·54	1·58	*3·2*
3·03	2·69	2·44	2·23	2·06	1·94	1·85	1·77	1·72	1·67	1·63	1·60	1·59	1·58	1·59	1·61	*3·3*
4·08	3·50	3·06	2·74	2·49	2·29	2·14	2·02	1·92	1·84	1·78	1·73	1·70	1·68	1·68	1·69	*3·4*
5·72	4·75	4·12	3·56	3·15	2·83	2·59	2·38	2·22	2·09	1·99	1·91	1·85	1·81	1·79	1·79	*3·5*
8·85	6·90	5·71	4·79	4·15	3·68	3·22	2·90	2·67	2·49	2·31	2·20	2·09	2·00	1·96	1·92	*3·6*
14·2	10·4	8·22	6·69	5·70	4·81	4·17	3·67	3·31	3·05	2·81	2·62	2·45	2·29	2·19	2·10	*3·7*
24·9	18·1	13·8	10·1	8·16	6·71	5·76	4·92	4·28	3·84	3·48	3·16	2·91	2·68	2·50	2·36	*3·8*
65·5	36·5	21·7	15·8	12·2	10·0	8·17	6·75	5·75	5·00	4·43	3·93	3·52	3·18	2·91	2·72	*3·9*
242	86·6	48·5	30·0	20·9	15·4	12·0	9·61	7·86	6·60	5·67	4·90	4·34	3·88	3·47	3·19	*4·0*
—	374	127	62·9	38·3	26·0	18·3	14·4	11·5	9·15	7·53	6·45	5·66	4·98	4·38	3·80	*4·1*
—	—	—	200	91·0	51·6	32·7	23·4	17·4	13·4	10·6	8·70	7·50	6·48	5·52	4·61	*4·2*
144	478	—	—	314	112	70·0	42·9	29·1	21·3	16·1	12·7	10·3	8·55	7·24	6·25	*4·3*
62·8	135	—	—	—	580	192	93·9	55·8	37·1	26·5	19·8	15·4	12·1	9·74	8·26	*4·4*
42·8	68·3	119	280	—	—	—	286	126	72·8	46·4	32·2	23·9	18·3	14·6	11·8	*4·5*
32·3	44·7	62·7	99·6	240	742	—	—	—	181	91·0	58·9	41·0	30·0	22·3	16·9	*4·6*
26·2	33·8	46·1	68·0	105	240	532	—	—	—	260	128	76·7	50·4	36·0	26·6	*4·7*
21·6	27·1	35·0	47·3	66·8	104	182	413	—	—	—	403	172	99·3	63·0	44·8	*4·8*
18·9	22·6	26·9	33·7	44·0	61·5	84·2	115	337	—	—	—	—	249	140	80·8	*4·9*
17·8	20·7	24·6	30·1	37·8	48·9	66·0	97·0	157	286	—	—	—	—	—	172	*5·0*
17·2	20·0	23·0	27·1	32·5	40·6	51·5	69·2	99·8	147	253	559	—	—	—	—	*5·1*
17·3	19·3	21·7	24·8	29·5	35·4	43·1	54·7	70·6	94·6	138	216	—	—	—	—	*5·2*
18·0	19·7	21·4	23·9	27·1	31·5	37·0	44·2	54·5	69·3	93·2	132	205	380	—	—	*5·3*
19·6	20·5	21·6	23·3	25·6	28·6	32·8	38·0	44·8	54·0	68·6	94·0	130	185	—	—	*5·4*
22·3	22·0	22·5	23·6	25·2	27·5	30·5	34·3	39·4	45·4	54·7	67·3	86·5	116	169	275	*5·5*
—	—	—	24·5	25·8	27·6	29·8	32·2	35·6	39·6	44·8	51·4	63·2	83·0	118	168	*5·6*
—	—	—	26·8	27·8	28·8	30·0	31·5	33·8	36·5	39·8	44·4	53·1	67·0	85·2	116	*5·7*
—	—	—	30·7	30·8	31·0	31·3	32·0	33·4	35·3	38·2	42·2	48·4	56·6	69·9	87·9	*5·8*
—	—	—	38·4	36·0	34·4	33·5	33·1	34·1	36·0	38·7	42·2	47·0	53·1	61·9	74·8	*5·9*
—	—	—	50·4	42·5	38·3	36·8	36·4	36·5	37·9	40·0	43·0	46·6	51·3	57·8	66·2	*6·0*
—	—	—	—	—	—	—	41·0	41·0	41·6	42·7	44·6	47·2	50·3	55·0	61·5	*6·1*
—	—	—	—	—	—	—	48·2	47·0	46·1	46·0	46·7	48·0	50·0	53·5	58·5	*6·2*
—	—	—	—	—	—	—	57·3	54·0	51·6	49·8	49·1	49·4	50·0	52·5	56·8	*6·3*
—	—	—	—	—	—	—	79·4	66·6	58·6	54·2	51·8	51·2	50·8	52·5	55·5	*6·4*
—	—	—	—	—	—	—	128	84·0	66·9	59·9	55·9	53·9	53·1	53·6	55·4	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	—	57·4	57·0	56·7	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	—	63·5	61·5	59·2	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	—	73·6	67·8	63·5	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	—	97·2	77·7	70·5	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	—	134	96·0	82·0	*7·0*

TABLE XLIV. *To find probable Frequency Type.*

Values of $1{\cdot}77\sqrt{N}\Sigma_1$ *for given values of* β_1, β_2 *(Semi-Minor Axis of Probability Ellipse).*

β_1

β_2	0	·05	·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75
2·0	0·0	0·5	0·7	0·8	0·8	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·5	0·4
2·1	0·0	0·5	0·7	0·8	0·9	0·9	0·8	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5
2·2	0·0	0·5	0·8	0·9	0·9	0·9	0·9	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·7	0·6
2·3	0·0	0·5	0·8	0·9	1·0	1·0	1·0	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·8	0·7
2·4	0·0	0·6	0·9	1·0	1·0	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9	0·9	0·9	0·8
2·5	0·0	0·6	0·9	1·0	1·1	1·1	1·1	1·1	1·1	1·1	1·0	1·0	1·0	1·0	1·0	0·9
2·6	0·0	0·6	1·0	1·1	1·1	1·2	1·2	1·2	1·1	1·1	1·1	1·1	1·1	1·0	1·0	1·0
2·7	0·0	0·7	1·0	1·2	1·2	1·3	1·3	1·3	1·2	1·2	1·2	1·2	1·1	1·1	1·1	1·1
2·8	0·0	0·7	1·1	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·3	1·2	1·2	1·2	1·2
2·9	0·0	0·8	1·2	1·3	1·3	1·4	1·4	1·4	1·4	1·4	1·4	1·4	1·3	1·3	1·3	1·3
3·0	0·0	0·8	1·2	1·4	1·4	1·5	1·5	1·5	1·5	1·5	1·5	1·5	1·4	1·4	1·4	1·4
3·1	0·0	0·9	1·3	1·5	1·5	1·6	1·6	1·6	1·6	1·6	1·6	1·6	1·6	1·5	1·5	1·5
3·2	0·0	1·0	1·4	1·6	1·6	1·7	1·7	1·7	1·7	1·7	1·7	1·7	1·7	1·6	1·6	1·6
3·3	·00	1·1	1·6	1·8	1·8	1·9	1·9	1·9	1·8	1·8	1·8	1·8	1·8	1·7	1·7	1·7
3·4	·00	1·2	1·7	1·9	1·9	2·0	2·0	2·0	2·0	2·0	1·9	1·9	1·9	1·9	1·9	1·8
3·5	·00	1·4	1·8	2·0	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·1	2·0	2·0	1·9
3·6	·00	1·5	2·0	2·1	2·2	2·3	2·3	2·3	2·3	2·3	2·3	2·2	2·2	2·1	2·1	2·1
3·7	·00	1·6	2·1	2·3	2·4	2·5	2·5	2·5	2·5	2·5	2·4	2·4	2·4	2·3	2·3	2·3
3·8	·00	1·8	2·3	2·5	2·6	2·7	2·7	2·7	2·7	2·7	2·6	2·6	2·6	2·5	2·5	2·4
3·9	·00	2·0	2·5	2·7	2·9	3·0	3·0	3·0	3·0	2·9	2·9	2·8	2·8	2·7	2·7	2·6
4·0	·00	2·2	2·8	3·0	3·2	3·3	3·3	3·3	3·3	3·2	3·1	3·1	3·0	2·9	2·9	2·8
4·1	—	—	3·0	3·3	3·5	3·6	3·6	3·6	3·6	3·5	3·4	3·4	3·3	3·2	3·2	3·1
4·2	—	—	3·3	3·6	3·8	3·9	4·0	4·0	3·9	3·8	3·8	3·7	3·6	3·5	3·5	3·4
4·3	—	—	3·6	4·0	4·2	4·3	4·4	4·4	4·3	4·2	4·1	4·0	3·9	3·8	3·8	3·7
4·4	—	—	4·0	4·4	4·7	4·8	4·8	4·8	4·7	4·6	4·5	4·4	4·3	4·2	4·1	4·0
4·5	—	—	4·4	4·9	5·2	5·3	5·3	5·3	5·2	5·1	5·0	4·9	4·7	4·6	4·5	4·4
4·6	—	—	—	—	—	—	5·9	5·9	5·8	5·7	5·6	5·4	5·2	5·1	5·0	4·8
4·7	—	—	—	—	—	—	6·7	6·7	6·6	6·4	6·2	6·0	5·8	5·7	5·6	5·3
4·8	—	—	—	—	—	—	7·6	7·6	7·4	7·2	6·9	6·6	6·4	6·3	6·2	5·9
4·9	—	—	—	—	—	—	8·5	8·5	8·4	8·1	7·8	7·5	7·2	7·0	6·8	6·5
5·0	—	—	—	—	—	—	11	10	9·7	9·3	8·9	8·5	8·1	7·7	7·4	7·1
5·1	—	—	—	—	—	—	—	—	—	—	—	—	9·3	8·6	8·1	7·7
5·2	—	—	—	—	—	—	—	—	—	—	—	—	11	9·5	8·9	8·6
5·3	—	—	—	—	—	—	—	—	—	—	—	—	12	11	10	9·6
5·4	—	—	—	—	—	—	—	—	—	—	—	—	14	13	12	11
5·5	—	—	—	—	—	—	—	—	—	—	—	—	16	15	14	13
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLIV—(*continued*).

Values of $1{\cdot}77\sqrt{N}\,\Sigma_1$ *for given values of* β_1, β_2 *(Semi-Minor Axis of Probability Ellipse).*

β_1

·8	·85	·9	·95	1·0	1·05	1·1	1·15	1·2	1·25	1·3	1·35	1·4	1·45	1·5	β_2
0·4	0·3	0·3	0·2	0·1	0·1	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	2·0
0·5	0·5	0·4	0·3	0·2	0·2	0·2	0·1	0·1	0·1	0·0	0·0	0·0	0·0	0·0	2·1
0·6	0·6	0·5	0·5	0·4	0·4	0·3	0·3	0·3	0·2	0·1	0·0	0·0	0·0	0·0	2·2
0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·4	0·4	0·3	0·2	0·1	0·1	0·0	0·0	2·3
0·8	0·8	0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·4	0·3	0·2	0·2	0·1	0·0	2·4
0·9	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·5	0·4	0·3	0·3	0·2	0·1	2·5
1·0	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·5	0·4	0·3	0·2	2·6
1·1	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·6	0·5	0·4	0·3	2·7
1·1	1·1	1·1	1·0	1·0	1·0	0·9	0·9	0·9	0·8	0·8	0·7	0·6	0·5	0·5	2·8
1·2	1·2	1·2	1·1	1·1	1·0	1·0	1·0	1·0	1·0	0·9	0·8	0·7	0·7	0·7	2·9
1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·1	1·0	1·0	0·9	0·9	0·8	0·8	3·0
1·4	1·4	1·4	1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	1·0	0·9	0·9	3·1
1·5	1·5	1·5	1·4	1·4	1·4	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	1·0	3·2
1·7	1·6	1·6	1·5	1·4	1·4	1·3	1·3	1·3	1·2	1·2	1·2	1·1	1·1	1·0	3·3
1·8	1·7	1·7	1·6	1·6	1·6	1·5	1·5	1·4	1·4	1·3	1·3	1·2	1·2	1·1	3·4
1·9	1·9	1·8	1·8	1·7	1·7	1·6	1·6	1·5	1·5	1·4	1·4	1·3	1·3	1·2	3·5
2·0	2·0	2·0	1·9	1·8	1·8	1·7	1·7	1·6	1·6	1·5	1·5	1·4	1·4	1·4	3·6
2·2	2·2	2·1	2·1	2·0	2·0	1·9	1·8	1·7	1·7	1·6	1·6	1·6	1·5	1·5	3·7
2·4	2·3	2·2	2·2	2·1	2·1	2·0	2·0	1·9	1·9	1·8	1·7	1·7	1·6	1·6	3·8
2·6	2·5	2·4	2·4	2·3	2·3	2·2	2·1	2·0	2·0	1·9	1·9	1·8	1·7	1·7	3·9
2·8	2·7	2·6	2·6	2·5	2·5	2·4	2·3	2·2	2·2	2·1	2·1	2·0	2·0	1·9	4·0
3·0	2·9	2·8	2·8	2·7	2·7	2·6	2·5	2·4	2·4	2·3	2·3	2·2	2·2	2·1	4·1
3·2	3·2	3·1	3·0	2·9	2·9	2·8	2·7	2·6	2·6	2·5	2·4	2·3	2·3	2·2	4·2
3·5	3·5	3·4	3·3	3·2	3·1	3·0	2·9	2·8	2·8	2·7	2·6	2·5	2·4	2·4	4·3
3·9	3·8	3·7	3·6	3·5	3·4	3·3	3·1	3·0	3·0	2·9	2·8	2·7	2·7	2·6	4·4
4·2	4·1	4·0	3·9	3·7	3·6	3·5	3·4	3·3	3·2	3·1	3·1	3·0	3·0	2·9	4·5
4·6	4·5	4·4	4·2	4·1	3·9	3·8	3·7	3·5	3·4	3·3	3·3	3·2	3·2	3·1	4·6
5·1	4·9	4·8	4·6	4·4	4·3	4·2	4·0	3·8	3·7	3·6	3·5	3·4	3·4	3·3	4·7
5·6	5·4	5·2	5·0	4·8	4·7	4·6	4·4	4·2	4·0	3·9	3·8	3·7	3·6	3·5	4·8
6·2	6·0	5·7	5·5	5·3	5·1	5·0	4·8	4·6	4·4	4·2	4·1	4·0	3·9	3·8	4·9
6·8	6·6	6·3	6·1	5·8	5·6	5·4	5·2	5·0	4·8	4·6	4·4	4·3	4·2	4·1	5·0
7·4	7·2	7·0	6·7	6·4	6·2	5·9	5·7	5·4	5·1	4·9	4·7	4·6	4·4	4·3	5·1
8·3	8·0	7·7	7·4	7·1	6·8	6·5	6·2	5·8	5·5	5·2	5·0	4·9	4·7	4·5	5·2
9·2	8·9	8·6	8·2	7·8	7·5	7·1	6·7	6·3	5·9	5·6	5·4	5·2	5·0	4·7	5·3
10	10	9·6	9·1	8·6	8·2	7·8	7·3	6·9	6·5	6·1	5·8	5·6	5·3	5·0	5·4
12	11	11	10	9·5	9·0	8·5	8·0	7·5	7·1	6·7	6·4	6·1	5·8	5·5	5·5
—	—	12	11	10	10	9·5	8·9	8·4	7·9	7·4	7·0	6·6	6·3	6·0	5·6
—	—	14	13	12	11	10	10	9·4	8·8	8·3	7·8	7·4	7·0	6·7	5·7
—	—	16	14	13	12	12	11	10	10	9·4	8·8	8·3	7·8	7·4	5·8
—	—	18	16	15	14	14	13	12	11	10	10	9·5	8·9	8·4	5·9
—	—	22	20	18	16	15	14	13	12	12	11	11	10	9·5	6·0
—	—	—	—	—	—	18	16	15	14	13	13	12	12	11	6·1
—	—	—	—	—	—	21	19	17	16	15	14	13	13	12	6·2
—	—	—	—	—	—	24	21	19	18	17	16	15	14	13	6·3
—	—	—	—	—	—	27	24	22	20	19	18	17	16	15	6·4
—	—	—	—	—	—	32	28	25	23	21	20	19	18	16	6·5
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·6
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·7
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·8
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·9
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7·0

TABLE XLV. *To find probable Frequency Type.*

Values of $1{\cdot}77\sqrt{N}\Sigma_2$ *for values of* β_1, β_2 *(Semi-Major Axis of Probability Ellipse).*

β_1

β_2	0	·05	·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75
2·0	1·6	1·9	2·2	2·5	2·8	3·1	3·4	3·7	4·1	4·4	4·7	5·0	5·4	5·7	6·1	6·4
2·1	1·7	2·1	2·4	2·7	2·9	3·2	3·5	3·8	4·1	4·4	4·6	4·9	5·3	5·6	6·0	6·3
2·2	1·9	2·3	2·6	2·9	3·1	3·4	3·7	3·9	4·1	4·4	4·6	4·9	5·2	5·5	5·9	6·2
2·3	2·1	2·5	2·8	3·1	3·3	3·5	3·8	4·0	4·2	4·4	4·6	4·9	5·2	5·5	5·8	6·1
2·4	2·4	2·8	3·1	3·3	3·5	3·7	3·9	4·1	4·3	4·5	4·7	4·9	5·1	5·4	5·7	6·0
2·5	2·8	3·3	3·5	3·7	3·8	4·0	4·1	4·3	4·5	4·7	4·8	5·0	5·2	5·4	5·7	5·9
2·6	3·3	3·8	4·0	4·2	4·3	4·5	4·6	4·7	4·8	4·9	5·0	5·1	5·3	5·5	5·7	6·0
2·7	3·8	4·4	4·6	4·7	4·9	5·0	5·1	5·2	5·3	5·4	5·4	5·4	5·5	5·6	5·8	6·1
2·8	4·4	5·0	5·3	5·4	5·6	5·7	5·8	5·9	5·9	5·9	5·8	5·8	5·8	5·9	6·0	6·2
2·9	5·1	5·7	6·0	6·2	6·3	6·4	6·5	6·6	6·5	6·5	6·4	6·3	6·3	6·3	6·4	6·5
3·0	5·8	6·5	6·9	7·1	7·2	7·3	7·3	7·3	7·2	7·1	7·0	6·9	6·8	6·8	6·8	6·8
3·1	6·7	7·5	8·0	8·3	8·4	8·4	8·4	8·3	8·2	8·0	7·9	7·8	7·7	7·6	7·5	7·4
3·2	7·8	8·7	9·2	9·6	9·8	9·8	9·7	9·5	9·4	9·2	9·0	8·8	8·6	8·4	8·3	8·1
3·3	9·2	10	11	11	11	11	11	11	11	11	10	10	9·6	9·4	9·2	8·9
3·4	11	11	12	12	13	13	13	12	12	12	11	11	11	11	10	10
3·5	13	13	14	14	15	15	14	14	14	14	13	12	12	12	11	11
3·6	15	16	17	17	17	17	16	16	16	16	15	14	14	14	13	13
3·7	18	19	20	20	20	20	19	19	18	18	17	17	16	16	15	14
3·8	21	23	24	24	24	23	22	22	21	21	20	19	18	18	17	16
3·9	25	27	28	28	28	27	26	25	24	24	23	22	21	20	19	18
4·0	29	32	34	33	32	31	30	29	28	27	26	24	23	22	21	20
4·1	—	—	43	41	39	37	36	34	32	31	30	28	26	25	24	23
4·2	—	—	55	51	48	45	42	40	38	36	34	32	30	29	27	26
4·3	—	—	69	62	57	53	50	48	45	43	40	37	35	33	31	29
4·4	—	—	87	76	69	64	60	56	53	50	46	43	40	38	36	34
4·5	—	—	113	98	86	78	71	66	62	57	53	49	46	43	41	39
4·6	—	—	—	—	—	—	90	79	73	67	61	57	53	50	48	45
4·7	—	—	—	—	—	—	110	99	90	80	72	67	62	58	55	52
4·8	—	—	—	—	—	—	140	124	110	97	86	79	73	68	64	60
4·9	—	—	—	—	—	—	200	160	135	119	105	95	86	79	74	69
5·0	—	—	—	—	—	—	384	234	182	153	130	115	102	92	85	79
5·1	—	—	—	—	—	—	—	—	—	—	—	—	124	110	101	93
5·2	—	—	—	—	—	—	—	—	—	—	—	—	148	134	120	109
5·3	—	—	—	—	—	—	—	—	—	—	—	—	188	166	146	129
5·4	—	—	—	—	—	—	—	—	—	—	—	—	245	203	177	154
5·5	—	—	—	—	—	—	—	—	—	—	—	—	405	297	230	194
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLV—(*continued*).

Values of $1{\cdot}77\sqrt{N}\,\Sigma_2$ *for values of* β_1, β_2 *(Semi-Major Axis of Probability Ellipse).*

β_1

·8	*·85*	*·9*	*·95*	*1·0*	*1·05*	*1·1*	*1·15*	*1·2*	*1·25*	*1·3*	*1·35*	*1·4*	*1·45*	*1·5*	β_2
6·8	7·2	7·6	7·9	8·3	8·7	9·1	9·5	9·9	10	11	11	11	12	12	*2·0*
6·7	7·0	7·4	7·8	8·1	8·5	8·9	9·2	9·6	10	11	11	11	11	12	*2·1*
6·5	6·8	7·2	7·6	8·0	8·3	8·6	9·0	9·4	9·8	10	10	11	11	12	*2·2*
6·4	6·7	7·1	7·4	7·8	8·1	8·5	8·8	9·2	9·6	9·9	10	11	11	12	*2·3*
6·3	6·6	6·9	7·3	7·6	8·0	8·3	8·7	9·0	9·4	9·8	10	10	11	11	*2·4*
6·2	6·5	6·8	7·1	7·5	7·8	8·2	8·5	8·9	9·2	9·6	10	10	11	11	*2·5*
6·2	6·5	6·8	7·1	7·4	7·7	8·0	8·3	8·7	9·0	9·4	9·9	10	11	11	*2·6*
6·3	6·5	6·8	7·0	7·3	7·6	7·9	8·2	8·5	8·8	9·2	9·7	10	10	11	*2·7*
6·4	6·6	6·8	7·0	7·2	7·5	7·7	8·0	8·3	8·7	9·1	9·5	9·8	10	10	*2·8*
6·6	6·7	6·8	7·0	7·2	7·4	7·6	7·9	8·2	8·6	8·9	9·3	9·6	10	10	*2·9*
6·9	7·0	7·1	7·2	7·3	7·5	7·7	7·9	8·2	8·5	8·8	9·1	9·4	9·7	10	*3·0*
7·3	7·3	7·3	7·4	7·5	7·6	7·8	8·0	8·2	8·5	8·7	9·0	9·3	9·6	9·9	*3·1*
7·9	7·9	7·8	7·8	7·8	7·9	8·1	8·2	8·3	8·5	8·7	8·9	9·2	9·5	9·8	*3·2*
8·7	8·5	8·4	8·3	8·3	8·3	8·3	8·4	8·5	8·7	8·9	9·1	9·3	9·5	9·7	*3·3*
9·5	9·3	9·1	9·0	8·8	8·8	8·8	8·8	8·9	9·0	9·1	9·2	9·4	9·5	9·7	*3·4*
11	10	10	9·8	9·6	9·5	9·4	9·3	9·3	9·3	9·3	9·4	9·5	9·6	9·8	*3·5*
12	11	11	11	10	10	10	10	9·9	9·9	9·8	9·7	9·7	9·8	9·9	*3·6*
13	12	12	12	11	11	11	11	11	10	10	10	10	10	10	*3·7*
15	14	14	13	13	13	12	12	12	11	11	10	10	10	11	*3·8*
17	16	16	15	14	14	14	13	13	12	12	11	11	11	11	*3·9*
19	18	18	17	16	16	15	14	14	13	13	12	12	12	12	*4·0*
22	21	20	19	18	17	17	16	15	15	14	14	13	13	13	*4·1*
24	23	22	21	20	19	18	18	17	16	15	15	14	14	14	*4·2*
28	26	25	23	22	21	20	20	19	18	17	16	15	15	15	*4·3*
32	30	29	27	25	24	23	22	21	20	19	18	17	17	17	*4·4*
37	35	33	31	29	27	26	25	24	23	22	21	20	19	18	*4·5*
43	40	37	35	33	31	29	28	27	25	24	23	22	21	20	*4·6*
49	45	42	39	37	35	33	32	30	28	27	26	24	23	22	*4·7*
56	52	48	45	42	40	38	36	34	32	31	29	27	26	25	*4·8*
64	59	55	51	48	46	42	40	38	36	35	33	31	29	28	*4·9*
73	68	63	59	55	52	49	46	43	41	39	37	35	33	32	*5·0*
85	78	72	67	63	59	55	52	49	46	43	41	39	37	36	*5·1*
99	90	82	77	72	67	63	58	55	52	49	46	43	41	40	*5·2*
114	104	95	88	82	76	71	66	62	58	55	51	48	46	44	*5·3*
136	123	112	102	95	87	80	75	70	65	61	57	54	51	49	*5·4*
167	147	132	119	108	100	92	85	79	74	69	64	60	57	54	*5·5*
—	—	160	141	126	115	105	96	89	83	78	73	68	64	60	*5·6*
—	—	206	169	148	132	120	110	102	95	88	82	76	72	67	*5·7*
—	—	258	206	175	150	136	126	116	108	100	93	87	81	75	*5·8*
—	—	318	255	215	190	168	150	136	125	115	107	99	92	85	*5·9*
—	—	446	332	273	228	200	178	161	147	134	123	113	104	98	*6·0*
—	—	—	—	—	—	264	215	190	171	157	144	130	120	112	*6·1*
—	—	—	—	—	—	345	268	230	207	184	167	150	138	127	*6·2*
—	—	—	—	—	—	480	364	294	250	215	194	174	159	144	*6·3*
—	—	—	—	—	—	680	477	370	299	252	224	201	181	165	*6·4*
—	—	—	—	—	—	1047	680	456	368	312	268	237	212	191	*6·5*
—	—	—	—	—	—	—	—	—	—	—	—	280	248	223	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	338	297	266	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	412	362	320	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	525	446	390	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	809	584	491	*7·0*

TABLE XLVI. *To find probable Frequency Type.*

Angle between Major-Axis and Axis of β_2 (Probability Ellipse) measured in degrees.

β_1

β_2	0	·05	·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75
2·0	0	12	23	28	31	33	35	36	37	38	39	40	41	41	42	42
2·1	0	11	21	25	28	30	32	34	35	37	38	39	40	40	41	41
2·2	0	10	19	23	26	28	30	32	33	35	36	38	39	39	40	40
2·3	0	10	18	22	25	27	28	30	32	34	35	37	38	38	39	39
2·4	0	9	17	20	23	25	26	29	31	33	34	35	36	37	38	38
2·5	0	8	15	18	21	23	25	27	29	31	33	34	35	36	37	37
2·6	0	7	14	17	20	22	24	26	28	30	31	33	34	35	35	36
2·7	0	7	13	16	19	21	23	25	26	28	29	31	32	33	34	35
2·8	0	6	12	15	17	19	21	23	25	27	28	30	31	32	33	33
2·9	0	6	11	14	16	18	20	22	23	25	26	28	29	30	31	32
3·0	0	5	10	13	15	17	19	21	22	24	25	27	28	29	30	31
3·1	0	5	9	12	14	16	18	20	21	23	24	26	27	28	29	30
3·2	0	5	9	12	14	16	17	19	20	21	22	24	25	26	27	28
3·3	0	4	8	11	13	15	16	18	19	20	21	22	24	25	26	27
3·4	0	4	8	10	12	14	15	17	18	19	20	21	22	23	24	25
3·5	0	3	7	9	11	13	14	15	17	18	19	20	21	22	23	24
3·6	0	3	6	8	10	12	13	14	15	16	18	19	20	21	22	23
3·7	0	3	5	7	9	11	12	13	14	15	16	17	18	19	20	21
3·8	0	3	5	7	9	10	11	12	13	14	15	16	17	18	19	20
3·9	0	2	4	6	8	9	10	11	12	13	14	15	16	17	18	19
4·0	0	2	4	6	7	8	9	10	11	12	13	14	15	16	17	18
4·1	—	—	3	5	6	7	8	9	10	11	12	13	14	15	16	17
4·2	—	—	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4·3	—	—	2	4	5	5	6	7	9	10	10	11	12	13	14	15
4·4	—	—	2	3	4	5	6	7	8	9	9	10	11	12	13	14
4·5	—	—	1	2	3	4	5	6	7	8	8	9	10	11	12	13
4·6	—	—	—	—	—	—	4	5	6	7	7	8	9	10	11	12
4·7	—	—	—	—	—	—	3	4	5	6	6	7	8	9	10	11
4·8	—	—	—	—	—	—	3	4	4	5	5	6	7	8	9	10
4·9	—	—	—	—	—	—	2	3	3	4	5	6	6	7	8	9
5·0	—	—	—	—	—	—	1	2	2	3	4	5	6	6	7	8
5·1	—	—	—	—	—	—	—	—	—	—	—	—	5	5	6	7
5·2	—	—	—	—	—	—	—	—	—	—	—	—	4	5	6	7
5·3	—	—	—	—	—	—	—	—	—	—	—	—	3	4	5	5
5·4	—	—	—	—	—	—	—	—	—	—	—	—	2	3	4	5
5·5	—	—	—	—	—	—	—	—	—	—	—	—	1	2	3	4
5·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE XLVI—(*continued*).

Angle between Major-Axis and Axis of β_2 (Probability Ellipse) measured in degrees.

β_1

·8	*·85*	*·9*	*·95*	*1·0*	*1·05*	*1·1*	*1·15*	*1·2*	*1·25*	*1·3*	*1·35*	*1·4*	*1·45*	*1·5*	β_2
43	43	44	44	45	45	46	46	46	46	47	47	48	48	49	*2·0*
42	42	43	44	44	44	45	45	45	46	46	17	47	48	48	*2·1*
41	41	42	42	43	43	44	44	45	45	46	46	46	47	47	*2·2*
40	40	41	41	42	42	43	43	44	44	45	45	46	46	47	*2·3*
39	39	40	40	41	41	42	42	43	43	44	44	45	45	46	*2·4*
38	38	39	39	40	40	41	41	42	42	43	43	44	44	45	*2·5*
37	37	38	39	39	39	40	40	41	41	42	42	43	43	44	*2·6*
36	36	37	38	38	39	39	40	41	41	42	42	43	43	44	*2·7*
34	35	36	36	37	38	38	39	40	40	41	41	42	42	43	*2·8*
33	34	35	35	36	37	38	38	39	39	40	40	41	41	42	*2·9*
32	33	34	34	35	36	37	37	38	38	39	39	40	40	41	*3·0*
30	31	32	33	34	35	35	36	37	38	38	39	39	40	40	*3·1*
29	30	31	32	33	34	34	35	36	37	38	38	39	39	39	*3·2*
28	29	30	31	32	32	33	34	35	36	37	37	38	38	38	*3·3*
26	27	28	29	30	31	32	33	34	35	36	36	37	38	38	*3·4*
25	26	27	28	29	30	31	32	33	34	35	35	36	36	37	*3·5*
24	25	26	27	28	29	30	31	32	33	34	34	35	35	36	*3·6*
22	23	24	25	26	27	28	29	31	32	33	33	34	34	35	*3·7*
21	22	23	24	25	26	27	28	29	30	31	32	33	33	34	*3·8*
20	21	22	23	24	25	26	27	28	29	30	31	32	32	33	*3·9*
19	20	21	22	23	24	25	26	27	28	29	30	31	31	32	*4·0*
18	19	20	21	22	23	24	25	26	27	28	29	30	30	31	*4·1*
17	18	19	19	20	21	22	23	25	26	27	28	29	30	30	*4·2*
16	17	18	19	19	20	21	22	23	24	25	26	27	28	29	*4·3*
15	16	17	18	18	19	20	21	22	23	24	25	26	27	28	*4·4*
14	15	16	16	17	18	19	20	21	22	23	24	25	25	26	*4·5*
13	14	15	15	16	17	18	19	20	21	22	23	24	25	25	*4·6*
12	13	14	14	15	16	17	18	19	20	21	22	23	24	24	*4·7*
11	12	13	14	15	15	16	17	18	19	20	21	22	23	23	*4·8*
10	11	12	13	14	14	15	16	17	18	19	20	21	21	22	*4·9*
9	10	11	12	13	13	14	15	16	17	18	19	20	20	21	*5·0*
8	9	10	11	12	13	13	15	16	16	17	18	19	20	20	*5·1*
7	8	9	10	11	12	13	14	15	15	16	17	18	19	19	*5·2*
6	7	8	9	10	11	12	13	14	14	15	16	17	18	18	*5·3*
6	7	8	8	9	10	11	12	13	13	14	15	16	17	17	*5·4*
5	6	7	7	8	9	10	11	12	12	13	14	15	16	17	*5·5*
—	—	6	7	7	8	9	10	11	12	13	13	14	15	16	*5·6*
—	—	5	6	6	7	8	9	10	11	12	12	13	14	15	*5·7*
—	—	4	5	5	6	7	8	9	10	11	11	12	13	14	*5·8*
—	—	3	4	5	5	6	7	8	9	10	10	11	12	13	*5·9*
—	—	2	3	4	4	5	6	7	8	9	9	10	11	12	*6·0*
—	—	—	—	—	—	—	—	6	7	8	9	9	10	11	*6·1*
—	—	—	—	—	—	—	—	5	6	7	8	8	9	10	*6·2*
—	—	—	—	—	—	—	—	4	5	6	6	7	8	9	*6·3*
—	—	—	—	—	—	—	—	3	4	5	6	6	7	8	*6·4*
—	—	—	—	—	—	—	—	2	3	4	5	6	6	7	*6·5*
—	—	—	—	—	—	—	—		—	—	—	5	6	7	*6·6*
—	—	—	—	—	—	—	—	—	—	—	—	4	5	6	*6·7*
—	—	—	—	—	—	—	—	—	—	—	—	3	4	5	*6·8*
—	—	—	—	—	—	—	—	—	—	—	—	2	3	4	*6·9*
—	—	—	—	—	—	—	—	—	—	—	—	1	3	4	*7·0*

Diagram XLVII determining the probability of a given Type of Frequency.

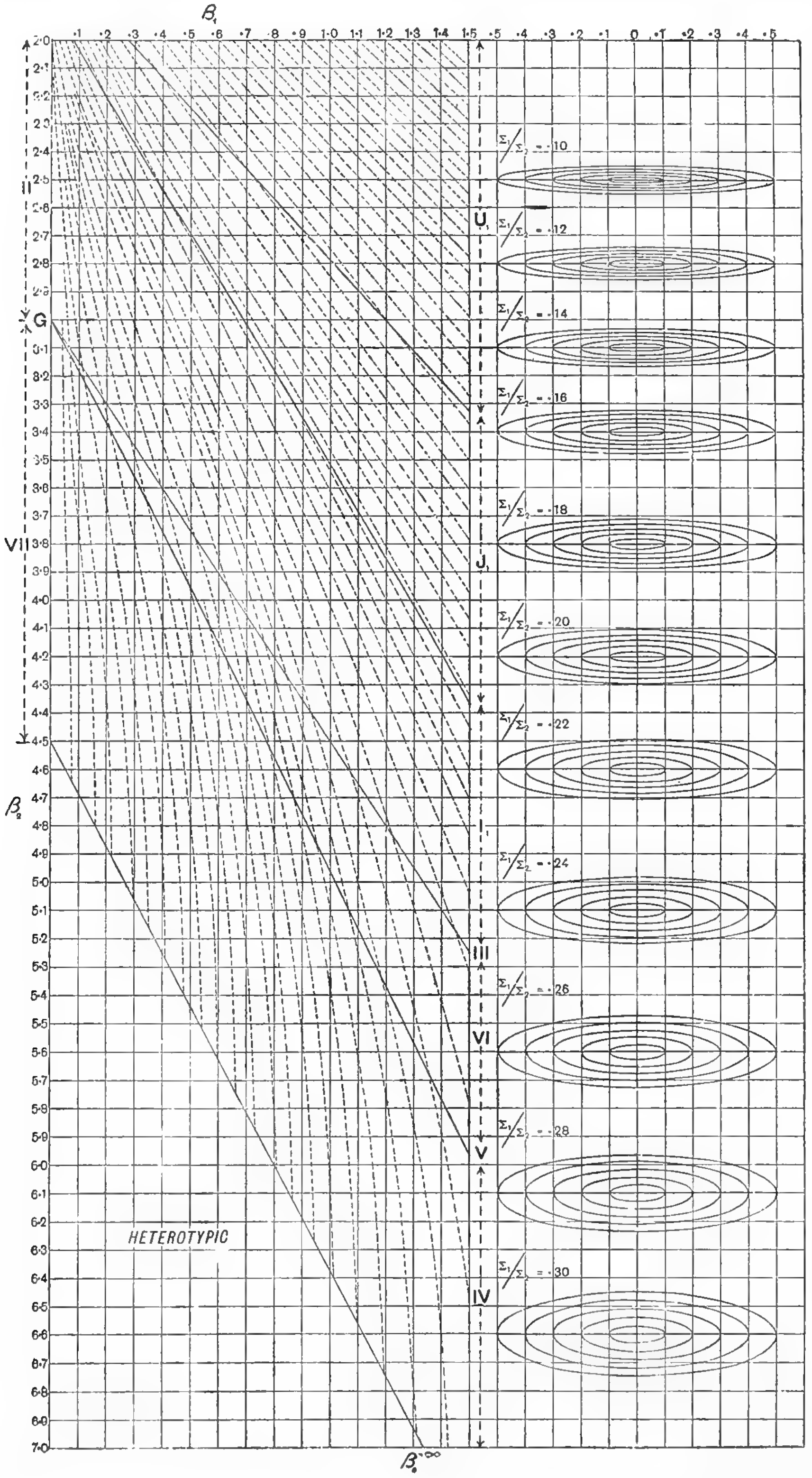


TABLE XLVIII. *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=6$	0	58·3333	31·8182	15·9091	7·0707		
$m=5$	1	26·5151	31·8182	26·5151	17·6768		
	2	10·6060	21·2121	26·5151	25·2525		
	3	3·5354	10·6060	18·9394	25·2525		
	4	·8838	3·7879	9·4697	17·6768		
	5	·1263	·7576	2·6515	7·0707		
$n=6$	0	53·8462	26·9231	12·2378	4·8951		
$m=6$	1	26·9231	29·3706	22·0280	13·0536		
	2	12·2378	22·0280	24·4755	20·3963		
	3	4·8951	13·0536	20·3962	23·3100		
	4	1·6317	6·1189	13·1119	20·3963		
	5	·4079	2·0979	6·1189	13·0536		
	6	·0582	·4079	1·6317	4·8951		
$n=7$	0	61·5385	35·8974	19·5804	9·7902		
$m=5$	1	25·6410	32·6340	29·3706	21·7560		
	2	9·3240	19·5804	26·1072	27·1950		
	3	2·7972	8·7024	16·3170	23·3100		
	4	·6216	2·7195	6·9930	13·5975		
	5	·0777	·4662	1·6317	4·3512		
$n=7$	0	57·1429	30·7692	15·3846	6·9930		
$m=6$	1	26·3736	30·7692	25·1748	16·7832		
	2	10·9890	20·9790	25·1748	23·3100		
	3	3·9960	11·1888	18·6480	23·3100		
	4	1·1988	4·6620	10·4895	17·4825		
	5	·2664	1·3986	4·1958	9·3240		
	6	·0333	·2331	·9324	2·7972		
$n=7$	0	53·3333	26·6667	12·3077	5·1282		
$m=7$	1	26·6667	28·7179	21·5385	13·0536		
	2	12·3077	21·5385	23·4965	19·5804		
	3	5·1282	13·0536	19·5804	21·7560		
	4	1·8648	6·5268	13·0536	19·0365		
	5	·5594	2·6107	6·8531	13·0536		
	6	·1243	·7615	2·6107	6·5268		
	7	·0155	·1243	·5594	1·8648		
$n=8$	0	64·2857	39·5604	23·0769	12·5874	6·2937	
$m=5$	1	24·7253	32·9670	31·4685	25·1748	17·4825	
	2	8·2418	17·9820	25·1748	27·9720	26·2238	
	3	2·2478	7·1928	13·9860	20·9790	26·2238	
	4	·4495	1·9980	5·2448	10·4895	17·4825	
	5	·0499	·2997	1·0489	2·7972	6·2937	
$n=8$	0	60·0000	34·2857	18·4615	9·2308	4·1958	
$m=6$	1	25·7143	31·6484	27·6923	20·1398	12·5874	
	2	9·8901	19·7802	25·1748	25·1748	20·9790	
	3	3·2967	9·5904	16·7832	22·3776	24·4755	
	4	·8991	3·5964	8·3916	14·6853	20·9790	
	5	·1798	·9590	2·9370	6·7133	12·5874	
	6	·0200	·1399	·5594	1·6783	4·1958	
$n=8$	0	56·2500	30·0000	15·0000	6·9231	2·8846	
$m=7$	1	26·2500	30·0000	24·2308	16·1538	9·1783	
	2	11·2500	20·7692	24·2308	22·0280	16·5210	
	3	4·3269	11·5385	18·3566	22·0280	21·4161	
	4	1·4423	5·2448	11·0140	17·1329	21·4161	
	5	·3934	1·8881	5·1399	10·2797	16·5210	
	6	·0787	·4895	1·7132	4·4056	9·1783	
	7	·0087	·0699	·3147	1·0489	2·8846	

TABLE XLVIII—(*continued*). *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=8$	0	52·9412	26·4706	12·3529	5·2941	2·0362	
$m=8$	1	26·4706	28·2353	21·1765	13·0317	6·7873	
	2	12·3529	21·1765	22·8054	19·0045	12·9576	
	3	5·2941	13·0317	19·0045	20·7322	18·1407	
	4	2·0362	6·7873	12·9576	18·1407	20·1563	
	5	·6787	2·9617	7·2563	12·9000	18·1407	
	6	·1851	1·0366	3·2250	7·2563	12·9576	
	7	·0370	·2633	1·0366	2·9617	6·7873	
	8	·0041	·0370	·1851	·6787	2·0362	
$n=9$	0	66·6667	42·8571	26·3736	15·3846	8·3916	
$m=5$	1	23·8095	32·9670	32·9670	27·9720	20·9790	
	2	7·3260	16·4835	23·9760	27·9720	27·9720	
	3	1·8315	5·9940	11·9880	18·6480	24·4755	
	4	·3330	1·4985	3·9960	8·1585	13·9860	
	5	·0333	·1998	·6993	1·8648	4·1958	
$n=9$	0	62·5000	37·5000	21·4286	11·5385	5·7692	
$m=6$	1	25·0000	32·1429	29·6703	23·0769	15·7343	
	2	8·9286	18·5439	24·7253	26·2238	23·6014	
	3	2·7472	8·2418	14·9850	20·9790	24·4755	
	4	·6868	2·8097	6·7433	12·2378	18·3566	
	5	·1249	·6743	2·0979	4·8951	9·4405	
	6	·0125	·0874	·3496	1·0489	2·6224	
$n=9$	0	58·8235	33·0882	17·6471	8·8235	4·0724	
$m=7$	1	25·7353	30·8824	26·4706	19·0045	11·8778	
	2	10·2941	19·8529	24·4344	23·7557	19·4364	
	3	3·6765	10·1810	16·9683	21·5961	22·6759	
	4	1·1312	4·2421	9·2554	15·1172	20·1563	
	5	·2828	1·3883	3·8873	8·0625	13·6055	
	6	·0514	·3239	1·1518	3·0234	6·4788	
	7	·0051	·0411	·1851	·6170	1·6968	
$n=9$	0	55·5555	29·4118	14·7059	6·8627	2·9412	
$m=8$	1	26·1438	29·4118	23·5294	15·6863	9·0498	
	2	11·4379	20·5882	23·5294	21·1161	15·8371	
	3	4·5752	11·7647	18·0995	21·1161	20·1563	
	4	1·6340	5·6561	11·3122	16·7969	20·1563	
	5	·5027	2·2624	5·7589	10·7500	16·1250	
	6	·1257	·7199	2·3036	5·3750	10·0782	
	7	·0229	·1645	·6582	1·9197	4·5249	
	8	·0023	·0206	·1028	·3771	1·1312	
$n=9$	0	52·6316	26·3158	12·3839	5·4180	2·1672	
$m=9$	1	26·3158	27·8638	20·8978	13·0031	6·9659	
	2	12·3839	20·8978	22·2910	18·5759	12·8602	
	3	5·4180	13·0031	18·5759	20·0047	17·5042	
	4	2·1672	6·9659	12·8602	17·5042	19·0955	
	5	·7740	3·2151	7·5018	12·7303	17·1859	
	6	·2381	1·2503	3·6372	7·6382	12·7303	
	7	·0595	·3897	1·4029	3·6372	7·5018	
	8	·0108	·0877	·3897	1·2503	3·2150	
	9	·0011	·0108	·0595	·2381	·7740	
$n=5$	0	54·5454	27·2727	12·1212	4·5454		
$m=5$	1	27·2727	30·3030	22·7273	12·9870		
	2	12·1212	22·7273	25·9740	21·6450		
	3	4·5454	12·9870	21·6450	25·9740		
	4	1·2987	5·4112	12·9870	22·7273		
	5	·2165	1·2987	4·5454	12·1212		

TABLE XLVIII—(*continued*).

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
$n=10$	*0*	68·7500	45·8333	29·4643	18·1318	10·5769	5·7692		
$m=5$	*1*	22·9167	32·7381	33·9972	30·2198	24·0385	17·3077		
	2	6·5476	15·1099	22·6648	27·4725	28·8461	26·9230		
	3	1·5110	5·0366	10·3022	16·4835	22·4359	26·9230		
	4	·2518	1·1447	3·0907	6·4103	11·2179	17·3077		
	5	·0229	·1373	·4807	1·2820	2·8846	5·7692		
$n=10$	*0*	52·3809	26·1905	12·4060	5·5138	2·2704	·8514		
$m=10$	*1*	26·1905	27·5689	20·6767	12·9736	7·0949	3·4056		
	2	12·4060	20·6767	21·8930	18·2441	12·7709	7·6625		
	3	5·5138	12·9736	18·2441	19·4604	17·0278	12·5744		
	4	2·2704	7·0949	12·7709	17·0278	18·3377	16·5039		
	5	·8514	3·4056	7·6625	12·5744	16·5039	18·0043		
	6	·2838	1·4190	3·9295	7·8590	12·5030	16·5039		
	7	·0811	·4990	1·6841	4·0826	7·8590	12·5744		
	8	·0187	·1403	·5741	1·6841	3·9295	7·6625		
	9	·0031	·0284	·1403	·4990	1·4190	3·4056		
	10	·0003	·0031	·0187	·0811	·2838	·8514		
$n=15$	*0*	76·1905	57·1429	42·1053	30·4094	21·4654	14·7575	9·8383	6·3246
$m=5$	*1*	19·0476	30·0752	35·0877	35·7757	33·5397	29·5149	24·5958	19·4604
	2	4·0100	10·0251	16·5119	22·3598	26·8318	29·5149	30·2717	29·1906
	3	·6683	2·3588	5·1599	8·9439	13·4159	18·1631	22·7038	26·5369
	4	·0786	·3685	1·0320	2·2360	4·1280	6·8111	10·3199	14·5953
	5	·0049	·0295	·1032	·2752	·6192	1·2384	2·2704	3·8921
$n=15$	*0*	61·5384	36·9231	21·5385	12·1739	6·6403	3·4783	1·7391	·8238
$m=10$	*1*	24·6154	30·7692	28·0936	22·1344	15·8103	10·4348	6·4073	3·6613
	2	9·2308	18·0602	22·9857	23·7154	21·3439	17·2997	12·8146	8·7226
	3	3·2107	8·7565	14·5941	18·9723	20·9694	20·5034	18·0913	14·5376
	4	1·0216	3·6485	7·6619	12·2322	16·3095	18·9958	19·7873	18·6566
	5	·2919	1·3135	3·3874	6·5238	10·3613	14·2469	17·4128	19·1897
	6	·0730	·4032	1·2546	2·8781	5·3965	8·7064	12·4377	15·9914
	7	·0153	·1024	·3795	1·0279	2·2614	4·2644	7·1073	10·6609
	8	·0025	·0203	·0889	·2827	·7269	1·5991	3·1094	5·4516
	9	·0003	·0028	·0145	·0538	·1615	·4146	·9423	1·9383
	10	·0000	·0002	·0012	·0054	·0189	·0565	·1508	·3661
$n=15$	*0*	51·6129	25·8065	12·4583	5·7842	2·5707	1·0876	·4351	·1631
$m=15$	*1*	25·8065	26·6963	20·0222	12·8538	7·4156	3·9155	1·9033	·8512
	2	12·4583	20·0222	20·7638	17·3032	12·4583	7·9941	4·6342	2·4375
	3	5·7842	12·8538	17·3032	17·9953	15·7459	12·0490	8·2152	5·0297
	4	2·5707	7·4156	12·4583	15·7459	16·4305	14·7874	11·7361	8·2991
	5	1·0876	3·9155	7·9941	12·0490	14·7874	15·4916	14·2006	11·5313
	6	·4351	1·9033	4·6342	8·2152	11·7361	14·2006	14·9480	13·8803
	7	·1631	·8512	2·4375	5·0297	8·2991	11·5313	13·8803	14·6968
	8	·0567	·3482	1·1607	2·7663	5·2415	8·3282	11·4309	13·7783
	9	·0181	·1289	·4965	1·3589	2·9443	5·3344	8·3350	11·4309
	10	·0052	·0426	·1882	·5889	1·4548	3·0006	5·3344	8·3282
	11	·0013	·0122	·0618	·2204	·6200	1·4548	2·9443	5·2415
	12	·0003	·0029	·0169	·0689	·2204	·5889	1·3589	2·7664
	13	·0001	·0006	·0037	·0170	·0618	·1882	·4965	1·1607
	14	·0000	·0001	·0006	·0029	·0122	·0426	·1290	·3482
	15	·0000	·0000	·0000	·0003	·0013	·0052	·0181	·0567

TABLE XLVIII—(*continued*). *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=20$, $m=5$	*0*	80·7692	64·6154	51·1538	40·0334	30·9349	23·5695
	1	16·1538	26·9231	33·3612	36·3940	36·8273	35·3542
	2	2·6923	7·0234	12·1313	17·3305	22·0964	26·0505
	3	·3512	1·2770	2·8884	5·1992	8·1408	11·5780
	4	·0319	·1520	·4333	·9577	1·8090	3·0647
	5	·0015	·0091	·0319	·0851	·1915	·3831
$n=20$, $m=10$	*0*	67·7419	45·1613	29·5884	19·0211	11·9763	7·3700
	1	22·5806	31·1457	31·7019	28·1795	23·0313	17·6880
	2	7·0078	15·0167	21·1346	24·3861	24·8738	23·2155
	3	2·0022	5·9325	10·8382	15·6071	19·3463	21·5333
	4	·5191	1·9965	4·5521	7·9661	11·7760	15·4158
	5	·1198	·5750	1·5932	3·3250	5·7809	8·8091
	6	·0240	·1398	·4618	1·1335	2·2940	4·0375
	7	·0040	·0278	·1079	·3084	·7210	1·4571
	8	·0005	·0043	·0193	·0636	·1707	·3946
	9	·0000	·0004	·0024	·0089	·0274	·0722
	10	·0000	·0000	·0002	·0006	·0023	·0068
$n=20$, $m=15$	*0*	58·3333	33·3333	18·6275	10·1604	5·3977	2·7859
	1	25·0000	29·4118	25·4011	19·0508	13·0590	8·3578
	2	10·2941	18·7166	22·2259	21·5090	18·2826	14·1218
	3	4·0553	10·1381	15·5343	18·6411	19·1232	17·4841
	4	1·5207	4·9056	9·3206	13·4987	16·3913	17·4841
	5	·5396	2·1584	4·9495	8·4849	12·0203	14·7942
	6	·1799	·8683	2·3569	4·7138	7·7053	10·8491
	7	·0558	·3190	1·0101	2·3310	4·3590	6·9744
	8	·0159	·1063	·3885	1·0256	2·1795	3·9421
	9	·0041	·0318	·1330	·3989	·9581	1·9511
	10	·0010	·0084	·0399	·1353	·3658	·8362
	11	·0002	·0019	·0102	·0391	·1188	·3041
	12	·0000	·0004	·0022	·0093	·0317	·0907
	13	·0000	·0001	·0004	·0017	·0065	·0209
	14	·0000	·0000	·0000	·0002	·0009	·0033
	15	·0000	·0000	·0000	·0000	·0001	·0003
$n=20$, $m=20$	*0*	51·2195	25·6098	12·4765	5·9099	2·7154	1·2068
	1	25·6098	26·2664	19·6998	12·7783	7·5427	4·1377
	2	12·4765	19·6998	20·2323	16·8602	12·2839	8·0929
	3	5·9099	12·7783	16·8602	17·3419	15·1742	11·7715
	4	2·7154	7·5427	12·2839	15·1742	15·6340	14·0706
	5	1·2068	4·1377	8·0929	11·7715	14·0706	14·5245
	6	·5172	2·1297	4·9048	8·2768	11·3473	13·3141
	7	·0839	1·0326	2·7589	5·3399	8·3213	11·0186
	8	·0315	·4719	1·4462	3·1817	5·5954	8·3131
	9	·0112	·2030	·7070	1·7554	3·4638	5·7473
	10	·0037	·0819	·3218	·8965	1·9757	3·6474
	11	·0012	·0308	·1358	·4226	1·0362	2·1221
	12	·0003	·0107	·0528	·1829	·4974	1·1274
	13	·0001	·0034	·0188	·0720	·2168	·5429
	14	·0000	·0010	·0060	·0255	·0848	·2345
	15	—	·0003	·0017	·0080	·0293	·0893
	16	—	·0000	·0004	·0022	·0087	·0293
	17	—	—	·0001	·0005	·0022	·0080
	18	—	—	·0000	·0001	·0004	·0017
	19	—	—	—	·0000	·0001	·0003
	20	—	—	—	—	·0000	·0000

TABLE XLVIII—(*continued*).

	Successes	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=25$	*0*	83·8710	69·8925	57·8421	47·5131	38·7144	31·2693
$m=5$	*1*	13·9785	24·1008	30·9868	35·1949	37·2254	37·5232
	2	1·9281	5·1645	9·1813	13·5365	17·8682	21·8885
	3	·2065	·7651	1·7656	3·2488	5·2115	7·6134
	4	·0153	·0736	·2119	·4738	·9064	1·5573
	5	·0006	·0035	·0123	·0329	·0741	·1483
$n=25$	*0*	72·2222	51·5873	36·4146	25·3798	17·4486	11·8200
$m=10$	*1*	20·6349	30·3455	33·1041	31·7248	28·1430	23·6401
	2	5·4622	12·4141	18·6211	23·0261	25·3287	25·6780
	3	1·3242	4·1380	8·0091	12·2806	16·3035	19·5642
	4	·2897	1·1680	2·8032	5·1875	8·1518	11·4125
	5	·0561	·2803	·8119	1·7786	3·2607	5·2673
	6	·0093	·0564	·1933	·4940	1·0451	1·9313
	7	·0013	·0092	·0368	·1086	·2628	·5518
	8	·0001	·0011	·0053	·0179	·0493	·1170
	9	·0000	·0001	·0005	·0020	·0062	·0165
	10	·0000	·0000	·0000	·0001	·0004	·0012
$n=25$	*0*	63·4146	39·6341	24·3902	14·7625	8·7777	5·1203
$m=15$	*1*	23·7805	30·4878	28·8832	23·9392	18·2869	13·1666
	2	8·5366	16·8485	21·8575	23·2742	21·9443	18·9753
	3	2·9204	7·8930	13·1550	17·2894	19·5777	19·9337
	4	·9472	3·2888	6·7654	10·6788	14·2383	16·8191
	5	·2894	1·2403	3·0643	5·6953	8·8100	11·9361
	6	·0827	·4256	1·2381	2·6697	4·7366	7·2943
	7	·0218	·1326	·4477	1·1072	2·2329	3·8807
	8	·0053	·0373	·1444	·4060	·9240	1·8017
	9	·0012	·0094	·0412	·1307	·3337	·7266
	10	·0002	·0020	·0102	·0364	·1038	·2515
	11	·0000	·0004	·0022	·0086	·0272	·0732
	12	·0000	·0001	·0004	·0016	·0058	·0173
	13	·0000	·0000	·0001	·0002	·0009	·0031
	14	—	·0000	·0000	·0000	·0001	·0004
	15	—	—	·0000	·0000	·0000	·0000
$n=25$	*0*	56·5217	31·4010	17·1278	9·1614	4·7988	2·4579
$m=20$	*1*	25·1208	28·5463	23·8993	17·4502	11·7044	7·3738
	2	10·8476	18·9202	21·6231	20·2168	16·6788	12·5733
	3	4·5409	10·8116	15·8218	18·1951	17·9618	15·8820
	4	1·8380	5·6036	10·0864	13·8796	16·0711	16·4186
	5	·7172	2·6897	5·7932	9·3504	12·5094	14·5943
	6	·2690	1·2069	3·0491	5·6861	8·6871	11·4669
	7	·0965	·5082	1·4833	3·1589	5·4604	8·0943
	8	·0330	·2009	·6695	1·6133	3·1317	5·1816
	9	·0107	·0744	·2806	·7592	1·6449	3·0226
	10	·0033	·0257	·1089	·3290	·7916	1·6088
	11	·0009	·0082	·0390	·1308	·3482	·7800
	12	·0002	·0024	·0128	·0475	·1393	·3429
	13	·0001	·0006	·0038	·0156	·0502	·1357
	14	·0000	·0002	·0010	·0046	·0162	·0477
	15	—	·0000	·0002	·0012	·0045	·0147
	16	—	—	·0001	·0002	·0011	·0039
	17	—	—	—	·0000	·0002	·0008
	18	—	—	—	—	·0000	·0001
	19	—	—	—	—	—	·0000
	20	—	—	—	—	—	—

TABLE XLVIII—(*continued*). *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=25$ $m=25$	*0*	50·9804	25·4902	12·4850	5·9824	2·8003	1·2784
	1	25·4902	26·0104	19·5078	12·7285	7·6094	4·2613
	2	12·4850	19·5078	19·9229	16·6024	12·1751	8·1352
	3	5·9824	12·7285	16·6024	16·9713	14·8499	11·6037
	4	2·8003	7·6094	12·1751	14·8499	15·1953	13·6757
	5	1·2784	4·2613	8·1352	11·6037	13·6757	14·0093
	6	·5682	2·2598	5·0451	8·2883	11·1185	12·8418
	7	·2453	1·1411	2·9344	5·4870	8·2990	10·7251
	8	·1027	·5502	1·6103	3·3951	5·7456	8·2555
	9	·0416	·2535	·8365	1·9732	3·7128	5·9003
	10	·0162	·1115	·4118	1·0801	2·2477	3·9335
	11	·0061	·0468	·1921	·5573	1·2771	2·4521
	12	·0022	·0187	·0848	·2709	·6811	1·4304
	13	·0007	·0070	·0353	·1238	·3406	·7802
	14	·0002	·0025	·0138	·0531	·1592	·3971
	15	—	·0008	·0051	·0212	·0693	·1879
	16	—	·0003	·0017	·0079	·0280	·0822
	17	—	·0001	·0005	·0027	·0104	·0330
	18	—	—	·0002	·0008	·0035	·0120
	19	—	—	·0000	·0002	·0010	·0039
	20	—	—	—	·0001	·0003	·0011
	21	—	—	—	—	·0001	·0003
	22	—	—	—	—	—	·0001
	23	—	—	—	—	—	—
	24	—	—	—	—	—	—
	25	—	—	—	—	—	—
$n=50$ $m=5$	*0*	91·0714	82·7922	75·1263	68·0389	61·4967	55·4676
	1	8·2792	15·3319	21·2621	26·1688	30·1454	33·2805
	2	·6133	1·7357	3·2711	5·1311	7·2349	9·5087
	3	·0347	·1335	·3207	·6157	1·0335	1·5848
	4	·0013	·0065	·0192	·0440	·0861	·1517
	5	·0000	·0001	·0005	·0014	·0033	·0066
$n=50$ $m=10$	*0*	83·6065	69·6721	57·8633	47·8869	39·4857	32·4346
	1	13·9344	23·6177	29·9293	33·6048	35·2551	35·3833
	2	2·1256	5·4972	9·4513	13·5019	17·3070	20·6402
	3	·2932	1·0287	2·2503	3·9278	5·9827	8·3080
	4	·0360	·1607	·4296	·8910	1·5803	2·5164
	5	·0039	·0210	·0668	·1614	·3282	·5921
	6	·0003	·0023	·0084	·0233	·0536	·1085
	7	·0000	·0002	·0008	·0026	·0067	·0152
	8	·0000	·0000	·0001	·0002	·0006	·0015
	9	·0000	·0000	·0000	·0000	·0000	·0001
	10	·0000	·0000	·0000	·0000	·0000	·0000
$n=50$ $m=15$	*0*	77·2727	59·4406	45·5092	34·6737	26·2849	19·8214
	1	17·8322	27·8628	32·5066	33·5552	32·3175	29·7321
	2	3·9008	9·2876	14·6804	19·2530	22·6222	24·6927
	3	·8049	2·5965	5·2143	8·3429	11·6306	14·7589
	4	·1558	·6385	1·5643	2·9695	4·8127	6·9910
	5	·0281	·1405	·4083	·9011	1·6718	2·7465
	6	·0047	·0278	·0939	·2371	·4976	·9155
	7	·0007	·0049	·0191	·0544	·1279	·2616
	8	·0001	·0008	·0034	·0109	·0284	·0642
	9	—	·0001	·0005	·0019	·0054	·0132
	10	—	—	·0001	·0003	·0009	·0024
	11	—	—	—	—	·0001	·0003
	12	—	—	—	—	—	·0000
	13, 14, 15	—	—	—	—	—	—

TABLE XLVIII—(*continued*).

Successes	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=50$ } *0*	71·8310	51·3078	36·4360	25·7195	18·0421	12·5748
$m=20$ } *1*	20·5231	29·7437	32·1494	30·7099	27·3365	23·2150
2	5·6513	12·4661	18·2340	22·1018	23·9720	24·1218
3	1·4959	4·4655	8·2882	12·2410	15·7316	18·3785
4	·3796	1·4377	3·2515	5·6902	8·4901	11·3384
5	·0920	·4247	1·1380	2·3122	3·9438	5·9480
6	·0212	·1161	·3613	·8391	1·6163	2·7262
7	·0046	·0295	·1049	·2751	·5926	1·1089
8	·0010	·0070	·0279	·0820	·1959	·4039
9	·0002	·0015	·0068	·0222	·0585	·1323
10	·0000	·0003	·0015	·0055	·0158	·0390
11	—	·0001	·0003	·0012	·0039	·0103
12	—	—	·0001	·0002	·0008	·0024
13	—	—	—	·0000	·0002	·0005
14	—	—	—	—	·0000	·0001
15—20	—	—	—	—	—	·0000
$n=50$ } *0*	67·1053	44·7368	29·6230	19·4782	12·7149	8·2378
$m=25$ } *1*	22·3684	30·2276	30·4346	27·0530	22·3854	17·6525
2	7·2546	14·9068	20·2898	22·8617	23·0250	21·4900
3	2·2857	6·3492	10·9546	15·0234	17·9083	19·3831
4	·6984	2·4592	5·1643	8·3826	11·5877	14·3204
5	·2066	·8853	2·2004	4·1420	6·5376	9·1130
6	·0590	·2994	·8629	1·8546	3·3018	5·1406
7	·0163	·0956	·3146	·7627	1·5167	2·6162
8	·0043	·0289	·1073	·2904	·6398	1·2147
9	·0011	·0083	·0343	·1029	·2494	·5181
10	·0003	·0022	·0103	·0340	·0901	·2038
11	·0001	·0006	·0029	·0105	·0302	·0741
12	·0000	·0001	·0008	·0030	·0094	·0249
13	·0000	·0000	·0002	·0008	·0027	·0077
14	—	·0000	·0000	·0002	·0007	·0022
15	—	—	·0000	·0000	·0002	·0006
16	—	—	—	·0000	·0000	·0001
17	—	—	—	—	·0000	·0000
18—25	—	—	—	—	—	·0000
$n=50$ } *0*	50·4950	25·2475	12·4963	6·1206	2·9657	1·4210
$m=50$ } *1*	25·2475	25·5026	19·1269	12·6198	7·7231	4·4875
2	12·4963	19·1269	19·3241	16·1034	11·9504	8·1873
3	6·1206	12·6198	16·1034	16·2729	14·2388	11·2686
4	2·9657	7·7231	11·9504	14·2388	14·3919	12·9527
5	1·4210	4·4875	8·1873	11·2686	12·9527	13·0951
6	·6731	2·5063	5·2821	8·2677	10·6753	12·0038
7	·3151	1·3552	3·2480	5·7108	8·2014	10·1734
8	·1457	·7126	1·9185	3·7517	5·9437	8·0780
9	·0665	·3654	1·0942	2·3606	4·0975	6·0662
10	·0300	·1831	·6049	1·4298	2·7034	4·3381
11	·0133	·0898	·3250	·8367	1·7147	2·9694
12	·0058	·0431	·1699	·4743	1·0490	1·9531
13	·0025	·0203	·0866	·2610	·6205	1·2381
14	·0011	·0093	·0431	·1396	·3557	·7582
15	·0004	·0042	·0209	·0726	·1978	·4493
16	·0002	·0019	·0099	·0368	·1068	·2580
17	·0001	·0008	·0046	·0182	·0561	·1437
18	—	·0003	·0021	·0088	·0286	·0777
19	—	·0001	·0009	·0041	·0142	·0408
20	—	—	·0004	·0019	·0069	·0208
21	—	—	·0002	·0008	·0032	·0103
22	—	—	·0001	·0004	·0015	·0050
23		—	—	·0002	·0007	·0023
24	—	—	—	·0001	·0003	·0010
25	—	—	—	·0000	·0001	·0005
26	—	—	—	—	·0000	·0002
27	—	—	—	—	·0000	·0001
28—50	—	—	—	—	—	·0000

TABLE XLVIII—(*continued*). *Percentage Frequency of Successes in a Second Sample "m" after drawing "p" Successes in a First Sample "n".*

Successes	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$
$n=100$ 0	90·9910	82·7191	75·1302	68·1737	61·8023	55·9719	50·6412	45·7719	41·3280	37·2763
$m=10$ 1	8·2719	15·1778	20·8695	25·4855	29·1520	31·9839	34·0855	35·5510	36·4659	36·9072
2	·6830	1·8972	3·5107	5·4097	7·4962	9·6874	11·9134	14·1158	16·2472	18·2690
3	·0506	·1891	·4416	·8243	1·3455	2·0065	2·8031	3·7269	4·7658	5·9052
4	·0033	·0156	·0442	·0971	·1829	·3098	·4857	·7174	1·0109	1·3708
5	·0002	·0011	·0036	·0090	·0194	·0368	·0641	·1044	·1609	·2374
6	·0000	·0001	·0002	·0007	·0016	·0034	·0065	·0115	·0194	·0309
7	—	·0000	·0000	·0000	·0001	·0002	·0005	·0010	·0017	·0030
8	—	—	—	—	·0000	·0000	·0000	·0001	·0001	·0002
9	—	—	—	—	—	—	—	·0000	·0000	·0000
10	—	—	—	—	—	—	—	—	—	—

	$p=10$	$p=15$	$p=20$	$p=25$	$p=30$	$p=35$	$p=40$	$p=45$	$p=50$
0	33·5855	19·6056	11·0992	6·0712	3·1945	1·6083	·7697	·3473	·1463
1	36·9441	33·0200	25·8982	18·5708	12·3788	7·7198	4·5082	2·4580	1·2434
2	20·1513	26·8727	28·8081	26·8613	22·5639	17·3696	12·3485	8·1229	4·9313
3	7·1284	13·8698	20·0784	24·1644	25·4567	24·1112	20·8229	16·5036	12·0167
4	1·8005	5·0127	9·6930	14·9554	19·6711	22·8554	23·9308	22·8255	19·9224
5	·3376	1·3220	3·3813	6·6468	10·8709	15·4516	19·5797	22·4513	23·4799
6	·0474	·2571	·8619	2·1464	4·3483	7·5418	11·5470	15·9030	19·9224
7	·0049	·0363	·1583	·4968	1·2424	2·6232	4·8456	8·0093	12·0167
8	·0003	·0036	·0200	·0788	·2425	·6221	1·3845	2·7446	4·9313
9	·0000	·0002	·0015	·0077	·0292	·0908	·2432	·5778	1·2434
10	—	·0000	·0001	·0004	·0017	·0062	·0199	·0567	·1463

Successes	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
$n=100$ 0	95·2830	90·7457	86·3830	82·1896	78·1607	74·2914	70·5768	67·0123	63·5933	60·3153	57·1739
$m=5$ 1	4·5373	8·7256	12·5800	16·1156	19·3467	22·2874	24·9514	27·3520	29·5020	31·4142	33·1007
2	·1745	·5083	·9867	1·5956	2·3216	3·1517	4·0737	5·0756	6·1463	7·2749	8·4512
3	·0051	·0199	·0488	·0957	·1642	·2573	·3780	·5287	·7117	·9287	1·1813
4	·0001	·0005	·0015	·0034	·0067	·0119	·0197	·0306	·0454	·0649	·0899
5	·0000	·0000	·0000	·0001	·0001	·0003	·0004	·0008	·0013	·0020	·0030
$n=100$ 0	87·0690	75·7121	65·7500	57·0221	49·3852	42·7116	36·8873	31·8110	27·3928	23·5527	20·2198
$m=15$ 1	11·3568	19·9243	26·1836	30·5476	33·3684	34·9458	35·5336	35·3456	34·5611	33·3293	31·7739
2	1·3947	3·7027	6·5459	9·6321	12·7407	15·7096	18·4248	20·8110	22·8233	24·4415	25·6636
3	·1604	·5730	1·2777	2·2767	3·5456	5·0426	6·7156	8·5076	10·3611	12·2207	14·0361
4	·0172	·0774	·2091	·4386	·7879	1·2724	1·9006	2·6738	3·5865	4·6273	5·7796
5	·0017	·0093	·0295	·0715	·1458	·2641	·4381	·6787	·9959	1·3973	1·8884
6	·0002	·0010	·0037	·0100	·0229	·0461	·0842	·1428	·2278	·3459	·5036
7	·0000	·0001	·0004	·0012	·0031	·0068	·0137	·0252	·0435	·0711	·1112
8	—	·0000	·0000	·0001	·0004	·0009	·0019	·0037	·0070	·0122	·0204
9	—	—	—	·0000	·0000	·0001	·0002	·0005	·0009	·0017	·0031
10	—	—	—	—	—	·0000	·0000	·0001	·0001	·0002	·0004
11—15	—	—	—	—	—	—	—	·0000	·0000	·0000	·0000
$n=100$ 0	83·4711	69·5592	57·8686	48·0604	39·8449	32·9751	27·2403	22·4613	18·4859	15·1848	12·4488
$m=20$ 1	13·9119	23·3813	29·4247	32·8618	34·3491	34·4088	33·4530	31·8036	29·7094	27·3600	24·8976
2	2·2212	5·6472	9·5567	13·4563	17·0252	20·0718	22·4994	24·2787	25·4270	25·9920	26·0397
3	·3388	1·1584	2·4716	4·2124	6·2724	8·5261	10·8479	13·1236	15·2562	17·1690	18·8065
4	·0492	·2122	·5480	1·0993	1·8873	2·9118	4·1535	5·5775	7·1382	8·7832	10·4578
5	·0068	·0354	·1077	·2490	·4853	·8394	1·3291	1·9649	2·7495	3·6775	4·7356
6	·0009	·0054	·0191	·0500	·1093	·2099	·3658	·5913	·8994	1·3010	1·8040
7	·0001	·0008	·0031	·0090	·0219	·0462	·0881	·1547	·2545	·3965	·5898
8	·0000	·0001	·0004	·0015	·0039	·0090	·0187	·0356	·0630	·1053	·1675
9	—	·0000	·0001	·0002	·0006	·0016	·0035	·0072	·0137	·0245	·0416
10	—	—	·0000	·0000	·0001	·0002	·0006	·0013	·0026	·0050	·0091
11	—	—	—	—	·0000	·0000	·0001	·0002	·0005	·0009	·0017
12	—	—	—	—	—	—	·0000	·0000	·0001	·0001	·0003
13—20	—	—	—	—	—	—	—	—	·0000	·0000	·0000

TABLE XLVIII—(*continued*).

Successes		$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
$n=100$	*0*	80·1587	64·1270	51·1981	40·7920	32·4330	25·7320	20·3711	16·0915	12·6823	9·9724	7·8232
$m=25$	*1*	16·0317	25·8576	31·2184	33·4361	33·5052	32·1649	29·9575	27·2737	24·3890	21·4922	18·7076
	2	3·1029	7·5681	12·2826	16·5799	20·1031	22·7047	24·3723	25·1757	25·2300	24·6693	23·6306
	3	·5802	1·9024	3·8912	6·3556	9·0661	11·8013	14·3734	16·6391	18·5020	19·9086	20·8423
	4	·1046	·4323	1·0701	2·0562	3·3806	4·9929	6·8150	8·7536	10·7117	12·5970	14·3291
	5	·0182	·0908	·2644	·5855	1·0922	1·8077	2·7378	3·8700	5·1757	6·6134	8·1327
	6	·0030	·0178	·0597	·1501	·3138	·5764	·9606	1·4841	2·1565	2·9790	3·9431
	7	·0005	·0033	·0125	·0351	·0815	·1647	·3000	·5035	·7910	1·1761	1·6693
	8	·0001	·0005	·0024	·0075	·0193	·0426	·0844	·1531	·2589	·4127	·6260
	9	·0000	·0001	·0004	·0015	·0042	·0100	·0215	·0421	·0763	·1299	·2099
	10	—	·0000	·0001	·0003	·0008	·0022	·0050	·0105	·0203	·0369	·0634
	11	—	—	·0000	·0001	·0001	·0004	·0011	·0024	·0049	·0095	·0173
	12	—	—	—	·0000	·0000	·0001	·0002	·0005	·0011	·0022	·0043
	13	—	—	—	—	—	·0000	·0000	·0001	·0002	·0005	·0009
	14	—	—	—	—	—	—	—	·0000	·0000	·0001	·0002
	15—25	—	—	—	—	—	—	—	—	—	·0000	·0000
$n=100$	*0*	66·8874	44·5916	29·6280	19·6185	12·9456	8·5121	5·5769	3·6405	2·3676	1·5339	·9900
$m=50$	*1*	22·2958	29·9273	30·0284	26·6919	22·1671	17·6113	13·5550	10·1832	7·5029	5·4395	3·8892
	2	7·3322	14·8625	20·0189	22·3956	22·4728	20·9746	18·5789	15·8126	13·0370	10·4710	8·2261
	3	2·3780	6·4708	10·9693	14·8274	17·4789	18·7745	18·8406	17·9434	16·3894	14·4635	12·3988
	4	·7603	2·6038	5·3333	8·4691	11·4896	13·9817	15·7005	16·5656	16·6252	16·0095	14·8876
	5	·2396	·9912	2·3852	4·3589	6·6996	9·1228	11·3492	13·1572	14·4085	15·0512	15·1066
	6	·0743	·3614	1·0008	2·0720	3·5636	5·3759	7·3484	9·2958	11·0430	12·4505	13·4281
	7	·0227	·1271	·3987	·9237	1·7600	2·9173	4·3512	5·9710	7·6559	9·2753	10·7081
	8	·0068	·0433	·1520	·3901	·8167	1·4771	2·3900	3·5398	4·8771	6·3248	7·7895
	9	·0020	·0143	·0557	·1572	·3590	·7044	1·2301	1·9578	2·8874	3·9946	5·2324
	10	·0006	·0046	·0197	·0607	·1504	·3185	·5978	1·0184	1·6022	2·3574	3·2752
	11	·0002	·0014	·0068	·0226	·0603	·1373	·2758	·5012	·8386	1·3088	1·9239
	12	—	·0004	·0022	·0081	·0232	·0566	·1213	·2345	·4161	·6871	1·0664
	13	—	·0001	·0007	·0028	·0086	·0224	·0510	·1046	·1965	·3425	·5601
	14	—	—	·0002	·0009	·0031	·0085	·0206	·0447	·0886	·1627	·2797
	15	—	—	·0001	·0003	·0011	·0031	·0080	·0183	·0382	·0738	·1332
	16	—	—	—	·0001	·0004	·0011	·0030	·0072	·0158	·0320	·0606
	17	—	—	—	—	·0001	·0004	·0011	·0027	·0063	·0133	·0264
	18	—	—	—	—	—	·0001	·0004	·0010	·0024	·0053	·0110
	19	—	—	—	—	—	—	·0001	·0003	·0009	·0020	·0044
	20	—	—	—	—	—	—	—	·0001	·0003	·0008	·0017
	21	—	—	—	—	—	—	—	—	·0001	·0003	·0006
	22	—	—	—	—	—	—	—	—	—	·0001	·0002
	23	—	—	—	—	—	—	—	—	—	—	·0001
	24—50	—	—	—	—	—	—	—	—	—	—	—

TABLE XLIX. *Logarithms of Factorials.*

1—250

$\log \underline{|n}$ from $n=1$ to $n=1000$.

n	$\log \underline{\|n}$	n	$\log \underline{\|n}$	n	$\log \underline{\|n}$	n	$\log \underline{\|n}$	n	$\log \underline{\|n}$
1	·000 0000	51	66·190 6450	101	159·974 3250	151	264·935 8704	201	377·200 0847
2	·301 0300	52	67·906 6484	102	161·982 9252	152	267·117 7139	202	379·505 4361
3	·778 1513	53	69·630 9243	103	163·995 7624	153	269·302 4054	203	381·812 9321
4	1·380 2112	54	71·363 3180	104	166·012 7958	154	271·489 9261	204	384·122 5623
5	2·079 1812	55	73·103 6807	105	168·033 9851	155	273·680 2578	205	386·434 3161
6	2·857 3325	56	74·851 8687	106	170·059 2909	156	275·873 3824	206	388·748 1834
7	3·702 4305	57	76·607 7436	107	172·088 6747	157	278·069 2820	207	391·064 1537
8	4·605 5205	58	78·371 1716	108	174·122 0985	158	280·267 9391	208	393·382 2170
9	5·559 7630	59	80·142 0236	109	176·159 5250	159	282·469 3363	209	395·702 3633
10	6·559 7630	60	81·920 1748	110	178·200 9176	160	284·673 4562	210	398·024 5826
11	7·601 1557	61	83·705 5047	111	180·246 2406	161	286·880 2821	211	400·348 8651
12	8·680 3370	62	85·497 8964	112	182·295 4586	162	289·089 7971	212	402·675 2009
13	9·794 2803	63	87·297 2369	113	184·348 5371	163	291·301 9847	213	405·003 5805
14	10·940 4084	64	89·103 4169	114	186·405 4419	164	293·516 8286	214	407·333 9943
15	12·116 4996	65	90·916 3303	115	188·466 1398	165	295·734 3125	215	409·666 4328
16	13·320 6196	66	92·735 8742	116	190·530 5978	166	297·954 4206	216	412·000 8865
17	14·551 0685	67	94·561 9490	117	192·598 7836	167	300·177 1371	217	414·337 3463
18	15·806 3410	68	96·394 4579	118	194·670 6656	168	302·402 4464	218	416·675 8027
19	17·085 0946	69	98·233 3070	119	196·746 2126	169	304·630 3331	219	419·016 2469
20	18·386 1246	70	100·078 4050	120	198·825 3938	170	306·860 7820	220	421·358 6695
21	19·708 3439	71	101·929 6634	121	200·908 1792	171	309·093 7781	221	423·703 0618
22	21·050 7666	72	103·786 9959	122	202·994 5390	172	311·329 3066	222	426·049 4148
23	22·412 4944	73	105·650 3187	123	205·084 4442	173	313·567 3527	223	428·397 7197
24	23·792 7057	74	107·519 5505	124	207·177 8658	174	315·807 9019	224	430·747 9677
25	25·190 6457	75	109·394 6117	125	209·274 7759	175	318·050 9400	225	433·100 1502
26	26·605 6190	76	111·275 4253	126	211·375 1464	176	320·296 4526	226	435·454 2586
27	28·036 9828	77	113·161 9160	127	213·478 9501	177	322·544 4259	227	437·810 2845
28	29·484 1408	78	115·054 0106	128	215·586 1601	178	324·794 8459	228	440·168 2193
29	30·946 5388	79	116·951 6377	129	217·696 7498	179	327·047 6989	229	442·528 0548
30	32·423 6601	80	118·854 7277	130	219·810 6932	180	329·302 9714	230	444·889 7827
31	33·915 0218	81	120·763 2127	131	221·927 9645	181	331·560 6500	231	447·253 3946
32	35·420 1717	82	122·677 0266	132	224·048 5384	182	333·820 7214	232	449·618 8826
33	36·938 6857	83	124·596 1047	133	226·172 3900	183	336·083 1725	233	451·986 2385
34	38·470 1646	84	126·520 3840	134	228·299 4948	184	338·347 9903	234	454·355 4544
35	40·014 2326	85	128·449 8029	135	230·429 8286	185	340·615 1620	235	456·726 5223
36	41·570 5351	86	130·384 3013	136	232·563 3675	186	342·884 6750	236	459·099 4343
37	43·138 7369	87	132·323 8206	137	234·700 0881	187	345·156 5166	237	461·474 1826
38	44·718 5205	88	134·268 3033	138	236·839 9672	188	347·430 6744	238	463·850 7596
39	46·309 5851	89	136·217 6933	139	238·982 9820	189	349·707 1362	239	466·229 1575
40	47·911 6451	90	138·171 9358	140	241·129 1100	190	351·985 8898	240	468·609 3687
41	49·524 4289	91	140·130 9772	141	243·278 3291	191	354·266 9232	241	470·991 3857
42	51·147 6782	92	142·094 7650	142	245·430 6174	192	356·550 2244	242	473·375 2011
43	52·781 1467	93	144·063 2480	143	247·585 9535	193	358·835 7817	243	475·760 8074
44	54·424 5993	94	146·036 3758	144	249·744 3160	194	361·123 5835	244	478·148 1972
45	56·077 8119	95	148·014 0994	145	251·905 6840	195	363·413 6181	245	480·537 3633
46	57·740 5697	96	149·996 3707	146	254·070 0368	196	365·705 8742	246	482·928 2984
47	59·412 6676	97	151·983 1424	147	256·237 3542	197	368·000 3404	247	485·320 9954
48	61·093 9088	98	153·974 3685	148	258·407 6159	198	370·297 0056	248	487·715 4470
49	62·784 1049	99	155·970 0037	149	260·580 8022	199	372·595 8586	249	490·111 6464
50	64·483 0749	100	157·970 0037	150	262·756 8934	200	374·896 8886	250	492·509 5864

TABLE XLIX—(*continued*).

n	log $\underline{\lvert n}$	n	log $\underline{\lvert n}$	n	log $\underline{\lvert n}$	n	log $\underline{\lvert n}$	n	log $\underline{\lvert n}$
251	494·909 2601	301	616·964 3695	351	742·637 2813	401	871·409 5586	451	1002·893 0675
252	497·310 6607	302	619·444 3765	352	745·183 8240	402	874·013 7846	452	1005·548 2059
253	499·713 7812	303	621·925 8191	353	747·731 5987	403	876·619 0896	453	1008·204 3041
254	502·118 6149	304	624·408 6927	354	750·280 6020	404	879·225 4710	454	1010·861 3600
255	504·525 1551	305	626·892 9925	355	752·830 8303	405	881·832 9260	455	1013·519 3714
256	506·933 3950	306	629·378 7140	356	755·382 2803	406	884·441 4521	456	1016·178 3362
257	509·343 3282	307	631·865 8523	357	757·934 9485	407	887·051 0465	457	1018·838 2524
258	511·754 9479	308	634·354 4031	358	760·488 8316	408	889·661 7066	458	1021·499 1179
259	514·168 2476	309	636·844 3615	359	763·043 9260	409	892·273 4300	459	1024·160 9306
260	516·583 2210	310	639·335 7232	360	765·600 2285	410	894·886 2138	460	1026·823 6884
261	518·999 8615	311	641·828 4836	361	768·157 7357	411	897·500 0556	461	1029·487 3893
262	521·418 1628	312	644·322 6382	362	770·716 4443	412	900·114 9528	462	1032·152 0313
263	523·838 1185	313	646·818 1825	363	773·276 3509	413	902·730 9029	463	1034·817 6123
264	526·259 7225	314	649·315 1122	364	775·837 4523	414	905·347 9032	464	1037·484 1303
265	528·682 9683	315	651·813 4227	365	778·399 7452	415	907·965 9513	465	1040·151 5832
266	531·107 8500	316	654·313 1098	366	780·963 2262	416	910·585 0447	466	1042·819 9692
267	533·534 3612	317	656·814 1691	367	783·527 8923	417	913·205 1807	467	1045·489 2860
268	535·962 4960	318	659·316 5962	368	786·093 7401	418	915·826 3570	468	1048·159 5319
269	538·392 2483	319	661·820 3869	369	788·660 7665	419	918·448 5710	469	1050·830 7047
270	540·823 6121	320	664·325 5369	370	791·228 9682	420	921·071 8203	470	1053·502 8026
271	543·256 5814	321	666·832 0419	371	793·798 3421	421	923·696 1024	471	1056·175 8235
272	545·691 1503	322	669·339 8978	372	796·368 8851	422	926·321 4149	472	1058·849 7655
273	548·127 3129	323	671·849 1003	373	798·940 5939	423	928·947 7552	473	1061·524 6266
274	550·565 0635	324	674·359 6453	374	801·513 4655	424	931·575 1211	474	1064·200 4050
275	553·004 3962	325	676·871 5287	375	804·087 4968	425	934·203 5100	475	1066·877 0986
276	555·445 3052	326	679·384 7463	376	806·662 6846	426	936·832 9196	476	1069·554 7056
277	557·887 7850	327	681·899 2940	377	809·239 0260	427	939·463 3475	477	1072·233 2239
278	560·331 8298	328	684·415 1679	378	811·816 5178	428	942·094 7913	478	1074·912 6518
279	562·777 4340	329	686·932 3638	379	814·395 1570	429	944·727 2486	479	1077·592 9873
280	565·224 5920	330	689·450 8777	380	816·974 9406	430	947·360 7170	480	1080·274 2286
281	567·673 2984	331	691·970 7057	381	819·555 8655	431	949·995 1943	481	1082·956 3737
282	570·123 5475	332	694·491 8438	382	822·137 9289	432	952·630 6780	482	1085·639 4207
283	572·575 3339	333	697·014 2880	383	824·721 1277	433	955·267 1659	483	1088·323 3678
284	575·028 6523	334	699·538 0345	384	827·305 4589	434	957·904 6557	484	1091·008 2132
285	577·483 4971	335	702·063 0793	385	829·890 9196	435	960·543 1449	485	1093·693 9549
286	579·939 8631	336	704·589 4186	386	832·477 5069	436	963·182 6314	486	1096·380 5912
287	582·397 7450	337	707·117 0485	387	835·065 2179	437	965·823 1128	487	1099·068 1202
288	584·857 1375	338	709·645 9652	388	837·654 0496	438	968·464 5869	488	1101·756 5400
289	587·318 0354	339	712·176 1649	389	840·243 9992	439	971·107 0515	489	1104·445 8488
290	589·780 4334	340	714·707 6438	390	842·835 0638	440	973·750 5041	490	1107·136 0449
291	592·244 3264	341	717·240 3982	391	845·427 2406	441	976·394 9427	491	1109·827 1264
292	594·709 7092	342	719·774 4243	392	848·020 5267	442	979·040 3650	492	1112·519 0915
293	597·176 5768	343	722·309 7184	393	850·614 9192	443	981·686 7687	493	1115·211 9384
294	599·644 9242	344	724·846 2768	394	853·210 4154	444	984·334 1517	494	1117·905 6654
295	602·114 7462	345	727·384 0959	395	855·807 0125	445	986·982 5117	495	1120·600 2706
296	604·586 0379	346	729·923 1720	396	858·404 7077	446	989·631 8466	496	1123·295 7523
297	607·058 7943	347	732·463 5015	397	861·003 4982	447	992·282 1541	497	1125·992 1086
298	609·533 0106	348	735·005 0807	398	863·603 3813	448	994·933 4321	498	1128·689 3380
299	612·008 6818	349	737·547 9062	399	866·204 3542	449	997·585 6784	499	1131·387 4385
300	614·485 8030	350	740·091 9742	400	868·806 4142	450	1000·238 8910	500	1134·086 4085

Table of log $\lfloor\underline{n}$ from $n=1$ to $n=1000$.

n	log $\lfloor\underline{n}$	n	log $\lfloor\underline{n}$	n	log $\lfloor\underline{n}$	n	log $\lfloor\underline{n}$	n	log $\lfloor\underline{n}$
501	1136·786 2463	*551*	1272·848 0029	*601*	1410·881 1614	*651*	1550·721 4519	*701*	1692·229 8994
502	1139·486 9500	*552*	1275·589 9419	*602*	1413·660 7579	*652*	1553·535 6995	*702*	1695·076 2365
503	1142·188 5180	*553*	1278·332 6671	*603*	1416·441 0752	*653*	1556·350 6126	*703*	1697·923 1918
504	1144·890 9485	*554*	1281·076 1768	*604*	1419·222 1122	*654*	1559·166 1904	*704*	1700·770 7644
505	1147·594 2399	*555*	1283·820 4698	*605*	1422·003 8676	*655*	1561·982 4317	*705*	1703·618 9536
506	1150·298 3904	*556*	1286·565 5446	*606*	1424·786 3402	*656*	1564·799 3355	*706*	1706·467 7583
507	1153·003 3984	*557*	1289·311 3998	*607*	1427·569 5289	*657*	1567·616 9009	*707*	1709·317 1777
508	1155·709 2621	*558*	1292·058 0340	*608*	1430·353 4324	*658*	1570·435 1268	*708*	1712·167 2109
509	1158·415 9798	*559*	1294·805 4458	*609*	1433·138 0497	*659*	1573·254 0122	*709*	1715·017 8572
510	1161·123 5500	*560*	1297·553 6338	*610*	1435·923 3796	*660*	1576·073 5561	*710*	1717·869 1155
511	1163·831 9709	*561*	1300·302 5967	*611*	1438·709 4208	*661*	1578·893 7576	*711*	1720·720 9851
512	1166·541 2409	*562*	1303·052 3330	*612*	1441·496 1722	*662*	1581·714 6156	*712*	1723·573 4651
513	1169·251 3583	*563*	1305·802 8414	*613*	1444·283 6327	*663*	1584·536 1291	*713*	1726·426 5546
514	1171·962 3214	*564*	1308·554 1205	*614*	1447·071 8011	*664*	1587·358 2972	*714*	1729·280 2529
515	1174·674 1286	*565*	1311·306 1690	*615*	1449·860 6762	*665*	1590·181 1188	*715*	1732·134 5589
516	1177·386 7783	*566*	1314·058 9854	*616*	1452·650 2569	*666*	1593·004 5931	*716*	1734·989 4719
517	1180·100 2688	*567*	1316·812 5684	*617*	1455·440 5420	*667*	1595·828 7189	*717*	1737·844 9911
518	1182·814 5986	*568*	1319·566 9168	*618*	1458·231 5305	*668*	1598·653 4954	*718*	1740·701 1155
519	1185·529 7660	*569*	1322·322 0290	*619*	1461·023 2212	*669*	1601·478 9215	*719*	1743·557 8444
520	1188·245 7693	*570*	1325·077 9039	*620*	1463·815 6129	*670*	1604·304 9963	*720*	1746·415 1769
521	1190·962 6070	*571*	1327·834 5400	*621*	1466·608 7045	*671*	1607·131 7188	*721*	1749·273 1122
522	1193·680 2775	*572*	1330·591 9360	*622*	1469·402 4948	*672*	1609·959 0881	*722*	1752·131 6494
523	1196·398 7792	*573*	1333·350 0907	*623*	1472·196 9829	*673*	1612·787 1031	*723*	1754·990 7877
524	1199·118 1105	*574*	1336·109 0026	*624*	1474·992 1675	*674*	1615·615 7630	*724*	1757·850 5262
525	1201·838 2698	*575*	1338·868 6704	*625*	1477·788 0475	*675*	1618·445 0668	*725*	1760·710 8642
526	1204·559 2556	*576*	1341·629 0929	*626*	1480·584 6218	*676*	1621·275 0135	*726*	1763·571 8009
527	1207·281 0662	*577*	1344·390 2687	*627*	1483·381 8894	*677*	1624·105 6022	*727*	1766·433 3353
528	1210·003 7001	*578*	1347·152 1965	*628*	1486·179 8490	*678*	1626·936 8319	*728*	1769·295 4667
529	1212·727 1558	*579*	1349·914 8751	*629*	1488·978 4997	*679*	1629·768 7016	*729*	1772·158 1942
530	1215·451 4316	*580*	1352·678 3031	*630*	1491·777 8402	*680*	1632·601 2106	*730*	1775·021 5170
531	1218·176 5262	*581*	1355·442 4792	*631*	1494·577 8696	*681*	1635·434 3577	*731*	1777·885 4344
532	1220·902 4378	*582*	1358·207 4022	*632*	1497·378 5866	*682*	1638·268 1420	*732*	1780·749 9455
533	1223·629 1650	*583*	1360·973 0708	*633*	1500·179 9904	*683*	1641·102 5627	*733*	1783·615 0495
534	1226·356 7063	*584*	1363·739 4836	*634*	1502·982 0796	*684*	1643·937 6189	*734*	1786·480 7455
535	1229·085 0600	*585*	1366·506 6395	*635*	1505·784 8533	*685*	1646·773 3094	*735*	1789·347 0329
536	1231·814 2248	*586*	1369·274 5371	*636*	1508·588 3105	*686*	1649·609 6335	*736*	1792·213 9107
537	1234·544 1991	*587*	1372·043 1752	*637*	1511·392 4499	*687*	1652·446 5903	*737*	1795·081 3782
538	1237·274 9814	*588*	1374·812 5525	*638*	1514·197 2706	*688*	1655·284 1787	*738*	1797·949 4345
539	1240·006 5702	*589*	1377·582 6678	*639*	1517·002 7714	*689*	1658·122 3979	*739*	1800·818 0790
540	1242·738 9639	*590*	1380·353 5198	*640*	1519·808 9514	*690*	1660·961 2470	*740*	1803·687 3107
541	1245·472 1612	*591*	1383·125 1073	*641*	1522·615 8094	*691*	1663·800 7251	*741*	1806·557 1289
542	1248·206 1605	*592*	1385·897 4290	*642*	1525·423 3445	*692*	1666·640 8312	*742*	1809·427 5328
543	1250·940 9603	*593*	1388·670 4837	*643*	1528·231 5554	*693*	1669·481 5644	*743*	1812·298 5216
544	1253·676 5592	*594*	1391·444 2702	*644*	1531·040 4413	*694*	1672·322 9239	*744*	1815·170 0946
545	1256·412 9557	*595*	1394·218 7871	*645*	1533·850 0010	*695*	1675·164 9087	*745*	1818·042 2508
546	1259·150 1483	*596*	1396·994 0334	*646*	1536·660 2335	*696*	1678·007 5179	*746*	1820·914 9897
547	1261·888 1357	*597*	1399·770 0077	*647*	1539·471 1378	*697*	1680·850 7507	*747*	1823·788 3103
548	1264·626 9162	*598*	1402·546 7089	*648*	1542·282 7128	*698*	1683·694 6061	*748*	1826·662 2119
549	1267·366 4886	*599*	1405·324 1357	*649*	1545·094 9575	*699*	1686·539 0833	*749*	1829·536 6937
550	1270·106 8513	*600*	1408·102 2870	*650*	1547·907 8709	*700*	1689·384 1813	*750*	1832·411 7549

TABLE XLIX—(*continued*).

751—1000

n	log $\underline{\lvert n}$	n	log $\underline{\lvert n}$	n	log $\underline{\lvert n}$	n	log $\underline{\lvert n}$	n	log $\underline{\lvert n}$
751	1835·287 3949	*801*	1979·790 7168	*851*	2125·649 5488	*901*	2272·784 2010	*951*	2421·123 8376
752	1838·163 6127	*802*	1982·694 8911	*852*	2128·579 9884	*902*	2275·739 4075	*952*	2424·102 4745
753	1841·040 4077	*803*	1985·599 6067	*853*	2131·510 9374	*903*	2278·695 0953	*953*	2427·081 5674
754	1843·917 7790	*804*	1988·504 8627	*854*	2134·442 3953	*904*	2281·651 2637	*954*	2430·061 1158
755	1846·795 7260	*805*	1991·410 6586	*855*	2137·374 3614	*905*	2284·607 9123	*955*	2433·041 1192
756	1849·674 2478	*806*	1994·316 9936	*856*	2140·306 8352	*906*	2287·565 0405	*956*	2436·021 5771
757	1852·553 3437	*807*	1997·223 8672	*857*	2143·239 8160	*907*	2290·522 6478	*957*	2439·002 4890
758	1855·433 0129	*808*	2000·131 2785	*858*	2146·173 3033	*908*	2293·480 7336	*958*	2441·983 8545
759	1858·313 2546	*809*	2003·039 2271	*859*	2149·107 2964	*909*	2296·439 2975	*959*	2444·965 6731
760	1861·194 0682	*810*	2005·947 7121	*860*	2152·041 7949	*910*	2299·398 3389	*960*	2447·947 9443
761	1864·075 4529	*811*	2008·856 7329	*861*	2154·976 7980	*911*	2302·357 8573	*961*	2450·930 6677
762	1866·957 4079	*812*	2011·766 2890	*862*	2157·912 3053	*912*	2305·317 8521	*962*	2453·913 8428
763	1869·839 9324	*813*	2014·676 3795	*863*	2160·848 3161	*913*	2308·278 3229	*963*	2456·897 4691
764	1872·723 0258	*814*	2017·587 0039	*864*	2163·784 8298	*914*	2311·239 2691	*964*	2459·881 5461
765	1875·606 6872	*815*	2020·498 1615	*865*	2166·721 8459	*915*	2314·200 6902	*965*	2462·866 0734
766	1878·490 9160	*816*	2023·409 8517	*866*	2169·659 3638	*916*	2317·162 5856	*966*	2465·851 0506
767	1881·375 7113	*817*	2026·322 0737	*867*	2172·597 3829	*917*	2320·124 9550	*967*	2468·836 4770
768	1884·261 0726	*818*	2029·234 8270	*868*	2175·535 9027	*918*	2323·087 7977	*968*	2471·822 3524
769	1887·146 9989	*819*	2032·148 1109	*869*	2178·474 9224	*919*	2326·051 1132	*969*	2474·808 6762
770	1890·033 4896	*820*	2035·061 9248	*870*	2181·414 4417	*920*	2329·014 9010	*970*	2477·795 4479
771	1892·920 5440	*821*	2037·976 2679	*871*	2184·354 4598	*921*	2331·979 1606	*971*	2480·782 6671
772	1895·808 1613	*822*	2040·891 1398	*872*	2187·294 9763	*922*	2334·943 8915	*972*	2483·770 3334
773	1898·696 3408	*823*	2043·806 5396	*873*	2190·235 9906	*923*	2337·909 0932	*973*	2486·758 4462
774	1901·585 0817	*824*	2046·722 4668	*874*	2193·177 5020	*924*	2340·874 7652	*974*	2489·747 0052
775	1904·474 3835	*825*	2049·638 9208	*875*	2196·119 5101	*925*	2343·840 9069	*975*	2492·736 0098
776	1907·364 2452	*826*	2052·555 9008	*876*	2199·062 0142	*926*	2346·807 5179	*976*	2495·725 4596
777	1910·254 6662	*827*	2055·473 4063	*877*	2202·005 0138	*927*	2349·774 5977	*977*	2498·715 3542
778	1913·145 6458	*828*	2058·391 4367	*878*	2204·948 5083	*928*	2352·742 1456	*978*	2501·705 6930
779	1916·037 1832	*829*	2061·309 9912	*879*	2207·892 4971	*929*	2355·710 1614	*979*	2504·696 4757
780	1918·929 2778	*830*	2064·229 0693	*880*	2210·836 9798	*930*	2358·678 6443	*980*	2507·687 7018
781	1921·821 9289	*831*	2067·148 6703	*881*	2213·781 9557	*931*	2361·647 5940	*981*	2510·679 3708
782	1924·715 1356	*832*	2070·068 7936	*882*	2216·727 4243	*932*	2364·617 0099	*982*	2513·671 4823
783	1927·608 8974	*833*	2072·989 4386	*883*	2219·673 3850	*933*	2367·586 8915	*983*	2516·664 0358
784	1930·503 2135	*834*	2075·910 6047	*884*	2222·619 8373	*934*	2370·557 2384	*984*	2519·657 0309
785	1933·398 0831	*835*	2078·832 2912	*885*	2225·566 7805	*935*	2373·528 0500	*985*	2522·650 4672
786	1936·293 5057	*836*	2081·754 4974	*886*	2228·514 2143	*936*	2376·499 3259	*986*	2525·644 3441
787	1939·189 4804	*837*	2084·677 2229	*887*	2231·462 1379	*937*	2379·471 0655	*987*	2528·638 6612
788	1942·086 0066	*838*	2087·600 4669	*888*	2234·410 5509	*938*	2382·443 2683	*988*	2531·633 4182
789	1944·983 0836	*839*	2090·524 2289	*889*	2237·359 4526	*939*	2385·415 9339	*989*	2534·628 6145
790	1947·880 7107	*840*	2093·448 5082	*890*	2240·308 8426	*940*	2388·389 0618	*990*	2537·624 2497
791	1950·778 8872	*841*	2096·373 3042	*891*	2243·258 7203	*941*	2391·362 6514	*991*	2540·620 3233
792	1953·677 6124	*842*	2099·298 6162	*892*	2246·209 0852	*942*	2394·336 7023	*992*	2543·616 8350
793	1956·576 8856	*843*	2102·224 4438	*893*	2249·159 9366	*943*	2397·311 2140	*993*	2546·613 7842
794	1959·476 7061	*844*	2105·150 7863	*894*	2252·111 2742	*944*	2400·286 1860	*994*	2549·611 1706
795	1962·377 0732	*845*	2108·077 6430	*895*	2255·063 0972	*945*	2403·261 6178	*995*	2552·608 9937
796	1965·277 9863	*846*	2111·005 0133	*896*	2258·015 4052	*946*	2406·237 5089	*996*	2555·607 2530
797	1968·179 4446	*847*	2113·932 8967	*897*	2260·968 1976	*947*	2409·213 8589	*997*	2558·605 9482
798	1971·081 4475	*848*	2116·861 2926	*898*	2263·921 4740	*948*	2412·190 6672	*998*	2561·605 0787
799	1973·983 9943	*849*	2119·790 2003	*899*	2266·875 2337	*949*	2415·167 9334	*999*	2564·604 6442
800	1976·887 0842	*850*	2122·719 6192	*900*	2269·829 4762	*950*	2418·145 6570	*1000*	2567·604 6442

TABLE L. *Table of Fourth-Moments of Subgroup-Frequencies.*

Ordinate 2—11. Frequency 1—50.

n	*x*=2	*x*=3	*x*=4	*x*=5	*x*=6	*x*=7	*x*=8	*x*=9	*x*=11	*n*
1	16	81	256	625	1296	2401	4096	6561	14641	*1*
2	32	162	512	1250	2592	4802	8192	13122	29282	*2*
3	48	243	768	1875	3888	7203	12288	19683	43923	*3*
4	64	324	1024	2500	5184	9604	16384	26244	58564	*4*
5	80	405	1280	3125	6480	12005	20480	32805	73205	*5*
6	96	486	1536	3750	7776	14406	24576	39366	87846	*6*
7	112	567	1792	4375	9072	16807	28672	45927	102487	*7*
8	128	648	2048	5000	10368	19208	32768	52488	117128	*8*
9	144	729	2304	5625	11664	21609	36864	59049	131769	*9*
10	160	810	2560	6250	12960	24010	40960	65610	146410	*10*
11	176	891	2816	6875	14256	26411	45056	72171	161051	*11*
12	192	972	3072	7500	15552	28812	49152	78732	175692	*12*
13	208	1053	3328	8125	16848	31213	53248	85293	190333	*13*
14	224	1134	3584	8750	18144	33614	57344	91854	204974	*14*
15	240	1215	3840	9375	19440	36015	61440	98415	219615	*15*
16	256	1296	4096	10000	20736	38416	65536	104976	234256	*16*
17	272	1377	4352	10625	22032	40817	69632	111537	248897	*17*
18	288	1458	4608	11250	23328	43218	73728	118098	263538	*18*
19	304	1539	4864	11875	24624	45619	77824	124659	278179	*19*
20	320	1620	5120	12500	25920	48020	81920	131220	292820	*20*
21	336	1701	5376	13125	27216	50421	86016	137781	307461	*21*
22	352	1782	5632	13750	28512	52822	90112	144342	322102	*22*
23	368	1863	5888	14375	29808	55223	94208	150903	336743	*23*
24	384	1944	6144	15000	31104	57624	98304	157464	351384	*24*
25	400	2025	6400	15625	32400	60025	102400	164025	366025	*25*
26	416	2106	6656	16250	33696	62426	106496	170586	380666	*26*
27	432	2187	6912	16875	34992	64827	110592	177147	395307	*27*
28	448	2268	7168	17500	36288	67228	114688	183708	409948	*28*
29	464	2349	7424	18125	37584	69629	118784	190269	424589	*29*
30	480	2430	7680	18750	38880	72030	122880	196830	439230	*30*
31	496	2511	7936	19375	40176	74431	126976	203391	453871	*31*
32	512	2592	8192	20000	41472	76832	131072	209952	468512	*32*
33	528	2673	8448	20625	42768	79233	135168	216513	483153	*33*
34	544	2754	8704	21250	44064	81634	139264	223074	497794	*34*
35	560	2835	8960	21875	45360	84035	143360	229635	512435	*35*
36	576	2916	9216	22500	46656	86436	147456	236196	527076	*36*
37	592	2997	9472	23125	47952	88837	151552	242757	541717	*37*
38	608	3078	9728	23750	49248	91238	155648	249318	556358	*38*
39	624	3159	9984	24375	50544	93639	159744	255879	570999	*39*
40	640	3240	10240	25000	51840	96040	163840	262440	585640	*40*
41	656	3321	10496	25625	53136	98441	167936	269001	600281	*41*
42	672	3402	10752	26250	54432	100842	172032	275562	614922	*42*
43	688	3483	11008	26875	55728	103243	176128	282123	629563	*43*
44	704	3564	11264	27500	57024	105644	180224	288684	644204	*44*
45	720	3645	11520	28125	58320	108045	184320	295245	658845	*45*
46	736	3726	11776	28750	59616	110446	188416	301806	673486	*46*
47	752	3807	12032	29375	60912	112847	192512	308367	688127	*47*
48	768	3888	12288	30000	62208	115248	196608	314928	702768	*48*
49	784	3969	12544	30625	63504	117649	200704	321489	717409	*49*
50	800	4050	12800	31250	64800	120050	204800	328050	732050	*50*

TABLE L—(*continued*).

Ordinate 12—19. Frequency 1—50.

n	*x*=12	*x*=13	*x*=14	*x*=15	*x*=16	*x*=17	*x*=18	*x*=19	*n*
1	20736	28561	38416	50625	65536	83521	104976	130321	*1*
2	41472	57122	76832	101250	131072	167042	209952	260642	*2*
3	62208	85683	115248	151875	196608	250563	314928	390963	*3*
4	82944	114244	153664	202500	262144	334084	419904	521284	*4*
5	103680	142805	192080	253125	327680	417605	524880	651605	*5*
6	124416	171366	230496	303750	393216	501126	629856	781926	*6*
7	145152	199927	268912	354375	458752	584647	734832	912247	*7*
8	165888	228488	307328	405000	524288	668168	839808	1042568	*8*
9	186624	257049	345744	455625	589824	751689	944784	1172889	*9*
10	207360	285610	384160	506250	655360	835210	1049760	1303210	*10*
11	228096	314171	422576	556875	720896	918731	1154736	1433531	*11*
12	248832	342732	460992	607500	786432	1002252	1259712	1563852	*12*
13	269568	371293	499408	658125	851968	1085773	1364688	1694173	*13*
14	290304	399854	537824	708750	917504	1169294	1469664	1824494	*14*
15	311040	428415	576240	759375	983040	1252815	1574640	1954815	*15*
16	331776	456976	614656	810000	1048576	1336336	1679616	2085136	*16*
17	352512	485537	653072	860625	1114112	1419857	1784592	2215457	*17*
18	373248	514098	691488	911250	1179648	1503378	1889568	2345778	*18*
19	393984	542659	729904	961875	1245184	1586899	1994544	2476099	*19*
20	414720	571220	768320	1012500	1310720	1670420	2099520	2606420	*20*
21	435456	599781	806736	1063125	1376256	1753941	2204496	2736741	*21*
22	456192	628342	845152	1113750	1441792	1837462	2309472	2867062	*22*
23	476928	656903	883568	1164375	1507328	1920983	2414448	2997383	*23*
24	497664	685464	921984	1215000	1572864	2004504	2519424	3127704	*24*
25	518400	714025	960400	1265625	1638400	2088025	2624400	3258025	*25*
26	539136	742586	998816	1316250	1703936	2171546	2729376	3388346	*26*
27	559872	771147	1037232	1366875	1769472	2255067	2834352	3518667	*27*
28	580608	799708	1075648	1417500	1835008	2338588	2939328	3648988	*28*
29	601344	828269	1114064	1468125	1900544	2422109	3044304	3779309	*29*
30	622080	856830	1152480	1518750	1966080	2505630	3149280	3909630	*30*
31	642816	885391	1190896	1569375	2031616	2589151	3254256	4039951	*31*
32	663552	913952	1229312	1620000	2097152	2672672	3359232	4170272	*32*
33	684288	942513	1267728	1670625	2162688	2756193	3464208	4300593	*33*
34	705024	971074	1306144	1721250	2228224	2839714	3569184	4430914	*34*
35	725760	999635	1344560	1771875	2293760	2923235	3674160	4561235	*35*
36	746496	1028196	1382976	1822500	2359296	3006756	3779136	4691556	*36*
37	767232	1056757	1421392	1873125	2424832	3090277	3884112	4821877	*37*
38	787968	1085318	1459808	1923750	2490368	3173798	3989088	4952198	*38*
39	808704	1113879	1498224	1974375	2555904	3257319	4094064	5082519	*39*
40	829440	1142440	1536640	2025000	2621440	3340840	4199040	5212840	*40*
41	850176	1171001	1575056	2075625	2686976	3424361	4304016	5343161	*41*
42	870912	1199562	1613472	2126250	2752512	3507882	4408992	5473482	*42*
43	891648	1228123	1651888	2176875	2818048	3591403	4513968	5603803	*43*
44	912384	1256684	1690304	2227500	2883584	3674924	4618944	5734124	*44*
45	933120	1285245	1728720	2278125	2949120	3758445	4723920	5864445	*45*
46	953856	1313806	1767136	2328750	3014656	3841966	4828896	5994766	*46*
47	974592	1342367	1805552	2379375	3080192	3925487	4933872	6125087	*47*
48	995328	1370928	1843968	2430000	3145728	4009008	5038848	6255408	*48*
49	1016064	1399489	1882384	2480625	3211264	4092529	5143824	6385729	*49*
50	1036800	1428050	1920800	2531250	3276800	4176050	5248800	6516050	*50*

TABLE L—(*continued*).

Ordinate 2—11. Frequency 51—100.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	$x=11$	n
51	816	4131	13056	31875	66096	122451	208896	334611	746691	51
52	832	4212	13312	32500	67392	124852	212992	341172	761332	52
53	848	4293	13568	33125	68688	127253	217088	347733	775973	53
54	864	4374	13824	33750	69984	129654	221184	354294	790614	54
55	880	4455	14080	34375	71280	132055	225280	360855	805255	55
56	896	4536	14336	35000	72576	134456	229376	367416	819896	56
57	912	4617	14592	35625	73872	136857	233472	373977	834537	57
58	928	4698	14848	36250	75168	139258	237568	380538	849178	58
59	944	4779	15104	36875	76464	141659	241664	387099	863819	59
60	960	4860	15360	37500	77760	144060	245760	393660	878460	60
61	976	4941	15616	38125	79056	146461	249856	400221	893101	61
62	992	5022	15872	38750	80352	148862	253952	406782	907742	62
63	1008	5103	16128	39375	81648	151263	258048	413343	922383	63
64	1024	5184	16384	40000	82944	153664	262144	419904	937024	64
65	1040	5265	16640	40625	84240	156065	266240	426465	951665	65
66	1056	5346	16896	41250	85536	158466	270336	433026	966306	66
67	1072	5427	17152	41875	86832	160867	274432	439587	980947	67
68	1088	5508	17408	42500	88128	163268	278528	446148	995588	68
69	1104	5589	17664	43125	89424	165669	282624	452709	1010229	69
70	1120	5670	17920	43750	90720	168070	286720	459270	1024870	70
71	1136	5751	18176	44375	92016	170471	290816	465831	1039511	71
72	1152	5832	18432	45000	93312	172872	294912	472392	1054152	72
73	1168	5913	18688	45625	94608	175273	299008	478953	1068793	73
74	1184	5994	18944	46250	95904	177674	303104	485514	1083434	74
75	1200	6075	19200	46875	97200	180075	307200	492075	1098075	75
76	1216	6156	19456	47500	98496	182476	311296	498636	1112716	76
77	1232	6237	19712	48125	99792	184877	315392	505197	1127357	77
78	1248	6318	19968	48750	101088	187278	319488	511758	1141998	78
79	1264	6399	20224	49375	102384	189679	323584	518319	1156639	79
80	1280	6480	20480	50000	103680	192080	327680	524880	1171280	80
81	1296	6561	20736	50625	104976	194481	331776	531441	1185921	81
82	1312	6642	20992	51250	106272	196882	335872	538002	1200562	82
83	1328	6723	21248	51875	107568	199283	339968	544563	1215203	83
84	1344	6804	21504	52500	108864	201684	344064	551124	1229844	84
85	1360	6885	21760	53125	110160	204085	348160	557685	1244485	85
86	1376	6966	22016	53750	111456	206486	352256	564246	1259126	86
87	1392	7047	22272	54375	112752	208887	356352	570807	1273767	87
88	1408	7128	22528	55000	114048	211288	360448	577368	1288408	88
89	1424	7209	22784	55625	115344	213689	364544	583929	1303049	89
90	1440	7290	23040	56250	116640	216090	368640	590490	1317690	90
91	1456	7371	23296	56875	117936	218491	372736	597051	1332331	91
92	1472	7452	23552	57500	119232	220892	376832	603612	1346972	92
93	1488	7533	23808	58125	120528	223293	380928	610173	1361613	93
94	1504	7614	24064	58750	121824	225694	385024	616734	1376254	94
95	1520	7695	24320	59375	123120	228095	389120	623295	1390895	95
96	1536	7776	24576	60000	124416	230496	393216	629856	1405536	96
97	1552	7857	24832	60625	125712	232897	397312	636417	1420177	97
98	1568	7938	25088	61250	127008	235298	401408	642978	1434818	98
99	1584	8019	25344	61875	128304	237699	405504	649539	1449459	99
100	1600	8100	25600	62500	129600	240100	409600	656100	1464100	100

TABLE L—(*continued*).

Ordinate 12—19. Frequency 51—100.

n	$x=12$	$x=13$	$x=14$	$x=15$	$x=16$	$x=17$	$x=18$	$x=19$	*n*
51	1057536	1456611	1959216	2581875	3342336	4259571	5353776	6646371	*51*
52	1078272	1485172	1997632	2632500	3407872	4343092	5458752	6776692	*52*
53	1099008	1513733	2036048	2683125	3473408	4426613	5563728	6907013	*53*
54	1119744	1542294	2074464	2733750	3538944	4510134	5668704	7037334	*54*
55	1140480	1570855	2112880	2784375	3604480	4593655	5773680	7167655	*55*
56	1161216	1599416	2151296	2835000	3670016	4677176	5878656	7297976	*56*
57	1181952	1627977	2189712	2885625	3735552	4760697	5983632	7428297	*57*
58	1202688	1656538	2228128	2936250	3801088	4844218	6088608	7558618	*58*
59	1223424	1685099	2266544	2986875	3866624	4927739	6193584	7688939	*59*
60	1244160	1713660	2304960	3037500	3932160	5011260	6298560	7819260	*60*
61	1264896	1742221	2343376	3088125	3997696	5094781	6403536	7949581	*61*
62	1285632	1770782	2381792	3138750	4063232	5178302	6508512	8079902	*62*
63	1306368	1799343	2420208	3189375	4128768	5261823	6613488	8210223	*63*
64	1327104	1827904	2458624	3240000	4194304	5345344	6718464	8340544	*64*
65	1347840	1856465	2497040	3290625	4259840	5428865	6823440	8470865	*65*
66	1368576	1885026	2535456	3341250	4325376	5512386	6928416	8601186	*66*
67	1389312	1913587	2573872	3391875	4390912	5595907	7033392	8731507	*67*
68	1410048	1942148	2612288	3442500	4456448	5679428	7138368	8861828	*68*
69	1430784	1970709	2650704	3493125	4521984	5762949	7243344	8992149	*69*
70	1451520	1999270	2689120	3543750	4587520	5846470	7348320	9122470	*70*
71	1472256	2027831	2727536	3594375	4653056	5929991	7453296	9252791	*71*
72	1492992	2056392	2765952	3645000	4718592	6013512	7558272	9383112	*72*
73	1513728	2084953	2804368	3695625	4784128	6097033	7663248	9513433	*73*
74	1534464	2113514	2842784	3746250	4849664	6180554	7768224	9643754	*74*
75	1555200	2142075	2881200	3796875	4915200	6264075	7873200	9774075	*75*
76	1575936	2170636	2919616	3847500	4980736	6347596	7978176	9904396	*76*
77	1596672	2199197	2958032	3898125	5046272	6431117	8083152	10034717	*77*
78	1617408	2227758	2996448	3948750	5111808	6514638	8188128	10165038	*78*
79	1638144	2256319	3034864	3999375	5177344	6598159	8293104	10295359	*79*
80	1658880	2284880	3073280	4050000	5242880	6681680	8398080	10425680	*80*
81	1679616	2313441	3111696	4100625	5308416	6765201	8503056	10556001	*81*
82	1700352	2342002	3150112	4151250	5373952	6848722	8608032	10686322	*82*
83	1721088	2370563	3188528	4201875	5439488	6932243	8713008	10816643	*83*
84	1741824	2399124	3226944	4252500	5505024	7015764	8817984	10946964	*84*
85	1762560	2427685	3265360	4303125	5570560	7099285	8922960	11077285	*85*
86	1783296	2456246	3303776	4353750	5636096	7182806	9027936	11207606	*86*
87	1804032	2484807	3342192	4404375	5701632	7266327	9132912	11337927	*87*
88	1824768	2513368	3380608	4455000	5767168	7349848	9237888	11468248	*88*
89	1845504	2541929	3419024	4505625	5832704	7433369	9342864	11598569	*89*
90	1866240	2570490	3457440	4556250	5898240	7516890	9447840	11728890	*90*
91	1886976	2599051	3495856	4606875	5963776	7600411	9552816	11859211	*91*
92	1907712	2627612	3534272	4657500	6029312	7683932	9657792	11989532	*92*
93	1928448	2656173	3572688	4708125	6094848	7767453	9762768	12119853	*93*
94	1949184	2684734	3611104	4758750	6160384	7850974	9867744	12250174	*94*
95	1969920	2713295	3649520	4809375	6225920	7934495	9972720	12380495	*95*
96	1990656	2741856	3687936	4860000	6291456	8018016	10077696	12510816	*96*
97	2011392	2770417	3726352	4910625	6356992	8101537	10182672	12641137	*97*
98	2032128	2798978	3764768	4961250	6422528	8185058	10287648	12771458	*98*
99	2052864	2827539	3803184	5011875	6488064	8268579	10392624	12901779	*99*
100	2073600	2856100	3841600	5062500	6553600	8352100	10497600	13032100	*100*

TABLE L—(*continued*).

Ordinate 2—7. Frequency 101—150.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	n
101	1616	8181	25856	63125	130896	242501	*101*
102	1632	8262	26112	63750	132192	244902	*102*
103	1648	8343	26368	64375	133488	247303	*103*
104	1664	8424	26624	65000	134784	249704	*104*
105	1680	8505	26880	65625	136080	252105	*105*
106	1696	8586	27136	66250	137376	254506	*106*
107	1712	8667	27392	66875	138672	256907	*107*
108	1728	8748	27648	67500	139968	259308	*108*
109	1744	8829	27904	68125	141264	261709	*109*
110	1760	8910	28160	68750	142560	264110	*110*
111	1776	8991	28416	69375	143856	266511	*111*
112	1792	9072	28672	70000	145152	268912	*112*
113	1808	9153	28928	70625	146448	271313	*113*
114	1824	9234	29184	71250	147744	273714	*114*
115	1840	9315	29440	71875	149040	276115	*115*
116	1856	9396	29696	72500	150336	278516	*116*
117	1872	9477	29952	73125	151632	280917	*117*
118	1888	9558	30208	73750	152928	283318	*118*
119	1904	9639	30464	74375	154224	285719	*119*
120	1920	9720	30720	75000	155520	288120	*120*
121	1936	9801	30976	75625	156816	290521	*121*
122	1952	9882	31232	76250	158112	292922	*122*
123	1968	9963	31488	76875	159408	295323	*123*
124	1984	10044	31744	77500	160704	297724	*124*
125	2000	10125	32000	78125	162000	300125	*125*
126	2016	10206	32256	78750	163296	302526	*126*
127	2032	10287	32512	79375	164592	304927	*127*
128	2048	10368	32768	80000	165888	307328	*128*
129	2064	10449	33024	80625	167184	309729	*129*
130	2080	10530	33280	81250	168480	312130	*130*
131	2096	10611	33536	81875	169776	314531	*131*
132	2112	10692	33792	82500	171072	316932	*132*
133	2128	10773	34048	83125	172368	319333	*133*
134	2144	10854	34304	83750	173364	321734	*134*
135	2160	10935	34560	84375	174960	324135	*135*
136	2176	11016	34816	85000	176256	326536	*136*
137	2192	11097	35072	85625	177552	328937	*137*
138	2208	11178	35328	86250	178848	331338	*138*
139	2224	11259	35584	86875	180144	333739	*139*
140	2240	11340	35840	87500	181440	336140	*140*
141	2256	11421	36096	88125	182736	338541	*141*
142	2272	11502	36352	88750	184032	340942	*142*
143	2288	11583	36608	89375	185328	343343	*143*
144	2304	11664	36864	90000	186624	345744	*144*
145	2320	11745	37120	90625	187920	348145	*145*
146	2336	11826	37376	91250	189216	350546	*146*
147	2352	11907	37632	91875	190512	352947	*147*
148	2368	11988	37888	92500	191808	355348	*148*
149	2384	12069	38144	93125	193104	357749	*149*
150	2400	12150	38400	93750	194400	360150	*150*

TABLE L—(*continued*).

Ordinate 8—14. Frequency 101—150.

n	$x=8$	$x=9$	$x=11$	$x=12$	$x=13$	$x=14$	n
101	413696	662661	1478741	2094336	2884661	3880016	*101*
102	417792	669222	1493382	2115072	2913222	3918432	*102*
103	421888	675783	1508023	2135808	2941783	3956848	*103*
104	425984	682344	1522664	2156544	2970344	3995264	*104*
105	430080	688905	1537305	2177280	2998905	4033680	*105*
106	434176	695466	1551946	2198016	3027466	4072096	*106*
107	438272	702027	1566587	2218752	3056027	4110512	*107*
108	442368	708588	1581228	2239488	3084588	4148928	*108*
109	446464	715149	1595869	2260224	3113149	4187344	*109*
110	450560	721710	1610510	2280960	3141710	4225760	*110*
111	454656	728271	1625151	2301696	3170271	4264176	*111*
112	458752	734832	1639792	2322432	3198832	4302592	*112*
113	462848	741393	1654433	2343168	3227393	4341008	*113*
114	466944	747954	1669074	2363904	3255954	4379424	*114*
115	471040	754515	1683715	2384640	3284515	4417840	*115*
116	475136	761076	1698356	2405376	3313076	4456256	*116*
117	479232	767637	1712997	2426112	3341637	4494672	*117*
118	483328	774198	1727638	2446848	3370198	4533088	*118*
119	487424	780759	1742279	2467584	3398759	4571504	*119*
120	491520	787320	1756920	2488320	3427320	4609920	*120*
121	495616	793881	1771561	2509056	3455881	4648336	*121*
122	499712	800442	1786202	2529792	3484442	4686752	*122*
123	503808	807003	1800843	2550528	3513003	4725168	*123*
124	507904	813564	1815484	2571264	3541564	4763584	*124*
125	512000	820125	1830125	2592000	3570125	4802000	*125*
126	516096	826686	1844766	2612736	3598686	4840416	*126*
127	520192	833247	1859407	2633472	3627247	4878832	*127*
128	524288	839808	1874048	2654208	3655808	4917248	*128*
129	528384	846369	1888689	2674944	3684369	4955664	*129*
130	532480	852930	1903330	2695680	3712930	4994080	*130*
131	536576	859491	1917971	2716416	3741491	5032496	*131*
132	540672	866052	1932612	2737152	3770052	5070912	*132*
133	544768	872613	1947253	2757888	3798613	5109328	*133*
134	548864	879174	1961894	2778624	3827174	5147744	*134*
135	552960	885735	1976535	2799360	3855735	5186160	*135*
136	557056	892296	1991176	2820096	3884296	5224576	*136*
137	561152	898857	2005817	2840832	3912857	5262992	*137*
138	565248	905418	2020458	2861568	3941418	5301408	*138*
139	569344	911979	2035099	2882304	3969979	5339824	*139*
140	573440	918540	2049740	2903040	3998540	5378240	*140*
141	577536	925101	2064381	2923776	4027101	5416656	*141*
142	581632	931662	2079022	2944512	4055662	5455072	*142*
143	585728	938223	2093663	2965248	4084223	5493488	*143*
144	589824	944784	2108304	2985984	4112784	5531904	*144*
145	593920	951345	2122945	3006720	4141345	5570320	*145*
146	598016	957906	2137586	3027456	4169906	5608736	*146*
147	602112	964467	2152227	3048192	4198467	5647152	*147*
148	606208	971028	2166868	3068928	4227028	5685568	*148*
149	610304	977589	2181509	3089664	4255589	5723984	*149*
150	614400	984150	2196150	3110400	4284150	5762400	*150*

TABLE L—(*continued*).

Ordinate 2—12. Frequency 151—200.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	$x=11$	$x=12$	n
151	2416	12231	38656	94375	195696	362551	618496	990711	2210791	3131136	151
152	2432	12312	38912	95000	196992	364952	622592	997272	2225432	3151872	152
153	2448	12393	39168	95625	198288	367353	626688	1003833	2240073	3172608	153
154	2464	12474	39424	96250	199584	369754	630784	1010394	2254714	3193344	154
155	2480	12555	39680	96875	200880	372155	634880	1016955	2269355	3214080	155
156	2496	12636	39936	97500	202176	374556	638976	1023516	2283996	3234816	156
157	2512	12717	40192	98125	203472	376957	643072	1030077	2298637	3255552	157
158	2528	12798	40448	98750	204768	379358	647168	1036638	2313278	3276288	158
159	2544	12879	40704	99375	206064	381759	651264	1043199	2327919	3297024	159
160	2560	12960	40960	100000	207360	384160	655360	1049760	2342560	3317760	160
161	2576	13041	41216	100625	208656	386561	659456	1056321	2357201	3338496	161
162	2592	13122	41472	101250	209952	388962	663552	1062882	2371842	3359232	162
163	2608	13203	41728	101875	211248	391363	667648	1069443	2386483	3379968	163
164	2624	13284	41984	102500	212544	393764	671744	1076004	2401124	3400704	164
165	2680	13365	42240	103125	213840	396165	675840	1082565	2415765	3421440	165
166	2656	13446	42496	103750	215136	398566	679936	1089126	2430406	3442176	166
167	2672	13527	42752	104375	216432	400967	684032	1095687	2445047	3462912	167
168	2688	13608	43008	105000	217728	403368	688128	1102248	2459688	3483648	168
169	2704	13689	43264	105625	219024	405769	692224	1108809	2474329	3504384	169
170	2720	13770	43520	106250	220320	408170	696320	1115370	2488970	3525120	170
171	2736	13851	43776	106875	221616	410571	700416	1121931	2503611	3545856	171
172	2752	13932	44032	107500	222912	412972	704512	1128492	2518252	3566592	172
173	2768	14013	44288	108125	224208	415373	708608	1135053	2532893	3587328	173
174	2784	14094	44544	108750	225504	417774	712704	1141614	2547534	3608064	174
175	2800	14175	44800	109375	226800	420175	716800	1148175	2562175	3628800	175
176	2816	14256	45056	110000	228096	422576	720896	1154736	2576816	3649536	176
177	2832	14337	45312	110625	229392	424977	724992	1161297	2591457	3670272	177
178	2848	14418	45568	111250	230688	427378	729088	1167858	2606098	3691008	178
179	2864	14499	45824	111875	231984	429779	733184	1174419	2620739	3711744	179
180	2880	14580	46080	112500	233280	432180	737280	1180980	2635380	3732480	180
181	2896	14661	46336	113125	234576	434581	741376	1187541	2650021	3753216	181
182	2912	14742	46592	113750	235872	436982	745472	1194102	2664662	3773952	182
183	2928	14823	46848	114375	237168	439383	749568	1200663	2679303	3794688	183
184	2944	14904	47104	115000	238464	441784	753664	1207224	2693944	3815424	184
185	2960	14985	47360	115625	239760	444185	757760	1213785	2708585	3836160	185
186	2976	15066	47616	116250	241056	446586	761856	1220346	2723226	3856896	186
187	2992	15147	47872	116875	242352	448987	765952	1226907	2737867	3877632	187
188	3008	15228	48128	117500	243648	451388	770048	1233468	2752508	3898368	188
189	3024	15309	48384	118125	244944	453789	774144	1240029	2767149	3919104	189
190	3040	15390	48640	118750	246240	456190	778240	1246590	2781790	3939840	190
191	3056	15471	48896	119375	247536	458591	782336	1253151	2796431	3960576	191
192	3072	15552	49152	120000	248832	460992	786432	1259712	2811072	3981312	192
193	3088	15633	49408	120625	250128	463393	790528	1266273	2825713	4002048	193
194	3104	15714	49664	121250	251424	465794	794624	1272834	2840354	4022784	194
195	3120	15795	49920	121875	252720	468195	798720	1279395	2854995	4043520	195
196	3136	15876	50176	122500	254016	470596	802816	1285956	2869636	4064256	196
197	3152	15957	50432	123125	255312	472997	806912	1292517	2884277	4084992	197
198	3168	16038	50688	123750	256608	475398	811508	1299078	2898918	4105728	198
199	3184	16119	50944	128375	257904	477799	815104	1305639	2913559	4126464	199
200	3200	16200	51200	125000	259200	480200	819200	1312200	2928200	4147200	200

TABLE L—(*continued*).

Ordinate 2—11. Frequency 201—250.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	$x=11$	n
201	3216	16281	51456	125625	260496	482601	823296	1318761	2942841	*201*
202	3232	16362	51712	126250	261792	485002	827392	1325322	2957482	*202*
203	3248	16443	51968	126875	263088	487403	831488	1331883	2972123	*203*
204	3264	16524	52224	127500	264384	489804	835584	1338444	2986764	*204*
205	3280	16605	52480	128125	265680	492205	839680	1345005	3001405	*205*
206	3296	16686	52736	128750	266976	494606	843776	1351566	3016046	*206*
207	3312	16767	52992	129375	268272	497007	847872	1358127	3030687	*207*
208	3328	16848	53248	130000	269568	499408	851968	1364688	3045328	*208*
209	3344	16929	53504	130625	270864	501809	856064	1371249	3059969	*209*
210	3360	17010	53760	131250	272160	504210	860160	1377810	3074610	*210*
211	3376	17091	54016	131875	273456	506611	864256	1384371	3089251	*211*
212	3392	17172	54272	132500	274752	509012	868352	1390932	3103892	*212*
213	3408	17253	54528	133125	276048	511413	872448	1397493	3118533	*213*
214	3424	17334	54784	133750	277344	513814	876544	1404054	3133174	*214*
215	3440	17415	55040	134375	278640	516215	880640	1410615	3147815	*215*
216	3456	17496	55296	135000	279936	518616	884736	1417176	3162456	*216*
217	3472	17577	55552	135625	281232	521017	888832	1423737	3177097	*217*
218	3488	17658	55808	136250	282528	523418	892928	1430298	3191738	*218*
219	3504	17739	56064	136875	283824	525819	897024	1436859	3206379	*219*
220	3520	17820	56320	137500	285120	528220	901120	1443420	3221020	*220*
221	3536	17901	56576	138125	286416	530621	905216	1449981	3235661	*221*
222	3552	17982	56832	138750	287712	533022	909312	1456542	3250302	*222*
223	3568	18063	57088	139375	289008	535423	913408	1463103	3264943	*223*
224	3584	18144	57344	140000	290304	537824	917504	1469664	3279584	*224*
225	3600	18225	57600	140625	291600	540225	921600	1476225	3294225	*225*
226	3616	18306	57856	141250	292896	542626	925696	1482786	3308866	*226*
227	3632	18387	58112	141875	294192	545027	929792	1489347	3323507	*227*
228	3648	18468	58368	142500	295488	547428	933888	1495908	3338148	*228*
229	3664	18549	58624	143125	296784	549829	937984	1502469	3352789	*229*
230	3680	18630	58880	143750	298080	552230	942080	1509030	3367430	*230*
231	3696	18711	59136	144375	299376	554631	946176	1515591	3382071	*231*
232	3712	18792	59392	145000	300672	557032	950272	1522152	3396712	*232*
233	3728	18873	59648	145625	301968	559433	954368	1528713	3411353	*233*
234	3744	18954	59904	146250	303264	561834	958464	1535274	3425994	*234*
235	3760	19035	60160	146875	304560	564235	962560	1541835	3440635	*235*
236	3776	19116	60416	147500	305856	566636	966656	1548396	3455276	*236*
237	3792	19147	60672	148125	307152	569037	970752	1554957	3469917	*237*
238	3808	19278	60928	148750	308448	571438	974848	1561518	3484558	*238*
239	3824	19359	61184	149375	309744	573839	978944	1568079	3499199	*239*
240	3840	19440	61440	150000	311040	576240	983040	1574640	3513840	*240*
241	3856	19521	61696	150625	312336	578641	987136	1581201	3528481	*241*
242	3872	19602	61952	151250	313632	581042	991232	1587762	3543122	*242*
243	3888	19683	62208	151875	314928	583443	995328	1594323	3557763	*243*
244	3904	19764	62464	152500	316224	585844	999424	1600884	3572404	*244*
245	3920	19845	62720	153125	317520	588245	1003520	1607445	3587045	*245*
246	3936	19926	62976	153750	318816	590646	1007616	1614006	3601686	*246*
247	3952	20007	63232	154375	320112	593047	1011712	1620567	3616327	*247*
248	3968	20088	63488	155000	321408	595448	1015808	1627128	3630968	*248*
249	3984	20169	63744	155625	322704	597849	1019904	1633689	3645609	*249*
250	4000	20250	64000	156250	324000	600250	1024000	1640250	3660250	*250*

TABLE L—(*continued*).

Ordinate 2—9. Frequency 251—300.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$	n
251	4016	20331	64256	156875	325296	602651	1028096	1646811	251
252	4032	20412	64512	157500	326592	605052	1032192	1653372	252
253	4048	20493	64768	158125	327888	607453	1036288	1659933	253
254	4064	20574	65024	158750	329184	609854	1040384	1666494	254
255	4080	20655	65280	159375	330480	612255	1044480	1673055	255
256	4096	20736	65536	160000	331776	614656	1048576	1679616	256
257	4112	20817	65792	160625	333072	617057	1052672	1686177	257
258	4128	20898	66048	161250	334368	619458	1056768	1692738	258
259	4144	20979	66304	161875	335664	621859	1060864	1699299	259
260	4160	21060	66560	162500	336960	624260	1064960	1705860	260
261	4176	21141	66816	163125	338256	626661	1069056	1712421	261
262	4192	21222	67072	163750	339552	629062	1073152	1718982	262
263	4208	21303	67328	164375	340848	631463	1077248	1725543	263
264	4224	21384	67584	165000	342144	633864	1081344	1732104	264
265	4240	21465	67840	165625	343440	636265	1085440	1738665	265
266	4256	21546	68096	166250	344736	638666	1089536	1745226	266
267	4272	21627	68352	166875	346032	641067	1093632	1751787	267
268	4288	21708	68608	167500	347328	643468	1097728	1758348	268
269	4304	21789	68864	168125	348624	645869	1101824	1764909	269
270	4320	21870	69120	168750	349920	648270	1105920	1771470	270
271	4336	21951	69376	169375	351216	650671	1110016	1778031	271
272	4352	22032	69632	170000	352512	653072	1114112	1784592	272
273	4368	22113	69888	170625	353808	655473	1118208	1791153	273
274	4384	22194	70144	171250	355104	657874	1122304	1797714	274
275	4400	22275	70400	171875	356400	660275	1126400	1804275	275
276	4416	22356	70656	172500	357696	662676	1130496	1810836	276
277	4432	22437	70912	173125	358992	665077	1134592	1817397	277
278	4448	22518	71168	173750	360288	667478	1138688	1823958	278
279	4464	22599	71424	174375	361584	669879	1142784	1830519	279
280	4480	22680	71680	175000	362880	672280	1146880	1837080	280
281	4496	22761	71936	175625	364176	674681	1150976	1843641	281
282	4512	22842	72192	176250	365472	677082	1155072	1850202	282
283	4528	22923	72448	176875	366768	679483	1159168	1856763	283
284	4544	23004	72704	177500	368064	681884	1163264	1863324	284
285	4560	23085	72960	178125	369360	684285	1167360	1869885	285
286	4576	23166	73216	178750	370656	686686	1171456	1876446	286
287	4592	23247	73472	179375	371952	689087	1175552	1883007	287
288	4608	23328	73728	180000	373248	691488	1179648	1889568	288
289	4624	23409	73984	180625	374544	693889	1183744	1896129	289
290	4640	23490	74240	181250	375840	696290	1187840	1902690	290
291	4656	23571	74496	181875	377136	698691	1191936	1909251	291
292	4672	23652	74752	182500	378432	701092	1196032	1915812	292
293	4688	23733	75008	183125	379728	703493	1200128	1922373	293
294	4704	23814	75264	183750	381024	705894	1204224	1928934	294
295	4720	23895	75520	184375	382320	708295	1208320	1935495	295
296	4736	23976	75776	185000	383616	710696	1212416	1942056	296
297	4752	24057	76032	185625	384912	713097	1216512	1948617	297
298	4768	24138	76288	186250	386208	715498	1220608	1955178	298
299	4784	24219	76544	186875	387504	717899	1224704	1961739	299
300	4800	24300	76800	187500	388800	720300	1228800	1968300	300

TABLE L—(*continued*).

Ordinate 2—8. Frequency 301—350.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	n
301	4816	24381	77056	188125	390096	722701	1232896	*301*
302	4832	24462	77312	188750	391392	725102	1236992	*302*
303	4848	24543	77568	189375	392688	727503	1241088	*303*
304	4864	24624	77824	190000	393984	729904	1245184	*304*
305	4880	24705	78080	190625	395280	732305	1249280	*305*
306	4896	24786	78336	191250	396576	734706	1253376	*306*
307	4912	24867	78592	191875	397872	737107	1257472	*307*
308	4928	24948	78848	192500	399168	739508	1261568	*308*
309	4944	25029	79104	193125	400464	741909	1265664	*309*
310	4960	25110	79360	193750	401760	744310	1269760	*310*
311	4976	25191	79616	194375	403056	746711	1273856	*311*
312	4992	25272	79872	195000	404352	749112	1277952	*312*
313	5008	25353	80128	195625	405648	751513	1282048	*313*
314	5024	25434	80384	196250	406944	753914	1286144	*314*
315	5040	25515	80640	196875	408240	756315	1290240	*315*
316	5056	25596	80896	197500	409536	758716	1294336	*316*
317	5072	25677	81152	198125	410832	761117	1298432	*317*
318	5088	25758	81408	198750	412128	763518	1302528	*318*
319	5104	25839	81664	199375	413424	765919	1306624	*319*
320	5120	25920	81920	200000	414720	768320	1310720	*320*
321	5136	26001	82176	200625	416016	770721	1314816	*321*
322	5152	26082	82432	201250	417312	773122	1318912	*322*
323	5168	26163	82688	201875	418608	775523	1323008	*323*
324	5184	26244	82944	202500	419904	777924	1327104	*324*
325	5200	26325	83200	203125	421200	780325	1331200	*325*
326	5216	26406	83456	203750	422496	782726	1335296	*326*
327	5232	26487	83712	204375	423792	785127	1339392	*327*
328	5248	26568	83968	205000	425088	787528	1343488	*328*
329	5264	26649	84224	205625	426384	789929	1347584	*329*
330	5280	26730	84480	206250	427680	792330	1351680	*330*
331	5296	26811	84736	206875	428976	794731	1355776	*331*
332	5312	26892	84992	207500	430272	797132	1359872	*332*
333	5328	26973	85248	208125	431568	799533	1363968	*333*
334	5344	27054	85504	208750	432864	801934	1368064	*334*
335	5360	27135	85760	209375	434160	804335	1372160	*335*
336	5376	27216	86016	210000	435456	806736	1376256	*336*
337	5392	27297	86272	210625	436752	809137	1380352	*337*
338	5408	27378	86528	211250	438048	811538	1384448	*338*
339	5424	27459	86784	211875	439344	813939	1388544	*339*
340	5440	27540	87040	212500	440640	816340	1392640	*340*
341	5456	27621	87296	213125	441936	818741	1396736	*341*
342	5472	27702	87552	213750	443232	821142	1400832	*342*
343	5488	27783	87808	214375	444528	823543	1404928	*343*
344	5504	27864	88064	215000	445824	825944	1409024	*344*
345	5520	27945	88320	215625	447120	828345	1413120	*345*
346	5536	28026	88576	216250	448416	830746	1417216	*346*
347	5552	28107	88832	216875	449712	833147	1421312	*347*
348	5568	28188	89088	217500	451008	835548	1425408	*348*
349	5584	28269	89344	218125	452304	837949	1429504	*349*
350	5600	28350	89600	218750	453600	840350	1433600	*350*

TABLE L—(*continued*).

Ordinate 2—7. Frequency 351—400.

n	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	n
351	5616	28431	89856	219375	454896	842751	*351*
352	5632	28512	90112	220000	456192	845152	*352*
353	5648	28593	90368	220625	457488	847553	*353*
354	5664	28674	90624	221250	458784	849954	*354*
355	5680	28755	90880	221875	460080	852355	*355*
356	5696	28836	91136	222500	461376	854756	*356*
357	5712	28917	91392	223125	462672	857157	*357*
358	5728	28998	91648	223750	463968	859558	*358*
359	5744	29079	91904	224375	465264	861959	*359*
360	5760	29160	92160	225000	466560	864360	*360*
361	5776	29241	92416	225625	467856	866761	*361*
362	5792	29322	92672	226250	469152	869162	*362*
363	5808	29403	92928	226875	470448	871563	*363*
364	5824	29484	93184	227500	471744	873964	*364*
365	5840	29565	93440	228125	473040	876365	*365*
366	5856	29646	93696	228750	474336	878766	*366*
367	5872	29727	93952	229375	475632	881167	*367*
368	5888	29808	94208	230000	476928	883568	*368*
369	5904	29889	94464	230625	478224	885969	*369*
370	5920	29970	94720	231250	479520	888370	*370*
371	5936	30051	94976	231875	480816	890771	*371*
372	5952	30132	95232	232500	482112	893172	*372*
373	5968	30213	95488	233125	483408	895573	*373*
374	5984	30294	95744	233750	484704	897974	*374*
375	6000	30375	96000	234375	486000	900375	*375*
376	6016	30456	96256	235000	487296	902776	*376*
377	6032	30537	96512	235625	488592	905177	*377*
378	6048	30618	96768	236250	489888	907578	*378*
379	6064	30699	97024	236875	491184	909979	*379*
380	6080	30780	97280	237500	492480	912380	*380*
381	6096	30861	97536	238125	493776	914781	*381*
382	6112	30942	97792	238750	495072	917182	*382*
383	6128	31023	98048	239375	496368	919583	*383*
384	6144	31104	98304	240000	497664	921984	*384*
385	6160	31185	98560	240625	498960	924385	*385*
386	6176	31266	98816	241250	500256	926786	*386*
387	6192	31347	99072	241875	501552	929187	*387*
388	6208	31428	99328	242500	502848	931588	*388*
389	6224	31509	99584	243125	504144	933989	*389*
390	6240	31590	99840	243750	505440	936390	*390*
391	6256	31671	100096	244375	506736	938791	*391*
392	6272	31752	100352	245000	508032	941192	*392*
393	6288	31833	100608	245625	509328	943593	*393*
394	6304	31914	100864	246250	510624	945994	*394*
395	6320	31995	101120	246875	511920	948395	*395*
396	6336	32076	101376	247500	513216	950796	*396*
397	6352	32157	101632	248125	514512	953197	*397*
398	6368	32238	101888	248750	515808	955598	*398*
399	6384	32319	102144	249375	517104	957999	*399*
400	6400	32400	102400	250000	518400	960400	*400*

TABLE LI. *Tables of* $e^{-m}m^x/x!$: *General Term of Poisson's Exponential Expansion ("Law of Small Numbers").*

x	*m*										*x*
	0·1	*0·2*	*0·3*	*0·4*	*0·5*	*0·6*	*0·7*	*0·8*	*0·9*	*1·0*	
0	·904837	·818731	·740818	·670320	·606531	·548812	·496585	·449329	·406570	·367879	*0*
1	·090484	·163746	·222245	·268128	·303265	·329287	·347610	·359463	·365913	·367879	*1*
2	·004524	·016375	·033337	·053626	·075816	·098786	·121663	·143785	·164661	·183940	*2*
3	·000151	·001092	·003334	·007150	·012636	·019757	·028388	·038343	·049398	·061313	*3*
4	·000004	·000055	·000250	·000715	·001580	·002964	·004968	·007669	·011115	·015328	*4*
5	—	·000002	·000015	·000057	·000158	·000356	·000696	·001227	·002001	·003066	*5*
6	—	—	·000001	·000004	·000013	·000036	·000081	·000164	·000300	·000511	*6*
7	—	—	—	—	·000001	·000003	·000008	·000019	·000039	·000073	*7*
8	—	—	—	—	—	—	·000001	·000002	·000004	·000009	*8*
9	—	—	—	—	—	—	—	—	—	·000001	*9*
x	*1·1*	*1·2*	*1·3*	*1·4*	*1·5*	*1·6*	*1·7*	*1·8*	*1·9*	*2·0*	*x*
0	·332871	·301194	·272532	·246597	·223130	.201897	·182684	·165299	·149569	·135335	*0*
1	·366158	·361433	·354291	·345236	·334695	.323034	·310562	·297538	·284180	·270671	*1*
2	·201387	·216860	·230289	·241665	·251021	.258428	·263978	·267784	·269971	·270671	*2*
3	·073842	·086744	·099792	·112777	·125510	·137828	·149587	·160671	·170982	·180447	*3*
4	·020307	·026023	·032432	·039472	·047067	·055131	·063575	·072302	·081216	·090224	*4*
5	·004467	·006246	·008432	·011052	·014120	·017642	·021615	·026029	·030862	·036089	*5*
6	·000819	·001249	·001827	·002579	·003530	·004705	·006124	·007809	·009773	·012030	*6*
7	·000129	·000214	·000339	·000516	·000756	·001075	·001487	·002008	·002653	·003437	*7*
8	·000018	·000032	·000055	·000090	·000142	·000215	·000316	·000452	·000630	·000859	*8*
9	·000002	·000004	·000008	·000014	·000024	·000038	·000060	·000090	·000133	·000191	*9*
10	—	·000001	·000001	·000002	·000004	·000006	·000010	·000016	·000025	·000038	*10*
11	—	—	—	—	—	·000001	·000002	·000003	·000004	·000007	*11*
12	—	—	—	—	—	—	—	—	·000001	·000001	*12*
x	*2·1*	*2·2*	*2·3*	*2·4*	*2·5*	*2·6*	*2·7*	*2·8*	*2·9*	*3·0*	*x*
0	·122456	·110803	·100259	·090718	·082085	·074274	·067206	·060810	·055023	·049787	*0*
1	·257159	·243767	·230595	·217723	·205212	·193111	·181455	·170268	·159567	·149361	*1*
2	·270016	·268144	·265185	·261268	·256516	·251045	·244964	·238375	·231373	·224042	*2*
3	·189012	·196639	·203308	·209014	·213763	·217572	·220468	·222484	·223660	·224042	*3*
4	·099231	·108151	·116902	·125409	·133602	·141422	·148816	·155739	·162154	·168031	*4*
5	·041677	·047587	·053775	·060196	·066801	·073539	·080360	·087214	·094049	·100819	*5*
6	·014587	·017448	·020614	·024078	·027834	·031867	·036162	·040700	·045457	·050409	*6*
7	·004376	·005484	·006773	·008255	·009941	·011836	·013948	·016280	·018832	·021604	*7*
8	·001149	·001508	·001947	·002477	·003106	·003847	·004708	·005698	·006827	·008102	*8*
9	·000268	·000369	·000498	·000660	·000863	·001111	·001412	·001773	·002200	·002701	*9*
10	·000056	·000081	·000114	·000158	·000216	·000289	·000381	·000496	·000638	·000810	*10*
11	·000011	·000016	·000024	·000035	·000049	·000068	·000094	·000126	·000168	·000221	*11*
12	·000002	·000003	·000005	·000007	·000010	·000015	·000021	·000029	·000041	·000055	*12*
13	—	·000001	·000001	·000001	·000002	·000003	·000004	·000006	·000009	·000013	*13*
14	—	—	—	—	—	·000001	·000001	·000001	·000002	·000003	*14*
15	—	—	—	—	—	—	—	—	—	·000001	*15*

TABLE LI—(*continued*).

x	*m*										*x*
	3·1	*3·2*	*3·3*	*3·4*	*3·5*	*3·6*	*3·7*	*3·8*	*3·9*	*4·0*	
0	·045049	·040762	·036883	·033373	·030197	·027324	·024724	·022371	·020242	·018316	*0*
1	·139653	·130439	·121714	·113469	·105691	·098365	·091477	·085009	·078943	·073263	*1*
2	·216461	·208702	·200829	·192898	·184959	·177058	·169233	·161517	·153940	·146525	*2*
3	·223677	·222616	·220912	·218617	·215785	·212469	·208720	·204588	·200122	·195367	*3*
4	·173350	·178093	·182252	·185825	·188812	·191222	·193066	·194359	·195119	·195367	*4*
5	·107477	·113979	·120286	·126361	·132169	·137680	·142869	·147713	·152193	·156293	*5*
6	·055530	·060789	·066158	·071604	·077098	·082608	·088102	·093551	·098925	·104196	*6*
7	·024592	·027789	·031189	·034779	·038549	·042484	·046568	·050785	·055115	·059540	*7*
8	·009529	·011116	·012865	·014781	·016865	·019118	·021538	·024123	·026869	·029770	*8*
9	·003282	·003952	·004717	·005584	·006559	·007647	·008854	·010185	·011643	·013231	*9*
10	·001018	·001265	·001557	·001899	·002296	·002753	·003276	·003870	·004541	·005292	*10*
11	·000287	·000368	·000467	·000587	·000730	·000901	·001102	·001337	·001610	·001925	*11*
12	·000074	·000098	·000128	·000166	·000213	·000270	·000340	·000423	·000523	·000642	*12*
13	·000018	·000024	·000033	·000043	·000057	·000075	·000097	·000124	·000157	·000197	*13*
14	·000004	·000006	·000008	·000011	·000014	·000019	·000026	·000034	·000044	·000056	*14*
15	·000001	·000001	·000002	·000002	·000003	·000005	·000006	·000009	·000011	·000015	*15*
16	—	—	—	·000001	·000001	·000001	·000001	·000002	·000003	·000004	*16*
17	—	—	—	—	—	—	—	—	·000001	·000001	*17*
x	*4·1*	*4·2*	*4·3*	*4·4*	*4·5*	*4·6*	*4·7*	*4·8*	*4·9*	*5·0*	*x*
0	·016573	·014996	·013569	·012277	·011109	·010052	·009095	·008230	·007447	·006738	*0*
1	·067948	·062981	·058345	·054020	·049990	·046238	·042748	·039503	·036488	·033690	*1*
2	·139293	·132261	·125441	·118845	·112479	·106348	·100457	·094807	·089396	·084224	*2*
3	·190368	·185165	·179799	·174305	·168718	·163068	·157383	·151691	·146014	·140374	*3*
4	·195127	·194424	·193284	·191736	·189808	·187528	·184925	·182029	·178867	·175467	*4*
5	·160004	·163316	·166224	·168728	·170827	·172525	·173830	·174748	·175290	·175467	*5*
6	·109336	·114321	·119127	·123734	·128120	·132270	·136167	·139798	·143153	·146223	*6*
7	·064040	·068593	·073178	·077775	·082363	·086920	·091426	·095862	·100207	·104445	*7*
8	·032820	·036011	·039333	·042776	·046329	·049979	·053713	·057517	·061377	·065278	*8*
9	·014951	·016805	·018793	·020913	·023165	·025545	·028050	·030676	·033416	·036266	*9*
10	·006130	·007058	·008081	·009202	·010424	·011751	·013184	·014724	·016374	·018133	*10*
11	·002285	·002695	·003159	·003681	·004264	·004914	·005633	·006425	·007294	·008242	*11*
12	·000781	·000943	·001132	·001350	·001599	·001884	·002206	·002570	·002978	·003434	*12*
13	·000246	·000305	·000374	·000457	·000554	·000667	·000798	·000949	·001123	·001321	*13*
14	·000072	·000091	·000115	·000144	·000178	·000219	·000268	·000325	·000393	·000472	*14*
15	·000020	·000026	·000033	·000042	·000053	·000067	·000084	·000104	·000128	·000157	*15*
16	·000005	·000007	·000009	·000012	·000015	·000019	·000025	·000031	·000039	·000049	*16*
17	·000001	·000902	·000002	·000003	·000004	·000005	·000007	·000009	·000011	·000014	*17*
18	—	—	·000001	·000001	·000001	·000001	·000002	·000002	·000003	·000004	*18*
19	—	—	—	—	—	—	—	·000001	·000001	·000001	*19*
x	*5·1*	*5·2*	*5·3*	*5·4*	*5·5*	*5·6*	*5·7*	*5·8*	*5·9*	*6·0*	*x*
0	·006097	·005517	·004992	·004517	·004087	·003698	·003346	·003028	·002739	·002479	*0*
1	·031093	·028686	·026455	·024390	·022477	·020708	·019072	·017560	·016163	·014873	*1*
2	·079288	·074584	·070107	·065852	·061812	·057982	·054355	·050923	·047680	·044618	*2*
3	·134790	·129279	·123856	·118533	·113323	·108234	·103275	·098452	·093771	·089235	*3*

TABLE LI—(*continued*).

x	*m*										*x*
	5·1	*5·2*	*5·3*	*5·4*	*5·5*	*5·6*	*5·7*	*5·8*	*5·9*	*6·0*	
4	·171857	·168063	·164109	·160020	·155819	·151528	·147167	·142755	·138312	·133853	*4*
5	·175294	·174785	·173955	·172821	·171401	·169711	·167770	·165596	·163208	·160623	*5*
6	·149000	·151480	·153660	·155539	·157117	·158397	·159382	·160076	·160488	·160623	*6*
7	·108557	·112528	·116343	·119987	·123449	·126717	·129782	·132635	·135268	·137677	*7*
8	·069205	·073143	·077077	·080991	·084871	·088702	·092470	·096160	·099760	·103258	*8*
9	·039216	·042261	·045390	·048595	·051866	·055192	·058564	·061970	·065398	·068838	*9*
10	·020000	·021976	·024057	·026241	·028526	·030908	·033382	·035943	·038585	·041303	*10*
11	·009273	·010388	·011591	·012882	·014263	·015735	·017298	·018952	·020696	·022529	*11*
12	·003941	·004502	·005119	·005797	·006537	·007343	·008216	·009160	·010175	·011264	*12*
13	·001546	·001801	·002087	·002408	·002766	·003163	·003603	·004087	·004618	·005199	*13*
14	·000563	·000669	·000790	·000929	·001087	·001265	·001467	·001693	·001946	·002228	*14*
15	·000191	·000232	·000279	·000334	·000398	·000472	·000557	·000655	·000766	·000891	*15*
16	·000061	·000075	·000092	·000113	·000137	·000165	·000199	·000237	·000282	·000334	*16*
17	·000018	·000023	·000029	·000036	·000044	·000054	·000067	·000081	·000098	·000118	*17*
18	·000005	·000007	·000008	·000011	·000014	·000017	·000021	·000026	·000032	·000039	*18*
19	·000001	·000002	·000002	·000003	·000004	·000005	·000006	·000008	·000010	·000012	*19*
20	—	—	·000001	·000001	·000001	·000001	·000002	·000002	·000003	·000004	*20*
21	—	—	—	—	—	—	—	·000001	·000001	·000001	*21*

x	*6·1*	*6·2*	*6·3*	*6·4*	*6·5*	*6·6*	*6·7*	*6·8*	*6·9*	*7·0*	*x*
0	·002243	·002029	·001836	·001662	·001503	·001360	·001231	·001114	·001008	·000912	*0*
1	·013682	·012582	·011569	·010634	·009772	·008978	·008247	·007574	·006954	·006383	*1*
2	·041729	·039006	·036441	·034029	·031760	·029629	·027628	·025751	·023990	·022341	*2*
3	·084848	·080612	·076527	·072595	·068814	·065183	·061702	·058368	·055178	·052129	*3*
4	·129393	·124948	·120530	·116151	·111822	·107553	·103351	·099225	·095182	·091226	*4*
5	·157860	·154936	·151868	·148674	·145369	·141969	·138490	·134946	·131351	·127717	*5*
6	·160491	·160100	·159461	·158585	·157483	·156166	·154648	·152939	·151053	·149003	*6*
7	·139856	·141803	·143515	·144992	·146234	·147243	·148020	·148569	·148895	·149003	*7*
8	·106640	·109897	·113018	·115994	·118815	·121475	·123967	·126284	·128422	·130377	*8*
9	·072278	·075707	·079113	·082484	·085811	·089082	·092286	·095415	·098457	·101405	*9*
10	·044090	·046938	·049841	·052790	·055777	·058794	·061832	·064882	·067935	·070983	*10*
11	·024450	·026456	·028545	·030714	·032959	·035276	·037661	·040109	·042614	·045171	*11*
12	·012429	·013669	·014986	·016381	·017853	·019402	·021028	·022728	·024503	·026350	*12*
13	·005832	·006519	·007263	·008064	·008926	·009850	·010837	·011889	·013005	·014188	*13*
14	·002541	·002887	·003268	·003687	·004144	·004644	·005186	·005774	·006410	·007094	*14*
15	·001033	·001193	·001373	·001573	·001796	·002043	·002317	·002618	·002949	·003311	*15*
16	·000394	·000462	·000540	·000629	·000730	·000843	·000970	·001113	·001272	·001448	*16*
17	·000141	·000169	·000200	·000237	·000279	·000327	·000382	·000445	·000516	·000596	*17*
18	·000048	·000058	·000070	·000084	·000101	·000120	·000142	·000168	·000198	·000232	*18*
19	·000015	·000019	·000023	·000028	·000034	·000042	·000050	·000060	·000072	·000085	*19*
20	·000005	·000006	·000007	·000009	·000011	·000014	·000017	·000020	·000025	·000030	*20*
21	·000001	·000002	·000002	·000003	·000003	·000004	·000005	·000007	·000008	·000010	*21*
22	—	—	·000001	·000001	·000001	·000001	·000002	·000002	·000003	·000003	*22*
23	—	—	—	—	—	—	—	·000001	·000001	·000001	*23*

TABLE LI—(*continued*).

x	*m*										*x*
	7·1	*7·2*	*7·3*	*7·4*	*7·5*	*7·6*	*7·7*	*7·8*	*7·9*	*8·0*	
0	·000825	·000747	·000676	·000611	·000553	·000500	·000453	·000410	·000371	·000335	*0*
1	·005858	·005375	·004931	·004523	·004148	·003803	·003487	·003196	·002929	·002684	*1*
2	·020797	·019352	·018000	·016736	·015555	·014453	·013424	·012464	·011569	·010735	*2*
3	·049219	·046444	·043799	·041282	·038889	·036614	·034455	·032407	·030465	·028626	*3*
4	·087364	·083598	·079934	·076372	·072916	·069567	·066326	·063193	·060169	·057252	*4*
5	·124057	·120382	·116703	·113031	·109375	·105742	·102142	·098581	·095067	·091604	*5*
6	·146800	·144458	·141989	·139405	·136718	·133940	·131082	·128156	·125171	·122138	*6*
7	·148897	·148586	·148074	·147371	·146484	·145421	·144191	·142802	·141264	·139587	*7*
8	·132146	·133727	·135118	·136318	·137329	·138150	·138783	·139232	·139499	·139587	*8*
9	·104249	·106982	·109596	·112084	·114440	·116660	·118737	·120668	·122449	·124077	*9*
10	·074017	·077027	·080005	·082942	·085830	·088661	·091427	·094121	·096735	·099262	*10*
11	·047774	·050418	·053094	·055797	·058521	·061257	·063999	·066740	·069473	·072190	*11*
12	·028267	·030251	·032299	·034408	·036575	·038796	·041066	·043381	·045736	·048127	*12*
13	·015438	·016754	·018137	·019586	·021101	·022681	·024324	·026029	·027794	·029616	*13*
14	·007829	·008616	·009457	·010353	·011304	·012312	·013378	·014502	·015684	·016924	*14*
15	·003706	·004136	·004603	·005107	·005652	·006238	·006867	·007541	·008260	·009026	*15*
16	·001644	·001861	·002100	·002362	·002649	·002963	·003305	·003676	·004078	·004513	*16*
17	·000687	·000788	·000902	·001028	·001169	·001325	·001497	·001687	·001895	·002124	*17*
18	·000271	·000315	·000366	·000423	·000487	·000559	·000640	·000731	·000832	·000944	*18*
19	·000101	·000119	·000141	·000165	·000192	·000224	·000259	·000300	·000346	·000397	*19*
20	·000036	·000043	·000051	·000061	·000072	·000085	·000100	·000117	·000137	·000159	*20*
21	·000012	·000015	·000018	·000021	·000026	·000031	·000037	·000043	·000051	·000061	*21*
22	·000004	·000005	·000006	·000007	·000009	·000011	·000013	·000015	·000018	·000022	*22*
23	·000001	·000002	·000002	·000002	·000003	·000004	·000004	·000005	·000006	·000008	*23*
24	—	—	·000001	·000001	·000001	·000001	·000001	·000002	·000002	·000003	*24*
25	—	—	—	—	—	—	—	·000001	·000001	·000001	*25*

x	*8·1*	*8·2*	*8·3*	*8·4*	*8·5*	*8·6*	*8·7*	*8·8*	*8·9*	*9·0*	*x*
0	·000304	·000275	·000249	·000225	·000203	·000184	·000167	·000151	·000136	·000123	*0*
1	·002459	·002252	·002063	·001889	·001729	·001583	·001449	·001326	·001214	·001111	*1*
2	·009958	·009234	·008560	·007933	·007350	·006808	·006304	·005836	·005402	·004998	*2*
3	·026885	·025239	·023683	·022213	·020826	·019517	·018283	·017120	·016025	·014994	*3*
4	·054443	·051740	·049142	·046648	·044255	·041961	·039765	·037664	·035656	·033737	*4*
5	·088198	·084854	·081576	·078368	·075233	·072174	·069192	·066289	·063467	·060727	*5*
6	·119067	·115967	·112847	·109716	·106581	·103449	·100328	·097224	·094143	·091090	*6*
7	·137778	·135848	·133805	·131659	·129419	·127094	·124693	·122224	·119696	·117116	*7*
8	·139500	·139244	·138823	·138242	·137508	·136626	·135604	·134446	·133161	·131756	*8*
9	·125550	·126866	·128025	·129026	·129869	·130554	·131084	·131459	·131682	·131756	*9*
10	·101696	·104031	·106261	·108382	·110388	·112277	·114043	·115684	·117197	·118580	*10*
11	·074885	·077550	·080179	·082764	·085300	·087780	·090197	·092547	·094823	·097020	*11*
12	·050547	·052993	·055457	·057935	·060421	·062909	·065393	·067868	·070327	·072765	*12*
13	·031495	·033426	·035407	·037435	·039506	·041617	·043763	·045941	·048147	·050376	*13*
14	·018222	·019578	·020991	·022461	·023986	·025565	·027196	·028877	·030608	·032384	*14*
15	·009840	·010703	·011615	·012578	·013592	·014657	·015773	·016941	·018161	·019431	*15*
16	·004981	·005485	·006025	·006604	·007221	·007878	·008577	·009318	·010102	·010930	*16*
17	·002373	·002646	·002942	·003263	·003610	·003985	·004389	·004823	·005289	·005786	*17*
18	·001068	·001205	·001356	·001523	·001705	·001904	·002121	·002358	·002615	·002893	*18*
19	·000455	·000520	·000593	·000673	·000763	·000862	·000971	·001092	·001225	·001370	*19*
20	·000184	·000213	·000246	·000283	·000324	·000371	·000423	·000481	·000545	·000617	*20*

TABLE LI—(*continued*).

x	*m*										*x*
	8·1	*8·2*	*8·3*	*8·4*	*8·5*	*8·6*	*8·7*	*8·8*	*8·9*	*9·0*	
21	·000071	·000083	·000097	·000113	·000131	·000152	·000175	·000201	·000231	·000264	*21*
22	·000026	·000031	·000037	·000043	·000051	·000059	·000069	·000081	·000093	·000108	*22*
23	·000009	·000011	·000013	·000016	·000019	·000022	·000026	·000031	·000036	·000042	*23*
24	·000003	·000004	·000005	·000006	·000007	·000008	·000009	·000011	·000013	·000016	*24*
25	·000001	·000001	·000002	·000002	·000002	·000003	·000003	·000004	·000005	·000006	*25*
26	—	—	—	·000001	·000001	·000001	·000001	·000001	·000002	·000002	*26*
27	—	—	—	—	—	—	—	—	·000001	·000001	*27*
x	*9·1*	*9·2*	*9·3*	*9·4*	*9·5*	*9·6*	*9·7*	*9·8*	*9·9*	*10·0*	*x*
0	·000112	·000101	·000091	·000083	·000075	·000068	·000061	·000055	·000050	·000045	*0*
1	·001016	·000930	·000850	·000778	·000711	·000650	·000594	·000543	·000497	·000454	*1*
2	·004624	·004276	·003954	·003655	·003378	·003121	·002883	·002663	·002459	·002270	*2*
3	·014025	·013113	·012256	·011452	·010696	·009987	·009322	·008698	·008114	·007567	*3*
4	·031906	·030160	·028496	·026911	·025403	·023969	·022606	·021311	·020082	·018917	*4*
5	·058069	·055494	·053002	·050593	·048266	·046020	·043855	·041770	·039763	·037833	*5*
6	·088072	·085091	·082154	·079262	·076421	·073632	·070899	·068224	·065609	·063055	*6*
7	·114493	·111834	·109147	·106438	·103714	·100981	·098246	·095514	·092790	·090079	*7*
8	·130236	·128609	·126883	·125065	·123160	·121178	·119123	·117004	·114827	·112599	*8*
9	·131683	·131467	·131113	·130623	·130003	·129256	·128388	·127405	·126310	·125110	*9*
10	·119832	·120950	·121935	·122786	·123502	·124086	·124537	·124857	·125047	·125110	*10*
11	·099133	·101158	·103090	·104926	·106661	·108293	·109819	·111236	·112542	·113736	*11*
12	·075176	·077555	·079895	·082192	·084440	·086634	·088770	·090843	·092847	·094780	*12*
13	·052623	·054885	·057156	·059431	·061706	·063976	·066236	·068481	·070707	·072908	*13*
14	·034205	·036067	·037968	·039904	·041872	·043869	·045892	·047937	·050000	·052077	*14*
15	·020751	·022121	·023540	·025006	·026519	·028076	·029677	·031319	·033000	·034718	*15*
16	·011802	·012720	·013683	·014691	·015746	·016846	·017992	·019183	·020419	·021699	*16*
17	·006318	·006884	·007485	·008123	·008799	·009513	·010266	·011058	·011891	·012764	*17*
18	·003194	·003518	·003867	·004242	·004644	·005074	·005532	·006021	·006540	·007091	*18*
19	·001530	·001704	·001893	·002099	·002322	·002563	·002824	·003105	·003408	·003732	*19*
20	·000696	·000784	·000880	·000986	·001103	·001230	·001370	·001522	·001687	·001866	*20*
21	·000302	·000343	·000390	·000442	·000499	·000563	·000633	·000710	·000795	·000889	*21*
22	·000125	·000144	·000165	·000189	·000215	·000245	·000279	·000316	·000358	·000404	*22*
23	·000049	·000057	·000067	·000077	·000089	·000102	·000118	·000135	·000154	·000176	*23*
24	·000019	·000022	·000026	·000030	·000035	·000041	·000048	·000055	·000064	·000073	*24*
25	·000007	·000008	·000010	·000011	·000013	·000016	·000018	·000022	·000025	·000029	*25*
26	·000002	·000003	·000003	·000004	·000005	·000006	·000007	·000008	·000010	·000011	*26*
27	·000001	·000001	·000001	·000001	·000002	·000002	·000002	·000003	·000004	·000004	*27*
28	—	—	—	—	·000001	·000001	·000001	·000001	·000001	·000001	*28*
29	—	—	—	—	—	—	—	—	—	·000001	*29*
x	*10·1*	*10·2*	*10·3*	*10·4*	*10·5*	*10·6*	*10·7*	*10·8*	*10·9*	*11·0*	*x*
0	·000041	·000037	·000034	·000030	·000028	·000025	·000023	·000020	·000018	·000017	*0*
1	·000415	·000379	·000346	·000317	·000289	·000264	·000241	·000220	·000201	·000184	*1*
2	·002095	·001934	·001784	·001646	·001518	·001400	·001291	·001190	·001097	·001010	*2*
3	·007054	·006574	·006125	·005705	·005313	·004946	·004603	·004283	·003984	·003705	*3*

TABLE LI—(*continued*).

x	*m*										*x*
	10·1	*10·2*	*10·3*	*10·4*	*10·5*	*10·6*	*10·7*	*10·8*	*10·9*	*11·0*	
4	·017811	·016764	·015773	·014834	·013946	·013107	·012313	·011564	·010856	·010189	*4*
5	·035979	·034199	·032492	·030855	·029287	·027786	·026350	·024978	·023667	·022415	*5*
6	·060565	·058139	·055777	·053482	·051252	·049089	·046991	·044960	·042995	·041095	*6*
7	·087387	·084716	·082072	·079458	·076878	·074334	·071830	·069367	·066949	·064577	*7*
8	·110326	·108013	·105668	·103296	·100902	·098493	·096072	·093646	·091218	·088794	*8*
9	·123810	·122415	·120931	·119364	·117720	·116003	·114219	·112375	·110475	·108526	*9*
10	·125048	·124863	·124559	·124139	·123606	·122963	·122215	·121365	·120418	·119378	*10*
11	·114817	·115782	·116633	·117368	·117987	·118492	·118882	·119159	·119323	·119378	*11*
12	·096637	·098415	·100110	·101719	·103239	·104667	·106003	·107243	·108386	·109430	*12*
13	·075080	·077218	·079318	·081375	·083385	·085344	·087248	·089094	·090877	·092595	*13*
14	·054165	·056259	·058355	·060450	·062539	·064618	·066683	·068730	·070754	·072753	*14*
15	·036471	·038256	·040071	·041912	·043777	·045663	·047567	·049485	·051415	·053352	*15*
16	·023022	·024388	·025795	·027243	·028729	·030252	·031810	·033403	·035026	·036680	*16*
17	·013678	·014633	·015629	·016666	·017744	·018863	·020022	·021220	·022458	·023734	*17*
18	·007675	·008292	·008943	·009629	·010351	·011108	·011902	·012732	·013600	·014504	*18*
19	·004080	·004451	·004848	·005271	·005720	·006197	·006703	·007237	·007802	·008397	*19*
20	·002060	·002270	·002497	·002741	·003003	·003285	·003586	·003908	·004252	·004618	*20*
21	·000991	·001103	·001225	·001357	·001502	·001658	·001827	·002010	·002207	·002419	*21*
22	·000455	·000511	·000573	·000642	·000717	·000799	·000889	·000987	·001093	·001210	*22*
23	·000200	·000227	·000257	·000290	·000327	·000368	·000413	·000463	·000518	·000578	*23*
24	·000084	·000096	·000110	·000126	·000143	·000163	·000184	·000208	·000235	·000265	*24*
25	·000034	·000039	·000045	·000052	·000060	·000069	·000079	·000090	·000103	·000117	*25*
26	·000013	·000015	·000018	·000021	·000024	·000028	·000032	·000037	·000043	·000049	*26*
27	·000005	·000006	·000007	·000008	·000009	·000011	·000013	·000015	·000017	·000020	*27*
28	·000002	·000002	·000003	·000003	·000004	·000004	·000005	·000006	·000007	·000008	*28*
29	·000001	·000001	·000001	·000001	·000001	·000002	·000002	·000002	·000003	·000003	*29*
30	—	—	—	—	—	·000001	·000001	·000001	·000001	·000001	*30*

x	*11·1*	*11·2*	*11·3*	*11·4*	*11·5*	*11·6*	*11·7*	*11·8*	*11·9*	*12·0*	*x*
0	·000015	·000014	·000012	·000011	·000010	·000009	·000008	·000008	·000007	·000006	*0*
1	·000168	·000153	·000140	·000128	·000116	·000106	·000097	·000089	·000081	·000074	*1*
2	·000931	·000858	·000790	·000727	·000670	·000617	·000568	·000522	·000481	·000442	*2*
3	·003445	·003202	·002976	·002764	·002568	·002385	·002214	·002055	·001907	·001770	*3*
4	·009559	·008965	·008406	·007879	·007382	·006915	·006476	·006062	·005674	·005309	*4*
5	·021221	·020082	·018997	·017963	·016979	·016043	·015153	·014307	·013504	·012741	*5*
6	·039259	·037487	·035778	·034130	·032544	·031017	·029549	·028137	·026782	·025481	*6*
7	·062253	·059979	·057755	·055584	·053465	·051400	·049388	·047432	·045530	·043682	*7*
8	·086376	·083970	·081579	·079206	·076856	·074529	·072231	·069962	·067725	·065523	*8*
9	·106531	·104496	·102427	·100328	·098204	·096060	·093900	·091728	·089548	·087364	*9*
10	·118249	·117036	·115743	·114374	·112935	·111430	·109863	·108239	·106562	·104837	*10*
11	·119324	·119164	·118899	·118533	·118068	·117508	·116854	·116110	·115281	·114368	*11*
12	·110375	·111220	·111964	·112607	·113149	·113591	·113933	·114175	·114320	·114363	*12*
13	·094243	·095820	·097322	·098747	·100093	·101358	·102539	·103636	·104647	·105570	*13*
14	·074721	·076656	·078553	·080409	·082219	·083982	·085694	·087350	·088950	·090489	*14*
15	·055294	·057236	·059177	·061110	·063035	·064946	·066841	·068716	·070567	·072391	*15*
16	·038360	·040065	·041793	·043541	·045306	·047086	·048877	·050678	·052484	·054293	*16*
17	·025047	·026396	·027780	·029198	·030648	·032129	·033639	·035176	·036739	·038325	*17*
18	·015446	·016424	·017440	·018492	·019581	·020706	·021865	·023060	·024288	·025550	*18*
19	·009023	·009682	·010372	·011095	·011852	·012641	·013465	·014322	·015212	·016137	*19*

TABLE LI—(*continued*).

x	m										x
	11·1	11·2	11·3	11·4	11·5	11·6	11·7	11·8	11·9	12·0	
20	·005008	·005422	·005860	·006324	·006815	·007332	·007877	·008450	·009051	·009682	20
21	·002647	·002892	·003153	·003433	·003732	·004050	·004388	·004748	·005129	·005533	21
22	·001336	·001472	·001620	·001779	·001951	·002136	·002334	·002547	·002774	·003018	22
23	·000645	·000717	·000796	·000882	·000975	·001077	·001187	·001307	·001435	·001575	23
24	·000298	·000335	·000375	·000419	·000467	·000521	·000579	·000642	·000712	·000787	24
25	·000132	·000150	·000169	·000191	·000215	·000242	·000271	·000303	·000339	·000378	25
26	·000057	·000065	·000074	·000084	·000095	·000108	·000122	·000138	·000155	·000174	26
27	·000023	·000027	·000031	·000035	·000041	·000046	·000053	·000060	·000068	·000078	27
28	·000009	·000011	·000012	·000014	·000017	·000019	·000022	·000025	·000029	·000033	28
29	·000004	·000004	·000005	·000006	·000007	·000008	·000009	·000010	·000012	·000014	29
30	·000001	·000002	·000002	·000002	·000003	·000003	·000003	·000004	·000005	·000005	30
31	—	·000001	·000001	·000001	·000001	·000001	·000001	·000002	·000002	·000002	31
32	—	—	—	—	—	—	—	·000001	·000001	·000001	32

x	12·1	12·2	12·3	12·4	12·5	12·6	12·7	12·8	12·9	13·0	x
0	·000006	·000005	·000005	·000004	·000004	·000003	·000003	·000003	·000002	·000002	0
1	·000067	·000061	·000056	·000051	·000047	·000042	·000039	·000035	·000032	·000029	1
2	·000407	·000374	·000344	·000317	·000291	·000268	·000246	·000226	·000208	·000191	2
3	·001641	·001522	·001412	·001309	·001213	·001124	·001042	·000965	·000894	·000828	3
4	·004966	·004643	·004341	·004057	·003791	·003541	·003307	·003088	·002882	·002690	4
5	·012017	·011330	·010679	·010062	·009477	·008924	·008400	·007905	·007436	·006994	5
6	·024233	·023037	·021892	·020794	·019744	·018740	·017781	·016864	·015988	·015153	6
7	·041889	·040151	·038467	·036836	·035258	·033733	·032259	·030837	·029464	·028141	7
8	·063358	·061230	·059142	·057095	·055091	·053129	·051212	·049339	·047511	·045730	8
9	·085181	·083000	·080828	·078665	·076515	·074381	·072266	·070171	·068100	·066054	9
10	103069	·101261	·099418	·097544	·095644	·093720	·091777	·089819	·087849	·085870	10
11	·113376	·112308	·111168	·109959	·108686	·107352	·105961	·104516	·103023	·101483	11
12	·114321	·114180	·113947	·113624	·113215	·112720	·112142	·111484	·110749	·109940	12
13	·106406	·107153	·107811	·108380	·108860	·109251	·109554	·109769	·109897	·109940	13
14	·091965	·093376	·094720	·095994	·097197	·098326	·099381	·100360	·101263	·102087	14
15	·074185	·075946	·077670	·079355	·080997	·082594	·084143	·085641	·087086	·088475	15
16	·056103	·057909	·059709	·061500	·063279	·065043	·066788	·068513	·070213	·071886	16
17	·039932	·041558	·043201	·044859	·046529	·048208	·049895	·051586	·053279	·054972	17
18	·026843	·028167	029521	·030903	·032312	·033746	·035204	·036683	·038183	·039702	18
19	·017095	·018086	·019111	·020168	·021258	·022379	·023531	·024713	·025925	·027164	19
20	·010342	·011033	·011753	·012504	·013286	·014099	·014942	·015816	·016721	·017657	20
21	·005959	·006409	·006884	·007383	·007908	·008459	·009036	·009640	·010272	·010930	21
22	·003278	·003554	·003849	·004162	·004493	·004845	·005216	·005609	·006023	·006459	22
23	·001724	·001885	·002058	·002244	·002442	·002654	·002880	·003122	·003378	·003651	23
24	·000869	·000958	·001055	·001159	·001272	·001393	·001524	·001665	·001816	·001977	24
25	·000421	·000468	·000519	·000575	·000636	·000702	·000774	·000852	·000937	·001028	25
26	·000196	·000219	·000246	·000274	·000306	·000340	·000378	·000420	·000465	·000514	26
27	·000088	·000099	·000112	·000126	·000142	·000159	·000178	·000199	·000222	·000248	27
28	·000038	·000043	·000049	·000056	·000063	·000071	·000081	·000091	·000102	·000115	28
29	·000016	·000018	·000021	·000024	·000027	·000031	·000035	·000040	·000046	·000052	29
30	·000006	·000007	·000009	·000010	·000011	·000013	·000015	·000017	·000020	·000022	30
31	·000002	·000003	·000003	·000004	·000005	·000005	·000006	·000007	·000008	·000009	31
32	·000001	·000001	·000001	·000002	·000002	·000002	·000002	·000003	·000003	·000004	32
33	—	—	—	·000001	·000001	·000001	·000001	·000001	·000001	·000002	33
34	—	—	—	—	—	—	—	—	—	·000001	34

TABLE LI—(*continued*).

x	*m*										*x*
	13·1	*13·2*	*13·3*	*13·4*	*13·5*	*13·6*	*13·7*	*13·8*	*13·9*	*14·0*	
0	·000002	·000002	·000002	·000002	·000001	·000001	·000001	·000001	·000001	·000001	*0*
1	·000027	·000024	·000022	·000020	·000019	·000017	·000015	·000014	·000013	·000012	*1*
2	·000175	·000161	·000148	·000136	·000125	·000115	·000105	·000097	·000089	·000081	*2*
3	·000766	·000709	·000657	·000608	·000562	·000520	·000481	·000445	·000411	·000380	*3*
4	·002510	·002341	·002183	·002035	·001897	·001768	·001648	·001535	·001429	·001331	*4*
5	·006575	·006180	·005807	·005455	·005123	·004810	·004514	·004236	·003974	·003727	*5*
6	·014356	·013596	·012872	·012183	·011526	·010902	·010308	·009743	·009206	·008696	*6*
7	·026867	·025639	·024458	·023322	·022230	·021181	·020173	·019207	·018280	·017392	*7*
8	·043994	·042304	·040661	·039064	·037512	·036007	·034547	·033132	·031762	·030435	*8*
9	·064036	·062046	·060088	·058161	·056269	·054410	·052588	·050802	·049054	·047344	*9*
10	·083887	·081901	·079916	·077936	·075963	·073998	·072046	·070107	·068185	·066282	*10*
11	·099901	·098281	·096626	·094940	·093227	·091489	·089730	·087953	·086162	·084359	*11*
12	·109059	·108109	·107094	·106017	·104880	·103687	·102441	·101146	·099804	·098418	*12*
13	·109898	·109773	·109566	·109279	·108914	·108473	·107957	·107370	·106713	·105989	*13*
14	·102833	·103500	·104087	·104595	·105024	·105373	·105644	·105836	·105951	·105989	*14*
15	·089807	·091080	·092291	·093439	·094522	·095539	·096488	·097369	·098181	·098923	*15*
16	·073530	·075141	·076717	·078255	·079753	·081208	·082618	·083981	·085295	·086558	*16*
17	·056661	·058345	·060019	·061683	·063333	·064966	·066580	·068173	·069741	·071283	*17*
18	·041237	·042786	·044348	·045920	·047500	·049086	·050675	·052266	·053856	·055442	*18*
19	·028432	·029725	·031043	·032385	·033750	·035135	·036539	·037962	·039400	·040852	*19*
20	·018623	·019619	·020644	·021698	·022781	·023892	·025030	·026193	·027383	·028597	*20*
21	·011617	·012332	·013074	·013846	·014645	·015473	·016329	·017213	·018125	·019064	*21*
22	·006917	·007399	·007904	·008433	·008987	·009565	·010168	·010797	·011452	·012132	*22*
23	·003940	·004246	·004571	·004913	·005275	·005656	·006057	·006478	·006921	·007385	*23*
24	·002151	·002336	·002533	·002743	·002967	·003205	·003457	·003725	·004008	·004308	*24*
25	·001127	·001233	·001348	·001470	·001602	·001744	·001895	·002056	·002229	·002412	*25*
26	·000568	·000626	·000689	·000758	·000832	·000912	·000998	·001091	·001191	·001299	*26*
27	·000275	·000306	·000340	·000376	·000416	·000459	·000507	·000558	·000613	·000674	*27*
28	·000129	·000144	·000161	·000180	·000201	·000223	·000248	·000275	·000305	·000337	*28*
29	·000058	·000066	·000074	·000083	·000093	·000105	·000117	·000131	·000146	·000163	*29*
30	·000025	·000029	·000033	·000037	·000042	·000047	·000053	·000060	·000068	·000076	*30*
31	·000011	·000012	·000014	·000016	·000018	·000021	·000024	·000027	·000030	·000034	*31*
32	·000004	·000005	·000006	·000007	·000008	·000009	·000010	·000012	·000013	·000015	*32*
33	·000002	·000002	·000002	·000003	·000003	·000004	·000004	·000005	·000006	·000006	*33*
34	·000001	·000001	·000001	·000001	·000001	·000001	·000002	·000002	·000002	·000003	*34*
35	—	—	—	—	—	·000001	·000001	·000001	·000001	·000001	*35*
x	*14·1*	*14·2*	*14·3*	*14·4*	*14·5*	*14·6*	*14·7*	*14·8*	*14·9*	*15·0*	*x*
0	·000001	·000001	·000001	·000001	·000001	—	—	—	—	—	*0*
1	·000011	·000010	·000009	·000008	·000007	·000007	·000006	·000006	·000005	·000005	*1*
2	·000075	·000069	·000063	·000058	·000053	·000049	·000045	·000041	·000038	·000034	*2*
3	·000352	·000325	·000300	·000277	·000256	·000237	·000219	·000202	·000186	·000172	*3*
4	·001239	·001153	·001073	·000999	·000929	·000864	·000803	·000747	·000694	·000645	*4*
5	·003494	·003275	·003070	·002876	·002694	·002523	·002362	·002211	·002069	·001936	*5*
6	·008212	·007752	·007316	·006902	·006510	·006139	·005787	·005454	·005138	·004839	*6*
7	·016541	·015726	·014946	·014199	·013486	·012804	·012152	·011530	·010937	·010370	*7*
8	·029153	·027913	·026715	·025559	·024443	·023367	·022330	·021331	·020370	·019444	*8*
9	·045673	·044040	·042447	·040894	·039380	·037907	·036472	·035078	·033723	·032407	*9*
10	·064399	·062537	·060700	·058887	·057101	·055343	·053614	·051915	·050247	·048611	*10*
11	·082547	·080730	·078910	·077089	·075270	·073456	·071648	·069850	·068062	·066287	*11*

TABLE LI—(*continued*).

x	*m*										*x*
	14·1	*14·2*	*14·3*	*14·4*	*14·5*	*14·6*	*14·7*	*14·8*	*14·9*	*15·0*	
12	·096993	·095530	·094034	·092507	·090951	·089371	·087769	·086148	·084510	·082859	*12*
13	·105200	·104349	·103437	·102469	·101446	·100371	·099247	·098076	·096862	·095607	*13*
14	·105951	·105839	·105654	·105396	·105069	·104672	·104209	·103681	·103089	·102436	*14*
15	·099594	·100195	·100723	·101181	·101567	·101881	·102125	·102298	·102402	·102436	*15*
16	·087768	·088923	·090021	·091063	·092045	·092967	·093827	·094626	·095361	·096034	*16*
17	·072795	·074277	·075724	·077135	·078509	·079842	·081133	·082380	·083581	·084736	*17*
18	·057023	·058596	·060158	·061708	·063243	·064761	·066259	·067735	·069187	·070613	*18*
19	·042317	·043793	·045277	·046768	·048264	·049763	·051263	·052762	·054257	·055747	*19*
20	·029834	·031093	·032373	·033673	·034992	·036327	·037678	·039044	·040422	·041810	*20*
21	·020031	·021025	·022045	·023090	·024161	·025256	·026375	·027517	·028680	·029865	*21*
22	·012838	·013570	·014329	·015114	·015924	·016761	·017623	·018511	·019424	·020362	*22*
23	·007870	·008378	·008909	·009462	·010039	·010640	·011264	·011911	·012584	·013280	*23*
24	·004624	·004957	·005308	·005677	·006065	·006472	·006899	·007345	·007812	·008300	*24*
25	·002608	·002816	·003036	·003270	·003518	·003780	·004057	·004348	·004656	·004980	*25*
26	·001414	·001538	·001670	·001811	·001962	·002123	·002294	·002475	·002668	·002873	*26*
27	·000739	·000809	·000884	·000966	·001054	·001148	·001249	·001357	·001473	·001596	*27*
28	·000372	·000410	·000452	·000497	·000546	·000598	·000656	·000717	·000784	·000855	*28*
29	·000181	·000201	·000223	·000247	·000273	·000301	·000332	·000366	·000403	·000442	*29*
30	·000085	·000095	·000106	·000118	·000132	·000147	·000163	·000181	·000200	·000221	*30*
31	·000039	·000044	·000049	·000055	·000062	·000069	·000077	·000086	·000096	·000107	*31*
32	·000017	·000019	·000022	·000025	·000028	·000032	·000035	·000040	·000045	·000050	*32*
33	·000007	·000008	·000009	·000011	·000012	·000014	·000016	·000018	·000020	·000023	*33*
34	·000003	·000003	·000004	·000005	·000005	·000006	·000007	·000008	·000009	·000010	*34*
35	·000001	·000001	·000002	·000002	·000002	·000002	·000003	·000003	·000004	·000004	*35*
36	—	·000001	·000001	·000001	·000001	·000001	·000001	·000001	·000002	·000002	*36*
37	—	—	—	—	—	—	—	·000001	·000001	·000001	*37*

TABLE LII.

Table of Poisson-Exponential for Cell Frequencies 1 *to* 30.

Cell Frequencies

	x	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Per cent. occurrence of values differing by *x* or more in defect from Actual.	*22*										
	21										
	20										
	19										
	18										
	17										
	16										
	15										
	14										
	13										
	12										
	11										
	10										·005
	9									·012	·050
	8								·034	·123	·277
	7							·091	·302	·623	1·033
	6						·248	·730	1·375	2·123	2·925
	5					·674	1·735	2·964	4·238	5·496	6·708
	4				1·832	4·043	6·197	8·177	9·963	11·569	13·014
	3			4·979	9·158	12·465	15·120	17·299	19·124	20·678	22·022
	2		13·534	19·915	23·810	26·503	28·506	30·071	31·337	32·390	33·282
	1	36·788	40·601	42·319	43·347	44·049	44·568	44·971	45·296	45·565	45·793
	Actual	36·788	27·067	22·404	19·537	17·547	16·062	14·900	13·959	13·176	12·511
Per cent. occurrence of values differing by *x* or more in excess from Actual.	*1*	26·424	32·332	35·277	37·116	38·404	39·370	40·129	40·745	41·259	41·696
	2	8·030	14·288	18·474	21·487	23·782	25·602	27·091	28·338	29·401	30·323
	3	1·899	5·265	8·392	11·067	13·337	15·276	16·950	18·411	19·699	20·845
	4	·366	1·656	3·351	5·113	6·809	8·392	9·852	11·192	12·422	13·554
	5	·059	·453	1·191	2·136	3·183	4·262	5·335	6·380	7·385	8·346
	6	·008	·110	·380	·813	1·370	2·009	2·700	3·418	4·146	4·875
	7	·001	·024	·110	·284	·545	·883	1·281	1·726	2·203	2·705
	8	·000	·005	·029	·092	·202	·363	·572	·823	1·110	1·428
	9	—	·001	·007	·027	·070	·140	·241	·372	·532	·719
	10	—	·000	·002	·008	·023	·051	·096	·159	·242	·346
	11	—	—	·000	·002	·007	·018	·036	·065	·105	·160
	12	—	—	—	·001	·002	·006	·013	·025	·044	·071
	13	—	—	—	·000	·001	·002	·005	·009	·017	·030
	14	—	—	—	—	·000	·001	·002	·003	·007	·013
	15	—	—	—	—	—	·000	·001	·001	·002	·006
	16	—	—	—	—	—	—	·000	·000	·001	·002
	17	—	—	—	—	—	—	—	—	·000	·001
	18	—	—	—	—	—	—	—	—	—	·001
	19	—	—	—	—	—	—	—	—	—	·000
	20	—	—	—	—	—	—	—	—	—	—
	21	—	—	—	—	—	—	—	—	—	—
	22	—	—	—	—	—	—	—	—	—	—
	23	—	—	—	—	—	—	—	—	—	—
	24	—	—	—	—	—	—	—	—	—	—
	25	—	—	—	—	—	—	—	—	—	—
	26	—	—	—	—	—	—	—	—	—	—
	27	—	—	—	—	—	—	—	—	—	—
	28	—	—	—	—	—	—	—	—	—	—

TABLE LII—(*continued*).

Cell Frequencies

	x	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*
Per cent. occurrence of values differing by *x* or more in defect from Actual.	*22*										
	21										
	20										—
	19									—	—
	18								—	—	—
	17							—	—	·000	·000
	16						—	·000	·000	·001	·002
	15					·000	·000	·001	·002	·004	·007
	14				·000	·001	·002	·004	·008	·015	·026
	13			·000	·001	·004	·009	·018	·032	·052	·078
	12		·001	·003	·009	·021	·040	·067	·104	·151	·209
	11	·002	·008	·022	·047	·086	·138	·206	·289	·387	·500
	10	·020	·052	·105	·181	·279	·401	·543	·706	·886	1·081
	9	·121	·229	·374	·553	·763	1·000	1·260	1·538	1·832	2·139
	8	·492	·760	1·073	1·423	1·800	2·199	2·612	3·037	3·467	3·901
	7	1·510	2·034	2·589	3·162	3·745	4·330	4·912	5·489	6·056	6·613
	6	3·752	4·582	5·403	6·206	6·985	7·740	8·467	9·167	9·840	10·486
	5	7·861	8·950	9·976	10·940	11·846	12·699	13·502	14·260	14·975	15·651
	4	14·319	15·503	16·581	17·568	18·475	19·312	20·087	20·808	21·479	22·107
	3	23·198	24·239	25·168	26·004	26·761	27·451	28·084	28·665	29·203	29·703
	2	34·051	34·723	35·317	35·846	36·322	36·753	37·146	37·505	37·836	38·142
	1	45·989	46·150	46·311	46·445	46·565	46·674	46·774	46·865	46·948	47·026
	Actual	11·938	11·437	10·994	10·599	10·244	9·922	9·629	9·360	9·112	8·884
Per cent. occurrence of values differing by *x* or more in excess from Actual.	*1*	42·073	42·404	42·695	42·956	43·191	43·404	43·597	43·776	43·939	44·091
	2	31·130	31·846	32·486	33·064	33·588	34·066	34·503	34·909	35·283	35·630
	3	21·871	22·798	23·639	24·408	25·114	25·765	26·367	26·928	27·451	27·939
	4	14·596	15·559	16·450	17·280	18·053	18·776	19·451	20·088	20·686	21·251
	5	9·261	10·129	10·953	11·736	12·478	13·184	13·852	14·491	15·099	15·677
	6	5·593	6·297	6·983	7·650	8·297	8·923	9·526	10·111	10·675	11·219
	7	3·219	3·742	4·266	4·791	5·311	5·825	6·329	6·826	7·313	7·789
	8	1·769	2·128	2·501	2·884	3·275	3·669	4·064	4·461	4·856	5·248
	9	·929	1·160	1·407	1·671	1·947	2·232	2·523	2·824	3·127	3·433
	10	·467	·607	·762	·933	1·117	1·312	1·516	1·732	1·954	2·182
	11	·225	·305	·396	·502	·619	·746	·882	1·030	1·185	1·348
	12	·104	·148	·201	·261	·331	·411	·497	·595	·699	·809
	13	·047	·069	·097	·131	·172	·219	·272	·333	·400	·473
	14	·020	·031	·046	·063	·086	·114	·144	·182	·223	·269
	15	·008	·014	·021	·030	·042	·057	·074	·096	·121	·149
	16	·003	·006	·009	·013	·020	·028	·036	·050	·064	·081
	17	·001	·002	·004	·006	·009	·014	·017	·025	·033	·042
	18	·001	·000	·002	·002	·004	·006	·008	·012	·017	·022
	19	·000	·000	·001	·001	·002	·003	·003	·006	·008	·011
	20	—	—	·001	·000	·001	·002	·002	·003	·004	·005
	21	—	—	·000	·000	·000	·001	·001	·002	·002	·003
	22	—	—	—	—	—	·000	·000	·001	·001	·001
	23	—	—	—	—	—	—	—	·000	·000	·001
	24	—	—	—	—	—	—	—	—	—	·000
	25	—	—	—	—	—	—	—	—	—	—
	26	—	—	—	—	—	—	—	—	—	—
	27	—	—	—	—	—	—	—	—	—	—
	28	—	—	—	—	—	—	—	—	—	—

TABLE LII—(*continued*).

Cell Frequencies

	x	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*
Per cent. occurrence of values differing by *x* or more in defect from Actual.	*22*	—	—	—	—	—	—	—	—	—	·000
	21	—	—	—	—	—	—	·000	·000	·000	·001
	20	—	—	—	—	·000	·000	·001	·001	·001	·002
	19	—	—	·000	·000	·001	·001	·002	·003	·004	·006
	18	·000	·000	·001	·001	·002	·004	·006	·009	·012	·017
	17	·001	·002	·003	·005	·008	·011	·016	·023	·031	·041
	16	·003	·006	·010	·015	·022	·031	·043	·056	·073	·092
	15	·012	·020	·030	·043	·059	·078	·102	·129	·160	·195
	14	·039	·058	·081	·109	·142	·180	·224	·273	·328	·387
	13	·111	·150	·198	·252	·314	·384	·460	·543	·632	·727
	12	·277	·355	·443	·540	·647	·762	·884	1·014	1·151	1·293
	11	·625	·763	·912	1·072	1·240	1·417	1·601	1·791	1·987	2·187
	10	1·290	1·512	1·743	1·983	2·229	2·482	2·739	3·000	3·263	3·528
	9	2·455	2·778	3·107	3·440	3·775	4·111	4·446	4·781	5·114	5·444
	8	4·336	4·769	5·200	5·626	6·048	6·463	6·872	7·274	7·669	8·057
	7	7·157	7·689	8·208	8·713	9·204	9·682	10·147	10·599	11·038	11·465
	6	11·107	11·704	12·277	12·827	13·358	13·867	14·357	14·830	15·285	15·724
	5	16·292	16·900	17·477	18·025	18·549	19·048	19·525	19·981	20·417	20·836
	4	22·696	23·250	23·771	24·263	24·730	25·172	25·591	25·990	26·371	26·734
	3	30·168	30·603	31·010	31·391	31·753	32·094	32·416	32·721	33·011	33·287
	2	38·426	38·691	38·938	39·168	39·387	39·593	39·786	39·970	40·143	40·308
	1	47·097	47·164	47·226	47·283	47·340	47·392	47·440	47·486	47·530	47·572
	Actual	8·671	8·473	8·288	8·115	7·952	7·799	7·654	7·517	7·387	7·264
Per cent. occurrence of values differing by *x* or more in excess from Actual.	*1*	44·232	44·363	44·485	44·603	44·708	44·810	44·906	44·997	45·083	45·165
	2	35·955	36·258	36·542	36·812	37·062	37·299	37·525	37·739	37·942	38·135
	3	28·397	28·828	29·235	29·620	29·982	30·326	30·653	30·965	31·262	31·546
	4	21·785	22·290	22·770	23·227	23·660	24·074	24·469	24·847	25·208	25·555
	5	16·230	16·758	17·264	17·748	18·211	18·655	19·083	19·493	19·888	20·269
	6	11·744	12·251	12·740	13·213	13·669	14·110	14·538	14·951	15·351	15·738
	7	8·254	8·709	9·153	9·585	10·007	10·418	10·819	11·210	11·591	11·962
	8	5·637	6·022	6·402	6·777	7·146	7·509	7·866	8·218	8·562	8·901
	9	3·742	4·052	4·362	4·670	4·978	5·284	5·588	5·890	6·188	6·484
	10	2·415	2·654	2·895	3·138	3·385	3·632	3·880	4·129	4·377	4·625
	11	1·517	1·692	1·873	2·057	2·246	2·438	2·633	2·831	3·030	3·230
	12	·927	1·051	1·181	1·315	1·456	1·599	1·747	1·899	2·053	2·210
	13	·552	·637	·727	·821	·921	1·025	1·134	1·247	1·362	1·481
	14	·320	·376	·437	·500	·570	·643	·720	·801	·885	·973
	15	·181	·217	·256	·298	·345	·394	·448	·504	·564	·626
	16	·100	·122	·147	·173	·204	·237	·272	·311	·352	·395
	17	·054	·067	·082	·098	·118	·139	·162	·188	·215	·245
	18	·028	·036	·045	·055	·067	·080	·095	·111	·129	·149
	19	·015	·019	·024	·030	·037	·045	·054	·065	·076	·089
	20	·007	·010	·013	·016	·020	·025	·031	·037	·044	·052
	21	·004	·005	·007	·009	·011	·014	·017	·021	·025	·030
	22	·002	·002	·004	·005	·006	·007	·009	·011	·014	·017
	23	·001	·001	·002	·003	·003	·004	·005	·006	·007	·010
	24	·001	·001	·001	·002	·002	·002	·003	·003	·004	·006
	25	·000	·000	·001	·001	·001	·001	·001	·002	·002	·003
	26	—	—	·000	·000	·000	·001	·001	·001	·001	·002
	27	—	—	—	—	—	·000	·000	·000	·000	·001
	28	—	—	—	—	—	—	—	—	—	·000

LENGTHS OF CIRCULAR ARCS

Deg.	Arc	Deg.	Arc	Deg.	Arc	′	Deg.	Arc	″	Deg.	Arc
1	·017 4533	61	1·064 6508	121	2·111 8484	1	·01667	·000 2909	1	·00028	·000 0048
2	·034 9066	62	1·082 1041	122	2·129 3017	2	·03333	·000 5818	2	·00056	·000 0097
3	·052 3599	63	1·099 5574	123	2·146 7550	3	·05000	·000 8727	3	·00083	·000 0145
4	·069 8132	64	1·117 0107	124	2·164 2083	4	·06667	·001 1636	4	·00111	·000 0194
5	·087 2665	65	1·134 4640	125	2·181 6616	5	·08333	·001 4544	5	·00139	·000 0242
6	·104 7198	66	1·151 9173	126	2·199 1149	6	·10000	·001 7453	6	·00167	·000 0291
7	·122 1730	67	1·169 3706	127	2·216 5682	7	·11667	·002 0362	7	·00194	·000 0339
8	·139 6263	68	1·186 8239	128	2·234 0214	8	·13333	·002 3271	8	·00222	·000 0388
9	·157 0796	69	1·204 2772	129	2·251 4747	9	·15000	·002 6180	9	·00250	·000 0436
10	·174 5329	70	1·221 7305	130	2·268 9280	10	·16667	·002 9089	10	·00278	·000 0485
11	·191 9862	71	1·239 1838	131	2·286 3813	11	·18333	·003 1998	11	·00306	·000 0533
12	·209 4395	72	1·256 6371	132	2·303 8346	12	·20000	·003 4907	12	·00333	·000 0582
13	·226 8928	73	1·274 0904	133	2·321 2879	13	·21667	·003 7815	13	·00361	·000 0630
14	·244 3461	74	1·291 5436	134	2·338 7412	14	·23333	·004 0724	14	·00389	·000 0679
15	·261 7994	75	1·308 9969	135	2·356 1945	15	·25000	·004 3633	15	·00417	·000 0727
16	·279 2527	76	1·326 4502	136	2·373 6478	16	·26667	·004 6542	16	·00444	·000 0776
17	·296 7060	77	1·343 9035	137	2·391 1011	17	·28333	·004 9451	17	·00472	·000 0824
18	·314 1593	78	1·361 3568	138	2·408 5544	18	·30000	·005 2360	18	·00500	·000 0873
19	·331 6126	79	1·378 8101	139	2·426 0077	19	·31667	·005 5269	19	·00528	·000 0921
20	·349 0659	80	1·396 2634	140	2·443 4610	20	·33333	·005 8178	20	·00556	·000 0970
21	·366 5191	81	1·413 7167	141	2·460 9142	21	·35000	·006 1087	21	·00583	·000 1018
22	·383 9724	82	1·431 1700	142	2·478 3675	22	·36667	·006 3995	22	·00611	·000 1067
23	·401 4257	83	1·448 6233	143	2·495 8208	23	·38333	·006 6904	23	·00639	·000 1115
24	·418 8790	84	1·466 0766	144	2·513 2741	24	·40000	·006 9813	24	·00667	·000 1164
25	·436 3323	85	1·483 5299	145	2·530 7274	25	·41667	·007 2722	25	·00694	·000 1212
26	·453 7856	86	1·500 9832	146	2·548 1807	26	·43333	·007 5631	26	·00722	·000 1261
27	·471 2389	87	1·518 4364	147	2·565 6340	27	·45000	·007 8540	27	·00750	·000 1309
28	·488 6922	88	1·535 8897	148	2·583 0873	28	·46667	·008 1449	28	·00778	·000 1357
29	·506 1455	89	1·553 3430	149	2·600 5406	29	·48333	·008 4358	29	·00806	·000 1406
30	·523 5988	90	1·570 7963	150	2·617 9939	30	·50000	·008 7266	30	·00833	·000 1454
31	·541 0521	91	1·588 2496	151	2·635 4472	31	·51667	·009 0175	31	·00861	·000 1503
32	·558 5054	92	1·605 7029	152	2·652 9005	32	·53333	·009 3084	32	·00889	·000 1551
33	·575 9587	93	1·623 1562	153	2·670 3538	33	·55000	·009 5993	33	·00917	·000 1600
34	·593 4119	94	1·640 6095	154	2·687 8070	34	·56667	·009 8902	34	·00944	·000 1648
35	·610 8652	95	1·658 0628	155	2·705 2603	35	·58333	·010 1811	35	·00972	·000 1697
36	·628 3185	96	1·675 5161	156	2·722 7136	36	·60000	·010 4720	36	·01000	·000 1745
37	·645 7718	97	1·692 9694	157	2·740 1669	37	·61667	·010 7629	37	·01028	·000 1794
38	·663 2251	98	1·710 4227	158	2·757 6202	38	·63333	·011 0538	38	·01056	·000 1842
39	·680 6784	99	1·727 8760	159	2·775 0735	39	·65000	·011 3446	39	·01083	·000 1891
40	·698 1317	100	1·745 3293	160	2·792 5268	40	·66667	·011 6355	40	·01111	·000 1939
41	·715 5850	101	1·762 7825	161	2·809 9801	41	·68333	·011 9264	41	·01139	·000 1988
42	·733 0383	102	1·780 2358	162	2·827 4334	42	·70000	·012 2173	42	·01167	·000 2036
43	·750 4916	103	1·797 6891	163	2·844 8867	43	·71667	·012 5082	43	·01194	·000 2085
44	·767 9449	104	1·815 1424	164	2·862 3400	44	·73333	·012 7991	44	·01222	·000 2133
45	·785 3982	105	1·832 5957	165	2·879 7933	45	·75000	·013 0900	45	·01250	·000 2182
46	·802 8515	106	1·850 0490	166	2·897 2466	46	·76667	·013 3809	46	·01278	·000 2230
47	·820 3047	107	1·867 5023	167	2·914 6999	47	·78333	·013 6717	47	·01306	·000 2279
48	·837 7580	108	1·884 9556	168	2·932 1531	48	·80000	·013 9626	48	0·1333	·000 2327
49	·855 2113	109	1·902 4089	169	2·949 6064	49	·81667	·014 2535	49	·01361	·000 2376
50	·872 6646	110	1·919 8622	170	2·967 0597	50	·83333	·014 5444	50	·01389	·000 2424
51	·890 1179	111	1·937 3155	171	2·984 5130	51	·85000	·014 8353	51	·01417	·000 2473
52	·907 5712	112	1·954 7688	172	3·001 9663	52	·86667	·015 1262	52	·01444	·000 2521
53	·925 0245	113	1·972 2221	173	3·019 4196	53	·88333	·015 4171	53	·01472	·000 2570
54	·942 4778	114	1·989 6753	174	3·036 8729	54	·90000	·015 7080	54	·01500	·000 2618
55	·959 9311	115	2·007 1286	175	3·054 3262	55	·91667	·015 9989	55	·01528	·000 2666
56	·977 3844	116	2·024 5819	176	3·071 7795	56	·93333	·016 2897	56	·01556	·000 2715
57	·994 8377	117	2·042 0352	177	3·089 2328	57	·95000	·016 5806	57	·01583	·000 2763
58	1·012 2910	118	2·059 4885	178	3·106 6861	58	·96667	·016 8715	58	·01611	·000 2812
59	1·029 7443	119	2·076 9418	179	3·124 1394	59	·98333	·017 1624	59	·01639	·000 2860
60	1·047 1976	120	2·094 3951	180	3·141 5927	60	1·00000	·017 4533	60	·01667	·000 2909

TABLE LIV. *The $G(r, \nu)$-Integrals.*

ϕ°	$r=1$			$r=2$			
	$\log F(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	0·301 0300	309		0·196 1199	0·196 1199	192	
1	·301 0609	928	619	·196 2052	·196 1391	575	383
2	·301 1538	1550	621	·196 4614	·196 1966	958	383
3	·301 3087	2175	625	·196 8890	·196 2924	1340	382
4	·301 5262	2805	630	·197 4890	·196 4264	1722	381
5	·301 8067	3441	636	·198 2627	·106 5985	2102	380
6	·302 1508	4086	645	·199 2118	·196 8087	2480	379
7	·302 5594	4740	654	·200 3385	·197 0567	2857	377
8	·303 0335	5406	666	·201 6452	·197 3424	3231	375
9	·303 5741	6085	679	·203 1349	·197 6655	3604	373
10	·304 1825	6778	693	·204 8110	·198 0260	3974	370
11	·304 8603	7488	710	·206 6774	·198 4234	4341	367
12	·305 6091	8216	728	·208 7382	·198 8574	4705	364
13	·306 4307	8964	749	·210 9985	·199 3280	5064	360
14	·307 3271	9735	771	·213 4631	·199 8344	5420	356
15	·308 3006	10532	796	·216 1383	·200 3764	5773	352
16	·309 3538	11353	822	·219 0303	·200 9537	6120	347
17	·310 4892	12206	853	·222 1462	·201 5657	6463	343
18	·311 7098	13091	885	·225 4936	·202 2120	6801	338
19	·313 0189	14011	919	·229 0807	·202 8921	7133	332
20	·314 4200	14969	958	·232 9167	·203 6054	7460	327
21	·315 9169	15968	999	·237 0114	·204 3514	7780	320
22	·317 5137	17013	1045	·241 3755	·205 1294	8093	313
23	·319 2150	18106	1093	·246 0203	·205 9387	8400	307
24	·321 0256	19252	1146	·250 9584	·206 7787	8700	300
25	·322 9507	20456	1204	·256 2034	·207 6487	8991	291
26	·324 9963	21722	1266	·261 7697	·208 5478	9275	284
27	·327 1685	23055	1334	·267 6733	·209 4753	9549	275
28	·329 4740	24462	1407	·273 9311	·210 4302	9816	266
29	·331 9202	25948	1486	·280 5618	·211 4118	1 0072	256
30	·334 5150	27521	1573	·287 5852	·212 4190	1 0319	247
31	·337 2672	29188	1667	·295 0232	·213 4509	1 0556	236
32	·340 1860	30958	1769	·302 8992	·214 5064	1 0782	226
33	·343 2818	32838	1881	·311 2388	·215 5846	1 0996	213
34	·346 5656	34840	2002	·320 0695	·216 6842	1 1199	203
35	·350 0496	36974	2134	·329 4214	·217 8041	1 1390	191
36	·353 7469	39252	2279	·339 3271	·218 9431	1 1569	178
37	·357 6722	41689	2436	·349 8221	·220 1000	1 1734	165
38	·361 8410	44298	2609	·360 9451	·221 2734	1 1886	152
39	·366 2708	47096	2799	·372 7382	·222 4621	1 2023	138
40	·370 9805	50103	3007	·385 2475	·223 6644	1 2148	125
41	·375 9908	53338	3235	·398 5232	·224 8791	1 2256	109
42	·381 3246	56824	3486	·412 6205	·226 1048	1 2350	94
43	·387 0070	60588	3764	·427 5995	·227 3397	1 2427	77
44	·393 0658	64658	4069	·443 5266	·228 5824	1 2489	62
45	·399 5316			·460 4745	·229 8313		

TABLE LIV—(*continued*).

$\phi°$	$r=3$				$r=4$				$r=5$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	0·124 9387	0·275 4537	137		0·071 1811	0·309 7418	105		0·028 0289	0·329 0589	84	
1	·125 0847	·275 4674	410	273	·071 3902	·309 7523	316	211	·028 3019	·329 0673	257	173
2	·125 5230	·275 5084	683	273	·072 0177	·309 7840	527	211	·029 1221	·329 0930	428	171
3	·126 2545	·275 5767	955	272	·073 0650	·309 8366	737	210	·030 4908	·329 1357	598	171
4	·127 2807	·275 6721	1226	271	·074 5342	·309 9103	946	209	·032 4110	·329 1956	768	170
5	·128 6039	·275 7948	1496	270	·076 4285	·310 0049	1154	208	·034 8865	·329 2723	937	169
6	·130 2270	·275 9444	1766	269	·078 7517	·310 1204	1361	207	·037 9224	·329 3661	1105	168
7	·132 1533	·276 1209	2032	267	·081 5088	·310 2565	1567	206	·041 5250	·329 4765	1272	167
8	·134 3870	·276 3242	2298	266	·084 7055	·310 4132	1771	204	·045 7016	·329 6037	1437	165
9	·136 9331	·276 5540	2561	263	·088 3486	·310 5904	1973	202	·050 4609	·329 7474	1601	164
10	·139 7969	·276 8101	2822	261	·092 4458	·310 7877	2174	201	·055 8130	·329 9075	1763	162
11	·142 9850	·277 0923	3080	258	·097 0060	·311 0051	2372	198	·061 7690	·330 0838	1923	160
12	·146 5043	·277 4003	3336	256	·102 0399	·311 2423	2567	196	·068 3415	·330 2761	2081	158
13	·150 3626	·277 7338	3588	252	·107 5555	·311 4990	2760	193	·075 5446	·330 4842	2237	156
14	·154 5688	·278 0926	3837	249	·113 5680	·311 7751	2950	190	·083 3937	·330 7079	2390	153
15	·159 1322	·278 4762	4082	245	·120 0895	·312 0701	3137	187	·091 9061	·330 9469	2541	151
16	·164 0636	·278 8844	4323	241	·127 1349	·312 3838	3321	184	·101 1002	·331 2010	2689	148
17	·169 3743	·279 3167	4560	237	·134 7199	·312 7159	3501	180	·110 9967	·331 4700	2833	144
18	·175 0768	·279 7726	4793	233	·142 8621	·313 0660	3677	176	·121 6176	·331 7532	2975	142
19	·181 1848	·280 2519	5020	228	·151 5802	·313 4337	3849	172	·132 9872	·332 0508	3114	138
20	·187 7130	·280 7539	5243	223	·160 8948	·313 8186	4018	168	·145 1317	·332 3621	3249	135
21	·194 6774	·281 2782	5460	217	·170 8281	·314 2204	4181	164	·158 0795	·332 6870	3380	131
22	·202 0955	·281 8243	5673	212	·181 4042	·314 6385	4341	160	·171 8614	·333 0250	3507	127
23	·209 9858	·282 3915	5879	206	·192 6491	·315 0726	4495	154	·186 5105	·333 3757	3630	123
24	·218 3688	·282 9794	6079	200	·204 5907	·315 5222	4645	150	·202 0627	·333 7387	3750	119
25	·227 2664	·283 5873	6272	194	·217 2596	·315 9866	4789	144	·218 5568	·334 1137	3864	115
26	·236 7023	·284 2145	6460	188	·230 6885	·316 4655	4927	138	·236 0346	·334 5001	3975	110
27	·246 7020	·284 8604	6640	180	·244 9127	·316 9583	5062	134	·254 5413	·334 8976	4081	106
28	·257 2933	·285 5244	6813	173	·259 9707	·317 4644	5189	127	·274 1255	·335 3057	4182	101
29	·268 5060	·286 2057	6978	165	·275 9034	·317 9833	5310	121	·294 8399	·335 7239	4277	96
30	·280 3725	·286 9035	7136	158	·292 7555	·318 5143	5426	116	·316 7413	·336 1516	4368	91
31	·292 9278	·287 6170	7286	150	·310 5754	·319 0569	5535	109	·339 8909	·336 5884	4455	87
32	·306 2096	·288 3456	7427	141	·329 4149	·319 6104	5637	102	·364 3553	·337 0339	4535	80
33	·320 2589	·289 0883	7560	133	·349 3304	·320 1741	5733	96	·390 2059	·337 4874	4610	75
34	·335 1201	·289 8443	7684	124	·370 3832	·320 7474	5822	89	·417 5203	·337 9485	4680	70
35	·350 8413	·290 6127	7800	115	·392 6390	·321 3297	5904	82	·446 3827	·338 4164	4744	64
36	·367 4747	·291 3927	7905	106	·416 1697	·321 9201	5980	75	·476 8841	·338 8908	4802	58
37	·385 0770	·292 1832	8002	97	·441 0529	·322 5181	6047	68	·509 1232	·339 3709	4854	52
38	·403 7099	·292 9834	8089	87	·467 3733	·323 1228	6107	60	·543 2072	·339 8563	4900	46
39	·423 4403	·293 7923	8166	77	·495 2227	·323 7335	6160	53	·579 2529	·340 3463	4940	40
40	·444 3416	·294 6089	8232	67	·524 7011	·324 3495	6205	45	·617 3872	·340 8404	4975	34
41	·466 4933	·295 4321	8289	57	·555 9177	·324 9700	6243	38	·657 7483	·341 3378	5003	28
42	·489 9829	·296 2610	8335	46	·588 9916	·325 5943	6272	30	·700 4872	·341 8381	5024	22
43	·514 9055	·297 0945	8371	36	·624 0530	·326 2216	6295	22	·745 7688	·342 3405	5040	16
44	·541 3658	·297 9316	8394	23	·661 2446	·326 8510	6307	12	·793 7739	·342 8446	5049	9
45	·569 4783	·298 7710			·700 7225	·327 4817			·844 6999	·343 3495		

TABLE LIV—(*continued*).

$\phi°$	$r=6$				$r=7$				$r=8$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^3
0	$\bar{1}$·991 9999	0·341 4849			$\bar{1}$·961 0819	0·350 1576			$\bar{1}$·934 0080	0·356 5570		
1	·992 3379	·341 4921	72	144	·961 4851	·350 1638	62	124	·934 4765	·356 5624	54	109
2	·993 3526	·341 5137	216	144	·962 6953	·350 1824	186	124	·935 8831	·356 5788	163	109
3	·995 0459	·341 5496	359	143	·964 7151	·350 2134	310	123	·938 2304	·356 6059	272	108
4	·997 4213	·341 5999	503	143	·967 5483	·350 2567	433	123	·941 5232	·356 6440	380	108
5	0·000 4836	·341 6644	645	142	·971 2008	·350 3123	556	122	·945 7679	·356 6928	488	107
6	·004 2390	·341 7432	787	141	·975 6796	·350 3801	678	122	·950 9729	·356 7524	596	107
7	·008 6950	·341 8360	928	140	·980 9940	·350 4601	800	121	·957 1486	·356 8226	702	106
8	·013 8607	·341 9428	1068	139	·987 1544	·350 5522	921	120	·964 3073	·356 9034	808	105
9	·019 7468	·342 0635	1207	137	·994 1735	·350 6562	1040	118	·972 4634	·356 9947	913	104
10	·026 3653	·342 1980	1345	136	0·002 0658	·350 7720	1158	117	·981 6333	·357 0964	1017	103
11	·033 7300	·342 3461	1481	134	·010 8465	·350 8995	1275	116	·991 8357	·357 2083	1120	101
12	·041 8562	·342 5075	1615	133	·020 5347	·351 0386	1391	114	0·003 0914	·357 3304	1221	100
13	·050 7609	·342 6823	1747	131	·031 1503	·351 1891	1505	112	·015 4237	·357 4625	1321	98
14	·060 4633	·342 8701	1878	128	·042 7157	·351 3508	1617	110	·028 8583	·357 6044	1419	97
15	·070 9839	·343 0707	2006	127	·055 2551	·351 5235	1727	109	·043 4233	·357 7560	1516	95
16	·082 3457	·343 2839	2133	123	·068 7956	·351 7071	1836	106	·059 1498	·357 9170	1611	93
17	·094 5734	·343 5095	2256	121	·083 3665	·351 9012	1942	104	·076 0714	·358 0874	1704	91
18	·107 6941	·343 7472	2377	119	·098 9997	·352 1058	2046	102	·094 2250	·358 2669	1795	89
19	·121 7375	·343 9968	2496	115	·115 7298	·352 3206	2148	99	·113 6505	·358 4553	1884	87
20	·136 7352	·344 2579	2611	112	·133 5946	·352 5452	2247	97	·134 3912	·358 6524	1971	84
21	·152 7219	·344 5302	2723	109	·152 6347	·352 7795	2343	94	·156 4939	.358 8579	2055	82
22	·169 7350	·344 8135	2833	106	·172 8941	·353 0232	2437	91	·180 0093	·359 0716	2137	80
23	·187 8149	·345 1074	2939	103	·194 4206	·353 2760	2528	88	·204 9923	·359 2933	2217	77
24	·207 0053	·345 4115	3041	99	·217 2653	·353 5375	2615	85	·231 5019	·359 5226	2293	74
25	·227 3532	·345 7256	3141	95	·241 4839	·353 8075	2700	82	·259 6019	·359 7594	2368	71
26	·248 9095	·346 0492	3236	92	·267 1362	·354 0857	2782	79	·289 3613	·360 0033	2439	68
27	·271 7291	·346 3819	3328	88	·294 2868	·354 3717	2860	75	·320 8543	·360 2540	2507	65
28	·295 8713	·346 7235	3415	84	·323 0053	·354 6652	2935	72	·354 1610	·360 5112	2572	62
29	·321 3998	·347 0734	3499	78	·353 3670	·354 9659	3007	67	·389 3678	·360 7747	2635	59
30	·348 3836	·347 4311	3577	76	·385 4529	·355 2732	3074	64	·426 5682	·361 0441	2694	56
31	·376 8974	·347 7965	3653	71	·419 3506	·355 5871	3138	60	·465 8626	·361 3191	2750	52
32	·407 0214	·348 1689	3724	66	·455 1549	·355 9070	3199	56	·507 3601	·361 5993	2802	49
33	·438 8428	·348 5480	3791	62	·492 9680	·356 2324	3255	53	·551 1780	·361 8844	2851	46
34	·472 4556	·348 9332	3852	57	·532 9005	·356 5632	3308	49	·597 4436	·362 1741	2897	42
35	·507 9618	·349 3242	3910	52	·575 0721	·356 8988	3356	44	·646 2944	·362 4681	2939	39
36	·545 4718	·349 7204	3962	47	·619 6127	·357 2388	3400	40	·697 8795	·362 7659	2978	35
37	·585 1052	·350 1213	4009	43	·666 6629	·357 5829	3441	36	·752 3605	·363 0671	3013	31
38	·626 9922	·350 5265	4052	38	·716 3753	·357 9306	3477	32	·809 9127	·363 3715	3044	28
39	·671 2739	·350 9354	4090	33	·768 9159	·358 2814	3509	28	·870 7267	·363 6787	3072	24
40	·718 1040	·351 3477	4122	28	·824 4653	·358 6350	3536	23	·935 0097	·363 9883	3096	20
41	·767 6502	·351 7627	4150	22	·883 2199	·358 9910	3559	19	1·002 9876	·364 2998	3116	16
42	·820 0951	·352 1799	4172	17	·945 3944	·359 3488	3578	14	·074 9063	·364 6130	3132	12
43	·875 6383	·352 5989	4190	12	1·011 2229	·359 7080	3592	10	·151 0352	·364 9274	3144	9
44	·934 4983	·353 0191	4202	7	·080 9618	·360 0682	3602	6	·231 6680	·365 2427	3153	5
45	·996 9145	·353 4399	4209		·154 8920	·360 4290	3608		·317 1271	·365 5584	3157	

TABLE LIV—(*continued*).

ϕ°	$r=9$				$r=10$				$r=11$			
	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·909 9294	0·361 4744			$\bar{1}$·888 2505	0·365 3717			$\bar{1}$·868 5367	0·368 5367		
1	·910 4635	·361 4793	49	97	·888 8502	·365 3761	44	87	·869 2023	·368 5407	40	80
2	·912 0669	·361 4938	146	97	·890 6508	·365 3892	131	87	·871 2002	·368 5527	120	79
3	·914 7427	·361 5180	242	97	·893 6556	·365 4111	219	87	·874 5345	·368 5725	199	79
4	·918 4961	·361 5520	339	96	·897 8705	·365 4416	306	87	·879 2114	·368 6004	278	79
5	·923 3346	·361 5955	435	96	·903 3037	·365 4808	392	86	·885 2402	·368 6361	357	78
6	·929 2675	·361 6485	531	95	·909 9658	·365 5287	479	86	·892 6324	·368 6796	435	78
7	·936 3067	·361 7111	626	94	·917 8700	·365 5851	564	85	·901 4027	·368 7310	514	77
8	·944 4661	·361 7831	720	94	·927 0317	·365 6500	649	84	·911 5681	·368 7900	591	77
9	·953 7619	·361 8644	813	92	·937 4692	·365 7233	733	83	·923 1487	·368 8568	668	76
10	·964 2128	·361 9550	906	92	·949 2033	·365 8050	817	82	·936 1674	·368 9311	743	75
11	·975 8398	·362 0547	997	90	·962 2575	·365 8949	899	82	·950 6505	·369 0129	818	74
12	·988 6667	·362 1635	1088	89	·976 6581	·365 9929	981	80	·966 6268	·369 1022	892	73
13	0·002 7197	·362 2812	1177	88	·992 4345	·366 0990	1061	79	·984 1288	·369 1987	965	72
14	·018 0278	·362 4075	1264	86	0·009 6193	·366 2129	1139	78	0·003 1923	·369 3024	1037	71
15	·034 6230	·362 5426	1350	84	·028 2479	·366 3346	1217	76	·023 8567	·369 4132	1108	69
16	·052 5403	·362 6860	1435	83	·048 3595	·366 4639	1293	75	·046 1651	·369 5308	1177	68
17	·071 8179	·362 8378	1518	81	·069 9966	·366 6007	1368	73	·070 1646	·369 6553	1245	66
18	·092 4974	·362 9976	1599	79	·093 2058	·366 7447	1441	72	·095 9063	·369 7864	1311	65
19	·114 6239	·363 1654	1678	77	·118 0374	·366 8960	1512	69	·123 4460	·369 9240	1376	63
20	·138 2465	·363 3409	1755	75	·144 5461	·367 0541	1581	68	·152 8439	·370 0679	1439	62
21	·163 4180	·363 5239	1830	73	·172 7910	·367 2190	1649	66	·184 1654	·370 2179	1500	60
22	·190 1960	·363 7142	1903	71	·202 8360	·367 3904	1714	64	·217 4809	·370 3739	1560	58
23	·218 6422	·363 9115	1973	68	·234 7504	·367 5682	1778	61	·252 8669	·370 5357	1618	56
24	·248 8237	·364 1157	2042	66	·268 6087	·367 7522	1839	59	·290 4057	·370 7030	1673	54
25	·280 8124	·364 3264	2107	63	·304 4913	·367 9420	1899	57	·330 1857	·370 8757	1727	52
26	·314 6864	·364 5435	2171	61	·342 4851	·368 1376	1956	55	·372 3033	·371 0536	1779	50
27	·350 5295	·364 7666	2231	58	·382 6838	·368 3386	2010	52	·416 8615	·371 2365	1829	47
28	·388 4323	·364 9955	2290	55	·425 1882	·368 5448	2062	50	·463 9718	·371 4241	1876	45
29	·428 4925	·365 2300	2345	52	·470 1076	·368 7559	2112	47	·513 7544	·371 6162	1921	43
30	·470 8156	·365 4697	2397	50	·517 5593	·368 9718	2159	44	·566 3390	·371 8125	1964	40
31	·515 5153	·365 7144	2447	47	·567 6703	·369 1922	2203	42	·621 8657	·372 0129	2004	38
32	·562 7146	·365 9636	2493	44	·620 5776	·369 4167	2245	39	·680 4854	·372 2171	2042	36
33	·612 5463	·366 2173	2537	40	·676 4293	·369 6451	2284	36	·742 3617	·372 4249	2078	33
34	·665 1541	·366 4750	2577	37	·735 3855	·369 8772	2321	33	·807 6710	·372 6359	2110	30
35	·720 6932	·366 7365	2615	34	·797 6194	·370 1126	2354	31	·876 6045	·372 8500	2141	28
36	·779 3322	·367 0013	2649	31	·863 3188	·370 3510	2385	28	·949 3690	·373 0669	2169	25
37	·841 2534	·367 2693	2680	28	·932 6869	·370 5923	2412	25	1·026 1888	·373 2862	2194	23
38	·906 6549	·367 5400	2707	24	1·005 9444	·370 8360	2437	22	·107 3073	·373 5078	2216	20
39	·975 7519	·367 8131	2731	21	·083 3313	·371 0819	2459	19	·192 9888	·373 7314	2236	17
40	1·048 7784	·368 0884	2752	18	·165 1080	·371 3297	2478	16	·283 5209	·373 9567	2253	14
41	·125 9892	·368 3654	2770	14	·251 5588	·371 5790	2494	13	·379 2165	·374 1834	2267	12
42	·207 6624	·368 6438	2784	11	·342 9931	·371 8296	2506	10	·480 4171	·374 4113	2279	9
43	·294 1013	·368 9233	2795	7	·439 7491	·372 0813	2516	6	·587 4953	·374 6400	2287	6
44	·385 6379	·369 2036	2803	4	·542 1966	·372 3335	2522	4	·700 8587	·374 8693	2293	3
45	·482 6360	·369 4842	2806		·650 7407	·372 5861	2526		·820 9540	·375 0990	2296	

TABLE LIV—(*continued*).

φ°	r=12				r=13				r=14			
	log F (r, ν)	log H (r, ν)	Δ	Δ²	log F (r, ν)	log H (r, ν)	Δ	Δ²	log F (r, ν)	log H (r, ν)	Δ	Δ²
0	1̄·850 4619	0·371 1582			1̄·833 7746	0·373 3653			1̄·818 2772	0·375 2489		
1	·851 1933	·371 1619	36	73	·834 5719	·373 3686	34	68	·819 1404	·375 2520	31	63
2	·853 3889	·371 1729	110	73	·836 9652	·373 3787	101	67	·821 7316	·375 2614	94	63
3	·857 0528	·371 1912	183	73	·840 9592	·373 3956	169	67	·826 0558	·375 2771	157	62
4	·862 1923	·371 2166	255	72	·846 5615	·373 4192	236	67	·832 1212	·375 2990	219	62
5	·868 8171	·371 2494	328	72	·853 7829	·373 4494	303	67	·839 9395	·375 3271	281	62
6	·876 9402	·371 2893	400	71	·862 6374	·373 4864	369	66	·849 5258	·375 3614	343	61
7	·886 5774	·371 3364	471	71	·873 1421	·373 5299	435	66	·860 8985	·375 4019	404	61
8	·897 7474	·371 3907	542	70	·885 3174	·373 5800	501	65	·874 0798	·375 4484	465	60
9	·910 4722	·371 4519	613	69	·899 1873	·373 6365	566	65	·889 0954	·375 5010	526	60
10	·924 7770	·371 5201	682	69	·914 7789	·373 6995	630	64	·905 9746	·375 5595	585	59
11	·940 6901	·371 5952	751	68	·932 1233	·373 7689	694	63	·924 7510	·375 6239	644	58
12	·958 2436	·371 6771	819	67	·951 2549	·373 8445	756	62	·945 4618	·375 6942	703	57
13	·977 4727	·371 7656	886	66	·972 2124	·373 9263	818	61	·968 1485	·375 7702	760	56
14	·998 4167	·371 8608	951	65	·995 0382	·374 0142	879	60	·992 8571	·375 8518	816	55
15	0·021 1187	·371 9624	1016	64	0·019 7792	·374 1081	938	59	0·019 6382	·375 9390	872	54
16	·045 6258	·372 0703	1080	62	·046 4865	·374 2078	997	58	·048 5469	·376 0317	926	53
17	·071 9896	·372 1845	1142	61	·075 2161	·374 3132	1055	56	·079 6435	·376 1297	980	52
18	·100 2660	·372 3048	1203	59	·106 0288	·374 4243	1111	55	·112 9939	·376 2329	1032	51
19	·130 5159	·372 4310	1262	58	·138 9908	·374 5409	1166	53	·148 6693	·376 3412	1083	50
20	·162 8054	·372 5630	1320	56	·174 1737	·374 6628	1219	52	·186 7470	·376 4544	1133	48
21	·197 2059	·372 7006	1376	55	·211 6551	·374 7899	1271	51	·227 3107	·376 5725	1181	47
22	·233 7945	·372 8437	1431	53	·251 5187	·374 9221	1322	49	·270 4509	·376 6952	1228	45
23	·272 6547	·372 9921	1484	51	·293 8552	·375 0591	1370	47	·316 2653	·376 8225	1273	44
24	·313 8765	·373 1456	1535	49	·338 7623	·375 2008	1417	46	·364 8594	·376 9542	1317	42
25	·357 5571	·373 3040	1584	47	·386 3455	·375 3472	1464	43	·416 3469	·377 0901	1359	40
26	·403 8013	·373 4672	1632	45	·436 7186	·375 4978	1506	42	·470 8506	·377 2301	1399	39
27	·452 7221	·373 6349	1677	43	·490 0042	·375 6527	1549	40	·528 5029	·377 3739	1439	37
28	·504 4412	·373 8069	1720	41	·546 3346	·375 8115	1589	38	·589 4465	·377 5215	1476	35
29	·559 0903	·373 9831	1762	39	·605 8526	·375 9742	1627	36	·653 8352	·377 6726	1511	33
30	·616 8111	·374 1631	1801	37	·668 7120	·376 1405	1663	34	·721 8353	·377 8270	1544	32
31	·677 7567	·374 3469	1838	35	·735 0790	·376 3102	1697	32	·793 6258	·377 9846	1576	30
32	·742 0923	·374 5341	1873	33	·805 1331	·376 4831	1729	30	·869 4003	·378 1452	1606	28
33	·809 9965	·374 7246	1905	30	·879 0680	·376 6589	1759	28	·949 3679	·378 3085	1634	26
34	·881 6625	·374 9182	1935	28	·957 0932	·376 8376	1787	26	1·033 7545	·378 4745	1659	24
35	·957 2989	·375 1145	1963	25	1·039 4354	·377 0188	1812	23	·122 8047	·378 6428	1683	22
36	1·037 1322	·375 3133	1988	23	·126 3402	·377 2024	1836	21	·216 7831	·378 8133	1705	19
37	·121 4073	·375 5144	2011	21	·218 0735	·377 3881	1857	19	·315 9768	·378 9857	1724	18
38	·210 3905	·375 7176	2032	18	·314 9240	·377 5757	1876	17	·420 6970	·379 1599	1742	15
39	·304 3704	·375 9226	2050	16	·417 2053	·377 7649	1893	14	·531 2818	·379 3356	1757	13
40	·403 6615	·376 1291	2065	13	·525 2853	·377 9556	1907	12	·648 0989	·379 5127	1771	11
41	·508 6058	·376 3370	2078	11	·639 4542	·378 1475	1919	10	·771 5487	·379 6909	1782	9
42	·619 5764	·376 5458	2089	8	·760 1977	·378 3403	1928	7	·902 0674	·379 8699	1791	7
43	·736 9806	·376 7555	2097	5	·887 9308	·378 5338	1936	5	2·040 1318	·380 0497	1797	5
44	·861 2638	·376 9657	2102	3	2·023 1367	·378 7279	1941	2	·186 2627	·380 2299	1802	2
45	·992 9140	·377 1762	2105		·166 3448	·378 9222	1943		·341 0309	·380 4103	1804	

TABLE LIV—(*continued*).

$\phi°$	$r=15$				$r=16$				$r=17$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·803 8114	0·376 8754			$\bar{1}$·790 2485	0·378 2941			$\bar{1}$·777 4825	0·379 5425		
1	·804 7405	·376 8783	29	59	·791 2436	·378 2969	28	55	·778 5436	·379 5450	26	52
2	·807 5297	·376 8871	88	58	·794 2308	·378 3051	82	55	·781 7289	·379 5528	78	52
3	·812 1842	·376 9018	146	58	·799 2158	·378 3188	137	55	·787 0445	·379 5657	129	51
4	·818 7130	·376 9222	205	58	·806 2081	·378 3380	192	54	·794 5004	·379 5838	181	51
5	·827 1285	·376 9485	262	58	·815 2211	·378 3626	246	54	·804 1110	·379 6070	232	51
6	·837 4469	·376 9805	321	57	·826 2719	·378 3927	300	54	·815 8945	·379 6352	283	51
7	·849 6881	·377 0183	378	57	·839 3819	·378 4281	354	53	·829 8736	·379 6686	333	50
8	·863 8758	·377 0617	435	56	·854 5764	·378 4688	408	53	·846 0751	·379 7070	384	50
9	·880 0376	·377 1108	491	56	·871 8848	·378 5149	460	52	·864 5306	·379 7503	433	49
10	·898 2051	·377 1655	547	55	·891 3410	·378 5661	513	52	·885 2758	·379 7986	483	49
11	·918 4141	·377 2256	602	54	·912 9831	·378 6226	564	51	·908 3516	·379 8517	531	48
12	·940 7047	·377 2912	656	54	·936 8541	·378 6841	615	50	·933 8035	·379 9096	579	47
13	·965 1215	·377 3622	710	53	·963 0016	·378 7507	666	49	·961 6821	·379 9723	627	47
14	·991 7138	·377 4384	762	52	·991 4782	·378 8222	715	49	·992 0436	·380 0396	673	46
15	0·020 5357	·377 5199	814	51	0·022 3418	·378 8985	764	48	0·024 9495	·380 1115	719	45
16	·051 6467	·377 6064	865	50	·055 6559	·378 9796	811	47	·060 4672	·380 1878	764	44
17	·085 1115	·377 6979	915	49	·091 4895	·379 0654	858	46	·098 6704	·380 2686	808	43
18	·121 0005	·377 7942	964	48	·129 9181	·379 1558	904	45	·139 6392	·380 3537	851	42
19	·159 3904	·377 8953	1011	46	·171 0234	·379 2506	948	43	·183 4606	·380 4430	893	41
20	·200 3641	·378 0011	1057	45	·214 8939	·379 3498	992	42	·230 2288	·380 5363	933	40
21	·244 0113	·378 1113	1102	44	·261 6258	·379 4532	1034	41	·280 0460	·380 6336	973	39
22	·290 4293	·378 2259	1146	43	·311 3226	·379 5606	1075	40	·333 0225	·380 7348	1012	37
23	·339 7229	·378 3448	1189	41	·364 0964	·379 6721	1114	38	·389 2774	·380 8397	1049	36
24	·392 0053	·378 4677	1229	39	·420 0681	·379 7874	1153	37	·448 9393	·380 9482	1085	35
25	·447 3985	·378 5946	1269	38	·479 3681	·379 9063	1190	35	·512 1471	·381 0602	1120	33
26	·506 0343	·378 7252	1307	36	·542 1372	·380 0289	1225	34	·579 0504	·381 1756	1153	32
27	·568 0547	·378 8595	1343	34	·608 5269	·380 1548	1259	33	·649 8104	·381 2941	1185	31
28	·633 6129	·378 9973	1377	33	·678 7007	·380 2839	1292	31	·724 6013	·381 4157	1216	29
29	·702 8740	·379 1383	1411	31	·752 8356	·380 4162	1323	29	·803 6106	·381 5402	1245	28
30	·776 0162	·379 2825	1442	30	·831 1213	·380 5514	1352	28	·887 0408	·381 6674	1273	26
31	·853 2318	·379 4296	1471	28	·913 7633	·380 6893	1380	26	·975 1103	·381 7973	1299	24
32	·934 7285	·379 5795	1499	26	1·000 9834	·380 8299	1405	24	1·068 0549	·381 9296	1323	23
33	1·020 7304	·379 7320	1525	24	·093 0211	·380 9729	1430	23	·166 1294	·382 0642	1346	21
34	·111 4801	·379 8869	1549	22	·190 1352	·381 1181	1452	21	·269 6092	·382 2009	1367	20
35	·207 2399	·380 0440	1571	20	·292 6060	·381 2654	1473	19	·378 7922	·382 3395	1387	18
36	·308 2938	·380 2031	1591	18	·400 7368	·381 4146	1492	17	·494 0009	·382 4800	1404	16
37	·414 9495	·380 3641	1610	17	·514 8561	·381 5655	1509	15	·615 5849	·382 6220	1420	14
38	·527 5412	·380 5267	1626	14	·635 3206	·381 7180	1525	13	·743 9235	·382 7655	1435	13
39	·646 4313	·380 6907	1640	12	·762 5175	·381 8718	1538	12	·879 4285	·382 9102	1448	11
40	·772 0144	·380 8560	1653	11	·896 8681	·382 0267	1549	10	2·022 5477	·383 0561	1458	9
41	·904 7199	·381 0223	1663	8	2·038 8307	·382 1826	1559	8	·173 7686	·383 2028	1468	7
42	2·045 0157	·381 1894	1671	6	·188 9051	·382 3393	1567	6	·333 6229	·383 3503	1475	6
43	·193 4131	·381 3572	1678	4	·347 6370	·382 4966	1573	4	·502 6906	·383 4984	1480	4
44	·350 4709	·381 5254	1682	2	·515 6232	·382 6543	1577	2	·681 6063	·383 6468	1484	2
45	·516 8011	·381 6938	1684		·693 5170	·382 8122	1579		·871 0649	·383 7953	1486	

TABLE LIV—(*continued*).

$\phi°$	$r=18$				$r=19$				$r=20$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ°	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·765 4249	0·380 6494			$\bar{1}$·754 0014	0·381 6376			$\bar{1}$·743 1485	0·382 5253		
1	·766 5520	·380 6518	24	49	·755 1945	·381 6399	23	46	·744 4077	·382 5275	22	44
2	·769 9355	·380 6591	73	49	·758 7762	·381 6469	70	46	·748 1876	·382 5341	66	44
3	·775 5818	·380 6713	122	49	·764 7532	·381 6584	116	46	·754 4955	·382 5451	110	44
4	·783 5015	·380 6884	171	48	·773 1369	·381 6746	162	46	·763 3431	·382 5605	154	44
5	·793 7099	·380 7103	219	48	·783 9431	·381 6954	208	46	·774 7474	·382 5802	197	43
6	·806 2262	·380 7370	267	48	·797 1925	·381 7207	253	45	·788 7299	·382 6042	241	43
7	·821 0746	·380 7685	315	47	·812 9104	·381 7505	299	45	·805 3174	·382 6326	284	43
8	·838 2835	·380 8048	362	47	·831 1269	·381 7849	343	45	·824 5417	·382 6652	327	42
9	·857 8863	·380 8457	409	47	·851 8772	·381 8237	388	44	·846 4398	·382 7021	369	42
10	·879 9209	·380 8913	456	46	·875 2015	·381 8669	432	44	·871 0541	·382 7432	411	41
11	·904 4307	·380 9415	502	45	·901 1455	·381 9144	476	43	·898 4326	·382 7883	452	41
12	·931 4638	·380 9962	547	45	·929 7603	·381 9663	519	42	·928 6293	·382 8376	493	40
13	·961 0741	·381 0554	592	44	·961 1026	·382 0224	561	42	·961 7039	·382 8909	533	40
14	·993 3209	·381 1190	636	43	·995 2351	·382 0826	603	41	·997 7224	·382 9482	573	39
15	0·028 2696	·381 1869	679	42	0·032 2269	·382 1470	643	40	0·036 7574	·383 0093	611	39
16	·065 9915	·381 2591	722	42	·072 1535	·382 2154	684	39	·078 8894	·383 0743	650	37
17	·106 5648	·381 3354	763	41	·115 0974	·382 2877	723	38	·124 2043	·383 1430	687	37
18	·150 0744	·381 4157	804	40	·161 1483	·382 3638	761	38	·172 7969	·383 2153	724	36
19	·196 6125	·381 5001	843	39	·210 4036	·382 4437	799	37	·224 7699	·383 2912	759	35
20	·246 2790	·381 5882	882	38	·262 9690	·382 5273	836	36	·280 2346	·383 3706	794	34
21	·299 1823	·381 6802	919	36	·318 9588	·382 6144	871	35	·339 3115	·383 4534	828	33
22	·355 4391	·381 7757	956	35	·378 4966	·382 7049	906	33	·402 1306	·383 5394	860	32
23	·415 1758	·381 8749	991	34	·441 7157	·382 7988	939	32	·468 8327	·383 6286	892	31
24	·478 5288	·381 9774	1025	33	·508 7603	·382 8960	971	31	·539 5695	·383 7209	923	30
25	·545 6451	·382 0832	1058	31	·579 7858	·382 9962	1002	30	·614 5047	·383 8162	952	28
26	·616 6833	·382 1921	1089	30	·654 9597	·383 0994	1032	29	·693 8148	·383 9142	981	27
27	·691 8145	·382 3041	1120	29	·734 4627	·383 2055	1061	27	·777 6902	·384 0150	1008	26
28	·771 2230	·382 4189	1148	27	·818 4896	·383 3143	1088	26	·866 3362	·384 1184	1034	25
29	·855 1078	·382 5365	1176	26	·907 2506	·383 4257	1114	25	·959 9740	·384 2243	1059	23
30	·943 6834	·382 6567	1202	25	1·000 9723	·383 5396	1139	23	1·058 8424	·384 3325	1082	22
31	1·037 1813	·382 7794	1227	23	·099 8993	·383 6558	1162	22	·163 1992	·384 4429	1104	21
32	·135 8513	·382 9043	1249	22	·204 2956	·383 7742	1184	20	·273 3224	·384 5554	1125	19
33	·239 9636	·383 0314	1271	20	·314 4464	·383 8946	1204	19	·389 5124	·384 6698	1144	18
34	·349 8099	·383 1605	1291	19	·430 6601	·384 0170	1223	17	·512 0941	·384 7860	1162	17
35	·465 7060	·383 2915	1310	17	·553 2702	·384 1411	1241	16	·641 4188	·384 9039	1179	15
36	·587 9937	·383 4241	1326	15	·682 6377	·384 2667	1257	14	·777 8668	·385 0233	1194	14
37	·717 0435	·383 5583	1342	14	·819 1539	·384 3938	1271	13	·921 8503	·385 1440	1208	12
38	·853 2571	·383 6938	1355	12	·963 2435	·384 5222	1284	11	2·073 8165	·385 2660	1220	11
39	·997 0711	·383 8305	1367	10	2·115 3673	·384 6518	1295	10	·234 2509	·385 3891	1231	9
40	2·148 9600	·383 9683	1377	9	·276 0267	·384 7823	1305	8	·403 6815	·385 5130	1240	8
41	·309 4403	·384 1069	1386	7	·445 7673	·384 9136	1313	7	·582 6831	·385 6378	1247	6
42	·479 0754	·384 2462	1393	5	·625 1841	·385 0455	1320	5	·771 8822	·385 7632	1254	5
43	·658 4799	·384 3860	1398	4	·814 9263	·385 1780	1324	3	·971 9629	·385 8890	1258	3
44	·848 3263	·384 5262	1402	2	3·015 7041	·385 3108	1328	2	3·183 6730	·386 0151	1261	2
45	3·049 3507	·384 6665	1403		·228 2952	·385 4437	1330		·407 8314	·386 1414	1263	

TABLE LIV—(*continued*).

$\phi°$	r=21				r=22				r=23			
	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·732 8121	0·383 3271	21		$\bar{1}$·722 9451	0·384 0548	20		$\bar{1}$·713 5069	0·384 7182	19	
1	734 1352	·383 3292	63	42	·724 3364	·384 0568	60	40	·714 9643	·384 7202	57	38
2	·738 1155	·383 3354	105	42	·728 5130	·384 0628	100	40	·719 3391	·384 7259	96	38
3	·744 7542	·383 3459	146	42	·735 4826	·384 0728	140	40	·726 6397	·384 7355	134	38
4	·754 0660	·383 3606	188	41	·745 2584	·384 0867	179	40	·736 8798	·384 7488	172	38
5	·766 0683	·383 3793	229	41	·757 8590	·384 1047	219	39	·750 0786	·384 7660	209	38
6	·780 7641	·383 4022	270	41	·773 3082	·384 1266	258	39	·766 2613	·384 7869	247	37
7	·798 2414	·383 4293	311	41	·791 6354	·384 1523	297	39	·785 4585	·384 8116	284	37
8	·818 4736	·383 4604	351	40	·812 8757	·384 1820	335	38	·807 7070	·384 8400	321	37
9	·841 5196	·383 4955	391	40	·837 0698	·384 2155	373	38	·833 0493	·384 8721	357	36
10	·867 4241	·383 5346	430	39	·864 2646	·384 2529	411	38	·861 5346	·384 9078	393	36
11	·896 2374	·383 5776	469	39	·894 5129	·384 2940	448	37	·893 2180	·384 9471	429	36
12	·928 0162	·383 6245	508	38	·927 8740	·384 3388	485	37	·928 1617	·384 9899	464	35
13	·962 8233	·383 6753	545	38	·964 4139	·384 3872	521	36	·966 4346	·385 0363	498	34
14	0·000 7283	·383 7299	582	37	0·004 2054	·384 4393	556	35	0·008 1129	·385 0861	532	34
15	·041 8074	·383 7881	619	36	·047 3287	·384 4949	591	35	·053 2804	·385 1393	565	33
16	·086 1444	·383 8500	654	36	·093 8713	·384 5540	625	34	·102 0289	·385 1958	598	33
17	·133 8306	·383 9154	689	35	·143 9291	·384 6164	658	33	·154 4586	·385 2556	629	32
18	·184 9653	·383 9843	723	34	·197 6061	·384 6822	690	32	·210 6783	·385 3185	660	31
19	·239 6563	·384 0566	756	33	·255 0156	·384 7513	722	31	·270 8064	·385 3845	691	30
20	·298 0208	·384 1322	788	32	·316 2801	·384 8235	753	31	·334 9713	·385 4536	720	29
21	·360 1851	·384 2111	820	31	·381 5322	·384 8987	782	30	·403 3116	·385 5256	748	28
22	·426 2861	·384 2930	850	30	·450 9155	·384 9770	811	29	·475 9774	·385 6004	776	28
23	·496 4716	·384 3780	879	29	·524 5848	·385 0581	839	28	·553 1308	·385 6780	803	27
24	·570 9011	·384 4659	907	28	·602 7073	·385 1420	866	27	·634 9466	·385 7583	828	26
25	·649 7464	·384 5566	934	27	·685 4632	·385 2286	892	26	·721 6136	·385 8411	853	25
26	·733 1933	·384 6500	960	26	·773 0472	·385 3178	916	25	·813 3351	·385 9264	877	24
27	·821 4416	·384 7460	985	25	·865 6689	·385 4094	940	24	·910 3304	·386 0141	899	23
28	·914 7070	·384 8445	1008	23	·963 5543	·385 5034	962	22	1·012 8362	·386 1040	921	22
29	1·013 2222	·384 9453	1031	22	1·066 9472	·385 5996	984	21	·121 1074	·386 1961	941	20
30	·117 2380	·385 0484	1052	21	·176 1108	·385 6980	1004	20	·235 4191	·386 2902	960	19
31	·227 0250	·385 1535	1071	20	·291 3286	·385 7984	1023	19	·356 0681	·386 3862	978	18
32	·342 8756	·385 2607	1090	18	·412 9072	·385 9007	1040	18	·483 3750	·386 4840	995	17
33	·465 1055	·385 3696	1107	17	·541 1773	·386 0047	1057	16	·617 6859	·386 5836	1011	16
34	·594 0558	·385 4803	1123	16	·676 4967	·386 1104	1072	15	·759 3748	·386 6846	1025	14
35	·730 0957	·385 5926	1137	14	·819 2523	·386 2175	1085	14	·908 8466	·386 7871	1038	13
36	·873 6248	·385 7063	1150	13	·969 8630	·386 3261	1098	12	2·066 5394	·386 8910	1050	12
37	2·025 0761	·385 8213	1162	12	2·128 7827	·386 4359	1109	11	·232 9279	·386 9960	1061	11
38	·184 9195	·385 9375	1172	10	·296 5039	·386 5468	1119	10	·408 5272	·387 1021	1070	10
39	·353 6651	·386 0547	1181	9	·473 5612	·386 6586	1127	8	·593 8968	·387 2091	1078	8
40	·531 8676	·386 1728	1188	7	·660 5361	·386 7713	1134	7	·789 6446	·387 3169	1085	7
41	·720 1308	·386 2916	1194	6	·858 0615	·386 8848	1140	6	·996 4325	·387 4254	1090	5
42	·919 1130	·386 4110	1198	4	3·066 8272	·386 9987	1144	4	3·214 9823	·387 5344	1094	4
43	3·129 5327	·386 5308	1201	3	·287 5865	·387 1131	1147	3	·446 0817	·387 6438	1097	3
44	·352 1756	·386 6510	1203	2	·521 1629	·387 2278	1148	1	·690 5919	·387 7535	1098	1
45	·587 9021	·386 7712			·768 4580	·387 3426			·949 4561	·387 8633		

TABLE LIV—(*continued*).

ϕ°	$r=24$				$r=25$				$r=26$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·704 4618	0·385 3256			$\bar{1}$·695 7781	0·385 8838			$\bar{1}$·687 4284	0·386 3984		
1	·705 9852	·385 3275	18	37	·697 3569	·385 8855	18	35	·689 0840	·386 4001	17	34
2	·710 5584	·385 3330	55	37	·702 1393	·385 8908	53	35	·694 0540	·386 4052	51	34
3	·718 1899	·385 3421	92	36	·710 1019	·385 8996	88	35	·702 3476	·386 4136	85	34
4	·728 8942	·385 3549	128	36	·721 2704	·385 9119	123	35	·713 9805	·386 4254	118	34
5	·742 6914	·385 3714	164	36	·735 6660	·385 9277	158	35	·728 9746	·386 4406	152	33
6	·759 6077	·385 3915	201	36	·753 3158	·385 9470	193	35	·747 3581	·386 4591	185	33
7	·779 6750	·385 4151	236	36	·774 2534	·385 9697	227	34	·769 1658	·386 4810	218	33
8	·802 9317	·385 4423	272	35	·798 5185	·385 9958	261	34	·794 4395	·386 5061	251	33
9	·829 4225	·385 4730	307	35	·826 1578	·386 0253	295	34	·823 2272	·386 5345	284	32
10	·859 1983	·385 5073	342	34	·857 2243	·386 0582	329	33	·855 5847	·386 5661	316	32
11	·892 3170	·385 5450	377	34	·891 7784	·386 0943	362	33	·891 5743	·386 6009	348	31
12	·928 8434	·385 5860	411	34	·929 8876	·386 1338	394	32	·931 2665	·386 6388	379	31
13	·968 8495	·385 6305	444	33	·971 6270	·386 1764	427	32	·974 7393	·386 6798	411	30
14	0·012 4148	·385 6782	477	32	0·017 0795	·386 2223	458	31	0·022 0791	·386 7239	440	30
15	·059 6268	·385 7292	510	32	·066 3362	·386 2712	489	31	·073 3806	·386 7710	471	29
16	·110 5814	·385 7834	542	31	·119 4971	·386 3232	520	30	·128 7480	·386 8210	500	29
17	·165 3831	·385 8406	573	30	·176 6711	·386 3782	550	29	·188 2945	·386 8738	529	28
18	·224 1458	·385 9009	603	30	·237 9768	·386 4361	579	29	·252 1435	·386 9295	557	27
19	·286 9928	·385 9642	633	29	·303 5430	·386 4969	608	28	·320 4290	·386 9880	584	27
20	·354 0583	·386 0304	662	28	·373 5093	·386 5604	635	27	·393 2963	·387 0491	611	26
21	·425 4870	·386 0994	690	27	·448 0267	·386 6267	662	26	·470 9025	·387 1128	637	25
22	·501 4356	·386 1711	717	26	·527 2584	·386 6955	689	25	·553 4176	·387 1790	662	24
23	·582 0734	·386 2455	744	26	·611 3809	·386 7669	714	25	·641 0250	·387 2477	687	24
24	·667 5830	·386 3225	769	25	·700 5843	·386 8408	739	24	·733 9226	·387 3187	710	23
25	·758 1612	·386 4018	794	24	·795 0741	·386 9170	762	23	·832 3242	·387 3920	733	22
26	·854 0205	·386 4836	818	23	·895 0715	·386 9955	785	22	·936 4600	·387 4674	755	21
27	·955 3899	·386 5676	840	22	1·000 8153	·387 0762	807	21	1·046 5783	·387 5450	776	20
28	1·062 5163	·386 6538	862	21	·112 5627	·387 1589	827	20	·162 9470	·387 6246	796	19
29	·175 6661	·386 7420	882	19	·230 5913	·387 2436	847	19	·285 8547	·387 7060	815	18
30	·295 1263	·386 8322	902	18	·355 2004	·387 3302	866	18	·415 6129	·387 7893	833	17
31	·421 2069	·386 9242	920	17	·486 7129	·387 4185	883	17	·552 5576	·387 8742	850	16
32	·554 2426	·387 0180	938	16	·625 4776	·387 5085	900	16	·697 0516	·387 9608	865	15
33	·694 5945	·387 1134	954	15	·771 8709	·387 6001	916	14	·849 4867	·388 0488	880	14
34	·842 6534	·387 2102	969	14	·926 3001	·387 6931	930	13	2·010 2864	·388 1382	894	13
35	·998 8417	·387 3085	982	13	2·089 2053	·387 7874	943	12	·179 9088	·388 2289	907	12
36	2·163 6169	·387 4080	995	11	·261 0633	·387 8829	955	11	·358 8499	·388 3208	919	11
37	·337 4747	·387 5086	1006	10	·442 3906	·387 9795	966	10	·547 6471	·388 4137	929	9
38	·520 9526	·387 6103	1017	9	·633 7476	·388 0771	976	9	·746 8833	·388 5075	938	8
39	·714 6347	·387 7128	1026	8	·835 7426	·388 1756	985	7	·957 1916	·388 6022	947	7
40	·919 1558	·387 8162	1033	6	3·049 0373	·388 2748	992	6	3·179 2602	·388 6975	954	6
41	3·135 2068	·387 9201	1040	5	·274 3517	·388 3746	998	5	·413 8384	·388 7935	960	5
42	·363 5411	·388 0246	1045	3	·512 4708	·388 4749	1003	4	·661 7426	·388 8900	964	4
43	·604 9809	·388 1295	1048	3	·764 2515	·388 5756	1007	2	·923 8647	·388 9868	968	2
44	·860 4254	·388 2346	1051	1	4·030 6307	·388 6765	1009	1	4·201 1786	·389 0838	970	1
45	4·130 8591	·388 3398	1052		·312 6342	·388 7775	1010		·494 7524	·389 1810	972	

TABLE LIV—(*continued*).

φ°	r=27				r=28				r=29			
	log *F* (*r*, *ν*)	log *H* (*r*, *ν*)	Δ	Δ²	log *F* (*r*, *ν*)	log *H* (*r*, *ν*)	Δ	Δ²	log *F* (*r*, *ν*)	log *H* (*r*, *ν*)	Δ	Δ²
0	$\bar{1}$·679 3877	0·386 8744			$\bar{1}$·671 6341	0·387 3160			$\bar{1}$·664 1478	0·387 7268		
1	·681 1094	·386 8760	16	33	·673 4219	·387 3176	16	31	·666 0017	·387 7283	15	30
2	·686 2778	·386 8809	49	33	·678 7887	·387 3223	47	31	·671 5669	·387 7328	46	30
3	·694 9025	·386 8891	81	32	·687 7445	·387 3301	79	31	·680 8538	·387 7404	76	30
4	·706 9998	·386 9005	114	32	·700 3062	·387 3411	110	31	·693 8799	·387 7510	106	30
5	·722 5923	·386 9151	146	32	·716 4973	·387 3552	141	31	·710 6695	·387 7647	136	30
6	·741 7096	·386 9329	178	32	·736 3483	·387 3724	172	31	·731 2544	·387 7813	166	30
7	·764 3877	·386 9539	210	32	·759 8968	·387 3927	203	30	·755 6734	·387 8008	196	30
8	·790 6698	·386 9781	242	31	·787 1876	·387 4160	233	30	·783 9728	·387 8234	225	29
9	·820 6063	·387 0055	273	31	·818 2728	·387 4424	264	30	·816 2068	·387 8488	254	29
10	·854 2546	·387 0359	304	31	·853 2121	·387 4717	293	30	·852 4372	·387 8772	283	29
11	·891 6799	·387 0694	335	30	·892 0731	·387 5041	323	29	·892 7340	·387 9084	312	28
12	·932 9552	·387 1059	365	30	·934 9315	·387 5393	352	29	·937 1757	·387 9424	340	28
13	·978 1615	·387 1454	395	29	·981 8715	·387 5774	381	28	·985 8495	·387 9792	368	27
14	0·027 3887	·387 1879	424	29	0·032 9863	·387 6183	409	28	0·038 8519	·388 0187	395	27
15	·080 7353	·387 2332	453	28	·088 3780	·387 6620	437	27	·096 2888	·388 0609	422	26
16	·138 3092	·387 2814	482	28	·148 1586	·387 7084	464	27	·158 2763	·388 1057	448	26
17	·200 2283	·387 3323	509	27	·212 4505	·387 7576	491	26	·224 9410	·388 1531	474	25
18	·266 6208	·387 3859	536	26	·281 3865	·387 8093	517	25	·296 4208	·388 2031	499	25
19	·337 6258	·387 4422	563	26	·355 1112	·387 8635	543	25	·372 8653	·388 2555	524	24
20	·413 3944	·387 5010	588	25	·433 7811	·387 9203	567	24	·454 4367	·388 3103	548	23
21	·494 0896	·387 5624	613	24	·517 5657	·387 9794	592	23	·541 3106	·388 3674	571	23
22	·579 8882	·387 6261	638	24	·606 6479	·388 0409	615	23	·633 6767	·388 4268	594	22
23	·670 9806	·387 6923	661	23	·701 2256	·388 1047	638	22	·731 7398	·388 4883	616	21
24	·767 5727	·387 7607	684	22	·801 5122	·388 1706	660	21	·835 7212	·388 5520	637	20
25	·869 8863	·387 8312	706	21	·907 7380	·388 2387	681	20	·945 8594	·388 6177	657	20
26	·978 1616	·387 9039	727	20	1·020 1512	·388 3088	701	19	1·062 4115	·388 6854	677	19
27	1·092 6538	·387 9786	747	19	·139 0194	·388 3808	720	18	·185 6549	·388 7550	696	17
28	·213 6439	·388 0552	766	18	·264 6312	·388 4547	739	18	·315 8886	·388 8263	713	17
29	·341 4310	·388 1337	784	17	·397 2979	·388 5303	756	17	·453 4351	·388 8993	730	16
30	·476 3386	·388 2138	802	16	·537 3550	·388 6077	773	16	·598 6420	·388 9740	747	15
31	·618 7157	·388 2956	818	15	·685 1648	·388 6865	789	15	·751 8846	·389 0501	762	14
32	·768 9393	·388 3790	833	14	·841 1182	·388 7669	804	14	·913 5680	·389 1277	776	13
33	·927 4164	·388 4638	848	13	2·005 6375	·388 8487	818	13	2·084 1297	·389 2067	789	12
34	2·094 5869	·388 5499	861	12	·179 1790	·388 9317	830	12	·264 0426	·389 2868	802	11
35	·270 9268	·388 6372	873	11	·362 2366	·389 0159	842	11	·453 8181	·389 3682	813	10
36	·456 9512	·388 7257	885	10	·555 3447	·389 1012	853	10	·654 0100	·389 4505	824	9
37	·653 2186	·388 8151	895	9	·759 0825	·389 1875	863	9	·865 2184	·389 5338	833	8
38	·860 3344	·388 9055	904	8	·974 0781	·389 2746	872	7	3·088 0940	·389 6179	841	7
39	3·078 9562	·388 9967	912	7	3·201 0137	·389 3625	879	7	·323 3436	·389 7028	849	6
40	·309 7991	·389 0885	918	6	·440 6310	·389 4511	886	5	·571 7357	·389 7883	855	5
41	·553 6412	·389 1809	924	5	·693 7374	·389 5402	891	4	·834 1065	·389 8744	860	4
42	·811 3309	·389 2738	929	3	·961 2127	·389 6298	896	3	4·111 3678	·389 9608	865	3
43	4·083 7948	·389 3670	932	2	4·244 0178	·389 7197	899	2	·404 5148	·390 0476	868	2
44	·372 0436	·389 4604	934	1	·543 2028	·389 8098	901	1	·714 6355	·390 1346	870	1
45	·677 1878	·389 5540	936		·859 9176	·389 9000	902		5·042 9213	·390 2217	871	

TABLE LIV—(*continued*).

ϕ^2	$r=30$				$r=31$				$r=32$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·656 9109	0·388 1099	15		$\bar{1}$·649 9073	0·388 4679	14		$\bar{1}$·643 1226	0·388 8034	14	
1	·658 8309	·388 1113	44	29	·651 8935	·388 4694	43	28	·645 1748	·388 8048	41	28
2	·664 5945	·388 1157	73	29	·657 8555	·388 4736	71	28	·651 3354	·388 8089	69	27
3	·674 2126	·388 1231	103	29	·667 8048	·388 4808	99	28	·661 6158	·388 8158	96	27
4	·687 7031	·388 1333	132	29	·681 7598	·388 4906	127	28	·676 0352	·388 8254	123	27
5	·705 0914	·388 1465	161	29	·699 7466	·388 5034	155	28	·694 6208	·388 8378	151	27
6	·726 4101	·388 1625	189	29	·721 7993	·388 5189	183	28	·717 4073	·388 8528	177	27
7	·751 6995	·388 1815	218	29	·747 9592	·388 5372	211	28	·744 4379	·388 8706	204	27
8	·781 0077	·388 2032	246	28	·778 2762	·388 5583	238	27	·775 7637	·388 8910	231	26
9	·814 3906	·388 2278	274	28	·812 8080	·388 5822	265	27	·811 4444	·388 9140	257	26
10	·851 9121	·388 2552	302	28	·851 6207	·388 6086	292	27	·851 5483	·388 9397	283	26
11	·893 6447	·388 2854	329	27	·894 7893	·388 6378	318	26	·896 1530	·388 9680	308	26
12	·939 6698	·388 3183	356	27	·942 3977	·388 6696	344	26	·945 3449	·388 9988	333	25
13	·990 0775	·388 3538	382	26	·994 5394	·388 7041	370	26	·999 2205	·389 0322	358	25
14	0·044 9676	·388 3920	408	26	0·051 3173	·388 7410	395	25	0·057 8864	·389 0680	383	24
15	·104 4498	·388 4328	433	25	·112 8450	·388 7805	419	25	·121 4595	·389 1062	406	24
16	·168 6443	·388 4762	458	25	·179 2464	·388 8225	444	24	·190 0681	·389 1469	430	23
17	·237 6820	·388 5220	483	24	·250 6573	·388 8668	467	23	·263 8522	·389 1899	453	23
18	·311 7056	·388 5703	507	24	·327 2249	·388 9136	490	23	·342 9638	·389 2351	475	22
19	·390 8701	·388 6209	530	23	·409 1093	·388 9626	513	22	·427 5684	·389 2826	497	22
20	·475 3432	·388 6739	552	22	·496 4842	·389 0138	534	22	·517 8452	·389 3323	518	21
21	·565 3065	·388 7291	574	22	·589 5372	·389 0673	556	21	·613 9879	·389 3840	538	20
22	·660 9566	·388 7865	595	21	·688 4714	·389 1228	576	20	·716 2063	·389 4379	558	20
23	·762 5053	·388 8460	616	20	·793 5058	·389 1804	596	20	·824 7266	·389 4937	577	19
24	·870 1816	·388 9076	635	20	·904 8771	·389 2400	615	19	·939 7929	·389 5514	596	19
25	·984 2323	·388 9711	654	19	1·022 8405	·389 3015	633	18	1·061 6692	·389 6109	613	18
26	1·104 9235	·389 0366	672	18	·147 6710	·389 3648	650	18	·190 6391	·389 6723	630	17
27	·232 5423	·389 1038	690	17	·279 6653	·389 4299	667	17	·327 0091	·389 7353	646	16
28	·367 3981	·389 1727	706	16	·419 1433	·389 4966	683	16	·471 1094	·389 8000	662	15
29	·509 8245	·389 2433	722	16	·566 4498	·389 5649	698	15	·623 2962	·389 8661	677	15
30	·660 1814	·389 3155	736	15	·721 9568	·389 6348	713	14	·783 9534	·389 9338	690	14
31	·818·8570	·389 3891	750	14	·886 0657	·389 7060	726	13	·953 4956	·390 0028	703	13
32	·986 2707	·389 4642	763	13	2·059 2096	·389 7786	738	13	2·132 3701	·390 0732	715	12
33	2·162 8749	·389 5405	775	12	·241 8567	·389 8525	750	12	·321 0601	·390 1447	727	11
34	·349 1593	·389 6180	786	11	·434 5127	·389 9275	761	11	·520 0879	·390 2174	737	10
35	·545 6529	·389 6966	796	10	·637 7246	·390 0035	770	10	·730 0183	·390 2911	746	9
36	·752 9288	·389 7762	805	9	·852 0847	·390 0806	779	9	·951 4628	·390 3657	755	9
37	·971 6080	·389 8567	813	8	3·078 2349	·390 1585	787	8	3·185 0841	·390 4412	763	8
38	3·202 3639	·389 9380	820	7	·316 8712	·390 2372	794	7	·431 6009	·390 5174	769	7
39	·445 9277	·390 0201	827	6	·568 7494	·390 3166	800	6	·691 7938	·390 5944	775	6
40	·703 0947	·390 1028	832	5	·834 6915	·390 3966	805	5	·966 5111	·390 6719	780	5
41	·974 7302	·390 1859	836	4	4·115 5919	·390 4771	809	4	4·256 6765	·390 7498	784	4
42	4·261 7776	·390 2695	839	3	·412 4256	·390 5580	812	3	·563 2967	·390 8282	787	3
43	·565 2668	·390 3534	841	2	·726 2571	·390 6392	814	2	·887 4707	·390 9069	789	2
44	·886 3234	·390 4375	842	1	5·058 2499	·390 7206	815	1	5·230 3999	·390 9857	789	1
45	5·226 1804	·390 5217			·409 6782	·390 8021			·593 3997	·391 0646		

TABLE LIV—(*continued*).

ϕ°	$r=33$				$r=34$				$r=35$			
	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·636 5434	0·389 1183			$\bar{1}$·630 1576	0·389 4146			$\bar{1}$·623 9542	0·389 6937		
1	·638 6617	·389 1197	13	27	·632 3421	·389 4159	13	26	·626 2048	·389 6949	13	25
2	·645 0208	·389 1237	40	27	·638 8996	·389 4198	39	26	·632 9608	·389 6987	38	25
3	·655 6323	·389 1304	67	26	·649 8424	·389 4262	65	26	·644 2348	·389 7050	63	25
4	·670 5163	·389 1397	93	26	·665 1908	·389 4353	91	26	·660 0478	·389 7138	88	25
5	·689 7005	·389 1516	120	26	·684 9738	·389 4469	116	26	·680 4295	·389 7251	113	25
6	·713 2210	·389 1662	146	26	·709 2283	·389 4611	142	25	·705 4180	·389 7389	138	25
7	·741 1222	·389 1835	172	26	·738 0001	·389 4778	167	25	·735 0604	·389 7551	162	24
8	·773 4568	·389 2033	198	26	·771 3436	·389 4970	192	25	·769 4129	·389 7738	187	24
9	·810 2865	·389 2256	224	25	·809 3225	·389 5187	217	25	·808 5407	·389 7948	211	24
10	·851 6818	·389 2505	249	25	·852 0090	·389 5429	242	24	·852 5188	·389 8183	235	24
11	·897 7225	·389 2780	274	25	·899 4858	·389 5695	266	24	·901 4317	·389 8442	259	23
12	·948 4980	·389 3078	299	24	·951 8449	·389 5985	290	24	·955 3744	·389 8724	282	23
13	0·004 1077	·389 3402	323	24	0·009 1887	·389 6299	314	23	0·014 4525	·389 9029	305	23
14	·064 6615	·389 3749	347	24	·071 6306	·389 6636	337	23	·078 7824	·389 9356	328	22
15	·130 2802	·389 4120	371	23	·139 2949	·389 6996	360	23	·148 4925	·389 9706	350	22
16	·201 0960	·389 4514	394	23	·212 3180	·389 7379	382	22	·223 7229	·390 0078	372	21
17	·277 2533	·389 4931	417	22	·290 8487	·389 7783	405	21	·304 6270	·390 0471	393	21
18	·358 9091	·389 5370	439	22	·375 0487	·389 8209	426	21	·391 3713	·390 0884	414	20
19	·446 2340	·389 5830	461	21	·465 0938	·389 8656	447	20	·484 1369	·390 1319	434	20
20	·539 4127	·389 6312	482	20	·561 1746	·389 9123	467	20	·583 1198	·390 1773	454	19
21	·638 6453	·389 6814	502	20	·663 4971	·389 9611	487	19	·688 5323	·390 2246	473	19
22	·744 1480	·389 7336	522	19	·772 2842	·390 0117	507	19	·800 6039	·390 2738	492	18
23	·856 1542	·389 7877	541	19	·887 7765	·390 0643	525	18	·919 5823	·390 3248	510	17
24	·974 9160	·389 8437	560	18	1·010 2336	·390 1186	543	17	1·045 7350	·390 3776	528	17
25	1·100 7050	·389 9014	578	17	·139 9357	·390 1746	561	17	·179 3502	·390 4321	545	16
26	·233 8145	·389 9609	595	17	·277 1848	·390 2324	577	16	·320 7390	·390 4881	561	16
27	·374 5602	·390 0220	611	16	·422 3065	·390 2917	593	15	·470 2366	·390 5458	576	15
28	·523 2830	·390 0847	627	15	·575 6518	·390 3525	609	14	·628 2047	·390 6049	591	14
29	·680 3501	·390 1489	642	14	·737 5995	·390 4148	623	14	·795 0329	·390 6654	605	13
30	·846 1578	·390 2145	656	13	·908 5576	·390 4785	637	13	·971 1417	·390 7273	619	13
31	2·021 1335	·390 2815	669	13	2·088 9669	·390 5435	650	12	2·156 9847	·390 7904	631	12
32	·205 7387	·390 3497	682	12	·279 3029	·390 6097	662	11	·353 0516	·390 8547	643	11
33	·400 4717	·390 4190	693	11	·480 0791	·390 6770	673	11	·559 8711	·390 9201	654	10
34	·605 8714	·390 4895	705	10	·691 8509	·390 7454	684	10	·778 0150	·390 9865	664	9
35	·822 5204	·390 5610	715	9	·915 2186	·390 8148	694	9	3·008 1016	·391 0539	674	9
36	3·051 0494	·390 6333	724	8	3·150 8322	·390 8850	703	8	·250 8000	·391 1222	682	8
37	·292 1420	·390 7065	732	7	·399 3963	·390 9561	711	7	·506 8356	·391 1912	690	7
38	·546 5396	·390 7805	739	6	·661 6747	·391 0278	718	6	·776 9950	·391 2609	697	6
39	·815 0471	·390 8551	746	6	·938 4971	·391 1002	724	5	4·062 1324	·391 3312	703	5
40	4·098 5399	·390 9302	751	5	4·230 7654	·391 1732	729	4	·363 1764	·391 4021	709	4
41	·397 9704	·391 0058	756	4	·539 4613	·391 2466	734	4	·681 1377	·391 4734	713	4
42	·714 3773	·391 0818	760	3	·865 6549	·391 3203	738	3	5·017 1183	·391 5450	716	3
43	5·048 3939	·391 1581	763	2	5·210 5144	·391 3943	740	2	·372 3207	·391 6169	719	2
44	·402 7596	·391 2345	765	1	·575 3166	·391 4686	742	1	·748 0596	·391 6890	721	1
45	·777 3311	·391 3111	766		·961 4600	·391 5429	743		6·145 7749	·391 7612	722	

TABLE LIV—(*continued*).

$\phi°$	$r=36$				$r=37$				$r=38$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·617 9231	0·389 9572	12		$\bar{1}$·612 0550	·0390 2063	12		$\bar{1}$·606 3413	0.390 4421	12	
1	·620 2399	·389 9584	37	24	·614 4378	·390 2074	36	24	·608 7902	·390 4433	35	23
2	·627 1943	·389 9620	61	24	·621 5908	·390 2110	60	24	·616 1417	·390 4468	58	23
3	·638 7995	·389 9682	85	24	·633 5272	·390 2170	83	24	·628 4093	·390 4526	81	23
4	·655 0771	·389 9767	110	24	·650 2694	·390 2253	107	24	·645 6161	·390 4607	104	23
5	·676 0575	·389 9877	134	24	·671 8485	·390 2360	130	23	·667 7940	·390 4711	127	23
6	·701 7800	·390 0011	158	24	·698 3051	·390 2490	154	23	·694 9847	·390 4837	149	23
7	·732 2932	·390 0168	182	24	·729 6890	·390 2643	177	23	·727 2393	·390 4987	172	23
8	·767 6546	·390 0350	205	24	·766 0594	·390 2820	200	23	·764 6187	·390 5159	194	22
9	·807 9316	·390 0555	228	23	·807 4855	·390 3020	222	23	·807 1940	·390 5353	216	22
10	·853 2010	·390 0783	251	23	·854 0464	·390 3242	245	22	·855 0464	·390 5569	238	22
11	·903 5502	·390 1035	274	23	·905 8318	·390 3486	267	22	·908 2680	·390 5808	260	22
12	·959 0766	·390 1309	296	22	·962 9420	·390 3753	288	22	·966 9620	·390 6067	281	21
13	0·019 8889	·390 1605	318	22	0·025 4886	·390 4041	310	21	0·031 2429	·390 6348	302	21
14	·086 1070	·390 1924	340	22	·093 5948	·390 4351	331	21	·101 2374	·390 6650	322	20
15	·157 8628	·390 2264	361	21	·167 3965	·390 4682	352	21	·177 0849	·390 6972	342	20
16	·235 3007	·390 2625	382	21	·247 0418	·390 5034	372	20	·258 9378	·390 7314	362	20
17	·318 5782	·390 3007	402	20	·332 6929	·390 5405	391	20	·346 9624	·390 7676	381	19
18	·407 8669	·390 3409	422	20	·424 5260	·390 5797	411	19	·441 3400	·390 8057	400	19
19	·503 3529	·390 3832	441	19	·522 7325	·390 6208	430	19	·542 2671	·390 8457	418	18
20	·605 2380	·390 4273	460	19	·627 5199	·390 6637	448	18	·649 9569	·390 8876	436	18
21	·713 7407	·390 4733	478	18	·739 1128	·390 7085	466	18	·764 6400	·390 9312	453	17
22	·829 0968	·390 5212	496	18	·857 7536	·390 7551	483	17	·886 5655	·390 9765	470	17
23	·951 5615	·390 5708	513	17	·983 7045	·390 8033	499	16	1·016 0028	·391 0235	486	16
24	1·081 4096	·390 6221	529	16	1·117 2483	·390 8532	515	16	·153 2423	·391 0721	502	16
25	·218 9381	·390 6750	545	16	·258 6900	·390 9048	530	15	·298 5974	·391 1223	517	15
26	·364 4667	·390 7296	560	15	·408 3585	·390 9578	545	15	·452 4059	·391 1739	531	14
27	·518 3405	·390 7856	575	14	·566 6085	·391 0123	559	14	·615 0321	·391 2270	545	14
28	·680 9313	·390 8431	588	14	·733 8222	·391 0682	573	13	·786 8688	·391 2815	557	13
29	·852 6402	·390 9019	601	13	·910 4119	·391 1255	585	13	·968 3394	·391 3372	570	12
30	2·033 8997	·390 9621	614	12	2·096 8223	·391 1840	597	12	2·159 9006	·391 3942	581	12
31	·225 1766	·391 0234	625	12	·293 5330	·391 2437	608	11	·362 0454	·391 4523	592	11
32	·426 9744	·391 0859	636	11	·501 0620	·391 3046	619	10	·575 3055	·391 5115	603	10
33	·639 8374	·391 1495	646	10	·719 9685	·391 3664	628	9	·800 2556	·391 5718	612	9
34	·864 3536	·391 2141	655	9	·950 8571	·391 4293	637	9	3·037 5167	·391 6330	621	9
35	3·101 1591	·391 2796	664	8	3·194 3816	·391 4930	646	8	·287 7604	·391 6950	629	8
36	·350 9424	·391 3460	671	8	·451 2499	·391 5576	653	7	·551 7138	·391 7579	636	7
37	·614 4497	·391 4131	678	7	·722 2290	·391 6229	659	6	·830 1647	·391 8215	642	6
38	·892 4902	·391 4809	684	6	4·008 1507	·391 6888	665	6	4·123 9677	·391 8857	648	6
39	4·185 9427	·391 5492	689	5	·309 9183	·391 7553	670	5	·434 0507	·391 9505	653	5
40	·495 7624	·391 6181	693	4	·628 5140	·391 8224	674	4	·761 4223	·392 0157	657	4
41	·822 9894	·391 6874	697	3	·965 0066	·391 8898	678	3	5·107 1807	·392 0814	660	3
42	5·168 7569	·391 7571	699	3	5·320 5613	·391 9576	680	3	·472 5225	·392 1474	662	2
43	·534 3023	·391 8270	701	2	·696 4499	·392 0256	682	2	·858 7544	·392 2136	664	2
44	·920 9782	·391 8971	702	1	6·094 0627	·392 0938	683	1	6·267 3043	·392 2800	665	1
45	6·330 2655	·391 9673			·514 9222	·392 1621			·699 7361	·392 3465		

TABLE LIV—(*continued*).

ϕ°	$r=39$				$r=40$				$r=41$			
	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·600 7740	0·390 6658			$\bar{1}$·595 3459	0·390 8782			$\bar{1}$·590 0501	0·391 0801		
1	603 2891	·390 6669	11	23	·597 9271	·390 8793	11	22	·592 6975	·391 0812	11	21
2	·610 8391	·390 6703	34	23	·605 6756	·390 8826	33	22	·600 6445	·391 0844	32	21
3	·623 4380	·390 6760	56	23	·618 6058	·390 8881	55	22	·613 9059	·391 0898	54	21
4	·641 1093	·390 6838	79	22	·636 7416	·390 8958	77	22	·632 5064	·391 0973	75	21
5	·663 8860	·390 6940	101	22	·660 1171	·390 9057	99	22	·656 4807	·391 1069	96	21
6	·691 8107	·390 7063	124	22	·688 7760	·390 9177	120	22	·685 8737	·391 1187	118	21
7	·724 9361	·390 7209	146	22	·722 7721	·390 9319	142	21	·720 7406	·391 1325	139	21
8	·763 3246	·390 7377	168	22	·762 1697	·390 9483	163	21	·761 1472	·391 1485	159	21
9	·807 0490	·390 7566	189	22	·807 0434	·390 9667	185	21	·807 1702	·391 1665	180	20
10	·856 1930	·390 7777	211	21	·857 4789	·390 9873	206	21	·858 8973	·391 1865	200	20
11	·910 8510	·390 8009	232	21	·913 5732	·391 0099	226	20	·916 4280	·391 2086	221	20
12	·971 1287	·390 8262	253	21	·975 4348	·391 0346	247	20	·979 8734	·391 2327	241	20
13	0·037 1440	·390 8535	274	20	0·043 1845	·391 0612	267	20	0·049 3576	·391 2587	260	19
14	·109 0268	·390 8829	294	20	·116 9556	·391 0899	287	19	·125 0171	·391 2867	280	19
15	·186 9202	·390 9143	314	20	·196 8949	·391 1205	306	19	·207 0023	·391 3165	299	19
16	·270 9806	·390 9477	334	19	·283 1630	·391 1530	325	19	·295 4780	·391 3483	317	18
17	·361 3789	·390 9829	353	19	·375 9350	·391 1874	344	18	·390 6238	·391 3818	335	18
18	·458 3010	·391 0201	371	18	·475 4017	·391 2236	362	18	·492 6351	·391 4172	353	17
19	·561 9488	·391 0591	390	18	·581 7701	·391 2616	380	17	·601 7243	·391 4542	371	17
20	·672 5410	·391 0998	408	17	·695 2648	·391 3014	397	17	·718 1215	·391 4930	388	16
21	·790 3143	·391 1423	425	17	·816 1285	·391 3428	414	16	·842 0755	·391 5334	404	16
22	·915 5247	·391 1865	442	16	·944 6237	·391 3859	431	16	·973 8556	·391 5754	420	16
23	1·048 4484	·391 2323	458	16	1·081 0339	·391 4305	447	15	1·113 7524	·391 6190	436	15
24	·189 3837	·391 2796	474	15	·225 6650	·391 4767	462	15	·262 0794	·391 6640	451	14
25	·338 6522	·391 3285	489	14	·378 8470	·391 5243	477	14	·419 1750	·391 7105	465	14
26	·496 6007	·391 3788	503	14	·540 9357	·391 5734	491	14	·585 4038	·391 7584	479	13
27	·663 6033	·391 4306	517	13	·712 3146	·391 6238	504	13	·761 1593	·391 8076	492	13
28	·840 0630	·391 4836	530	13	·893 3974	·391 6756	517	12	·946 8652	·391 8581	505	12
29	2·026 4145	·391 5379	543	12	2·084 6300	·391 7285	530	12	2·142 9789	·391 9097	517	11
30	·223 1268	·391 5935	555	11	·286 4933	·391 7827	541	11	·349 9933	·391 9626	528	11
31	·430 7056	·391 6501	566	11	·499 5062	·391 8379	552	10	·568 4405	·392 0164	539	10
32	·649 6970	·391 7078	577	10	·724 2290	·391 8942	563	10	·798 8947	·392 0713	549	9
33	·880 6908	·391 7665	587	9	·961 2666	·391 9514	572	9	3·041 9761	·392 1272	558	9
34	3·124 3244	·391 8261	596	8	3·211 2729	·392 0095	581	8	·298 3551	·392 1839	567	8
35	·381 2874	·391 8866	605	8	·474 9551	·392 0685	590	8	·568 7567	·392 2414	575	8
36	·652 3259	·391 9479	612	7	·753 0789	·392 1282	597	7	·853 9658	·392 2997	583	7
37	·938 2488	·392 0098	619	6	4·046 4738	·392 1886	604	6	4·154 8329	·392 3586	589	6
38	4·239 9332	·392 0724	626	5	·356 0397	·392 2496	610	5	·472 2803	·392 4181	595	5
39	·558 3315	·392 1355	631	5	·682 7535	·392 3112	615	5	·807 3096	·392 4782	600	5
40	·894 4793	·392 1991	636	4	5·027 6774	·392 3732	620	4	5·161 0098	·392 5386	605	4
41	5·249 5035	·392 2631	640	3	·391 9676	·392 4355	624	3	·534 5660	·392 5995	609	3
42	·624 6328	·392 3274	643	2	·776 8842	·392 4982	627	2	·929 2701	·392 6607	612	2
43	6·021 2079	·392 3919	645	2	6·183 8028	·392 5612	629	1	6·346 5322	·392 7221	614	1
44	·440 6950	·392 4566	647	1	·614 2272	·392 6242	630	0	·787 8938	·392 7836	615	1
45	·884 6991	·392 5213	648		7·069 8037	·392 6874	631		7·255 0429	·392 8452	616	

TABLE LIV—(*continued*).

ϕ°	$r=42$				$r=43$				$r=44$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·584 8804	0·391 2724			$\bar{1}$·579 8310	0·391 4556			$\bar{1}$·574 8962	0·391 6305		
1	·587 5939	·391 2734	10	21	·582 6106	·391 4566	10	21	·577 7420	·391 6315	10	20
2	·595 7394	·391 2766	31	21	·590 9546	·391 4597	31	20	·586 2845	·391 6345	30	20
3	·609 3321	·391 2818	52	21	·604 8785	·391 4648	51	20	·600 5397	·391 6395	50	20
4	·628 3972	·391 2891	73	21	·624 4083	·391 4720	72	20	·620 5341	·391 6465	70	20
5	·652 9704	·391 2985	94	21	·649 5803	·391 4812	92	20	·646 3049	·391 6554	90	20
6	·683 0975	·391 3100	115	21	·680 4416	·391 4924	112	20	·677 9004	·391 6664	109	20
7	·718 8352	·391 3235	135	20	·717 0501	·391 5056	132	20	·715 3798	·391 6793	129	20
8	·760 2510	·391 3391	156	20	·759 4750	·391 5208	152	20	·758 8138	·391 6942	149	19
9	·807 4232	·391 3567	176	20	·807 7965	·391 5380	172	19	·808 2846	·391 7109	168	19
10	·860 4419	·391 3763	196	20	·862 1068	·391 5571	191	19	·863 8866	·391 7296	187	19
11	·919 4089	·391 3978	216	19	·922 5103	·391 5781	210	19	·925 7265	·391 7502	206	19
12	·984 4383	·391 4213	235	19	·989 1236	·391 6011	230	19	·993 9238	·391 7726	224	18
13	0·055 6569	·391 4467	254	19	0·062 0767	·391 6259	248	19	0·068 6113	·391 7769	243	18
14	·133 2048	·391 4740	273	19	·141 5130	·391 6526	267	18	·149 9361	·391 8229	261	18
15	·217 2361	·391 5032	292	18	·227 5903	·391 6810	285	18	·238 0595	·391 8508	278	17
16	·307 9195	·391 5341	310	18	·320 4814	·391 7113	303	17	·333 1584	·391 8803	296	17
17	·405 4390	·391 5669	328	17	·420 3748	·391 7433	320	17	·435 4256	·391 9116	313	17
18	·509 9950	·391 6014	345	17	·527 4755	·391 7770	337	17	·545 0710	·391 9445	329	16
19	·621 8050	·391 6376	362	16	·642 0063	·391 8123	353	16	·662 3227	·391 9791	346	16
20	·741 1047	·391 6754	378	16	·764 2086	·391 8493	370	16	·787 4277	·392 0152	361	15
21	·868 1492	·391 7149	395	16	·894 3438	·391 8878	385	15	·920 6532	·392 0528	377	15
22	1·003 2143	·391 7559	410	15	1·032 6937	·391 9279	401	15	1·062 2883	·392 0920	391	14
23	·146 5976	·391 7984	425	15	·179 5637	·391 9694	415	14	·212 6450	·392 1326	406	14
24	·298 6206	·391 8424	440	14	·335 2826	·392 0124	430	14	·372 0600	·392 1746	420	13
25	·459 6298	·391 8878	454	13	·500 2054	·392 0567	443	13	·540 8966	·392 2179	433	13
26	·629 9989	·391 9345	467	13	·674 7149	·392 1024	457	13	·719 5463	·392 2625	446	12
27	·810 1308	·391 9826	480	12	·859 2234	·392 1493	469	12	·908 4315	·392 3084	459	12
28	2·000 4600	·392 0318	493	12	2·054 1759	·392 1974	481	12	2·108 0073	·392 3554	470	11
29	·201 4548	·392 0823	504	11	·260 0519	·392 2467	493	11	·318 7645	·392 4035	482	11
30	·413 6204	·392 1338	516	10	·477 3687	·392 2970	504	10	·541 2327	·392 4528	492	10
31	·637 5019	·392 1864	526	10	·706 6845	·392 3484	514	10	·775 9829	·392 5030	502	9
32	·873 6876	·392 2400	536	9	·948 6018	·392 4007	523	9	3·023 6317	·392 5541	512	9
33	3·122 8129	·392 2945	545	9	3·203 7711	·392 4540	533	8	·284 8450	·392 6062	520	8
34	·385 5647	·392 3499	554	8	·472 8957	·392 5081	541	8	·560 3426	·392 6590	528	8
35	·662 6857	·392 4060	562	7	·756 7363	·392 5629	548	7	·850 9027	·392 7126	536	7
36	·954 9802	·392 4629	569	7	4·056 1162	·392 6185	556	6	4·157 3682	·392 7669	543	6
37	4·263 3195	·392 5204	575	6	·371 9277	·392 6747	562	6	·480 6520	·392 8218	549	6
38	·588 6485	·392 5785	581	5	·705 1384	·392 7314	567	5	·821 7444	·392 8773	555	5
39	·931 9935	·392 6371	586	4	5·056 7991	·392 7886	572	4	5·181 7208	·392 9332	559	4
40	5·294 4700	·392 6962	590	4	·428 0520	·392 8463	577	3	·561 7502	·392 9896	564	4
41	·677 2923	·392 7556	594	3	·820 1404	·392 9044	580	3	·963 1049	·393 0463	567	3
42	6·081 7839	·392 8153	597	2	6·234 4197	·392 9627	583	2	6·387 1719	·393 1033	570	2
43	·509 3896	·392 8752	599	1	·672 3690	·393 0212	585	2	·835 4649	·393 1605	572	1
44	·961 6887	·392 9353	601	1	7·135 6055	·393 0799	587	1	7·309 6389	·393 2178	573	1
45	7·440 4103	·392 9954	601		·625 8999	·393 1386	587		·811 5060	·393 2752	574	

TABLE LIV—(*continued*).

ϕ°	$r=45$				$r=46$				$r=47$			
	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2	$\log F(r, \nu)$	$\log H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·570 0711	0·391 7975	10		$\bar{1}$·565 3509	0·391 9572	10		$\bar{1}$·560 7311	0·392 1100	9	
1	·572 9830	·391 7984	29	20	·568 3289	·391 9581	29	19	·563 7753	·392 1110	28	19
2	·581 7241	·391 8014	49	20	·577 2685	·391 9610	48	19	·572 9134	·392 1138	47	19
3	·596 3106	·391 8063	68	19	·592 1863	·391 9658	67	19	·588 1625	·392 1185	66	19
4	·616 7696	·391 8131	88	19	·613 1100	·391 9725	86	19	·609 5508	·392 1250	84	19
5	·643 1393	·391 8219	107	19	·640 0785	·391 9811	105	19	·637 1182	·392 1334	103	18
6	·675 4689	·391 8326	126	19	·673 1423	·391 9915	124	19	·670 9162	·392 1437	121	18
7	·713 8192	·391 8452	145	19	·712 3634	·392 0039	142	19	·711 0082	·392 1557	139	18
8	·758 2623	·391 8598	164	19	·757 8157	·392 0181	161	18	·757 4696	·392 1697	157	18
9	·808 8824	·391 8762	183	19	·809 5852	·392 0342	179	18	·810 3885	·392 1854	175	18
10	·865 7761	·391 8944	201	18	·867 7706	·392 0520	197	18	·869 8656	·392 2029	193	18
11	·929 0524	·391 9145	219	18	·932 4834	·392 0717	215	18	·936 0149	·392 2221	210	17
12	·998 8337	·391 9365	237	18	0·003 8487	·392 0932	232	18	0·008 9642	·392 2431	227	17
13	0·075 2558	·391 9602	255	18	·082 0054	·392 1164	249	17	·088 8555	·392 2658	244	17
14	·158 4691	·391 9857	272	17	·167 1071	·392 1413	266	17	·175 8458	·392 2902	261	17
15	·248 6386	·392 0129	289	17	·259 3228	·392 1679	283	17	·270 1076	·392 3163	277	16
16	·345 9453	·392 0418	306	17	·358 8372	·392 1962	299	16	·371 8299	·392 3440	293	16
17	·450 5863	·392 0724	322	16	·465 8522	·392 2261	315	16	·481 2188	·392 3732	308	16
18	·562 7765	·392 1046	338	16	·580 5873	·392 2576	331	16	·598 4987	·392 4041	323	15
19	·682 7492	·392 1383	353	15	·703 2808	·392 2906	346	15	·723 9132	·392 4364	338	15
20	·810 7568	·392 1737	368	15	·834 1911	·392 3252	360	15	·857 7263	·392 4702	353	14
21	·947 0729	·392 2105	383	15	·973 5979	·392 3612	375	14	1·000 2236	·392 5055	367	14
22	1·091 9931	·392 2488	397	14	1·121 8031	·392 3987	388	14	·151 7141	·392 5421	380	13
23	·245 8365	·392 2885	411	14	·279 1333	·392 4375	402	13	·312 5311	·392 5801	393	13
24	·408 9476	·392 3295	424	13	·445 9406	·392 4776	414	13	·483 0345	·392 6194	406	13
25	·581 6979	·392 3719	436	13	·622 6047	·392 5191	427	12	·663 6124	·392 6600	418	12
26	·764 4880	·392 4155	448	12	·809 5352	·392 5618	438	12	·854 6834	·392 7018	429	12
27	·957 7499	·392 4603	460	12	2·007 1738	·392 6056	450	11	2·056 6988	·392 7447	440	11
28	2·161 9490	·392 5063	471	11	·215 9964	·392 6506	461	11	·270 1448	·392 7887	451	11
29	·377 5876	·392 5534	481	10	·436 5163	·392 6967	471	10	·495 5462	·392 8338	461	10
30	·605 2071	·392 6015	491	10	·669 2873	·392 7437	480	10	·733 4685	·392 8799	470	9
31	·845 3918	·392 6506	500	9	·914 9064	·392 7918	489	9	·984 5222	·392 9269	479	9
32	3·098 7723	·392 7006	509	9	3·174 0186	·392 8407	498	8	3·249 3662	·392 9748	487	8
33	·366 0297	·392 7515	517	8	·447 3201	·392 8905	505	8	·528 7119	·393 0235	495	8
34	·647 9002	·392 8032	524	7	·735 5636	·392 9410	513	7	·823 3284	·393 0729	502	7
35	·945 1800	·392 8556	531	7	4·039 5631	·392 9923	519	6	4·134 0476	·393 1231	508	6
36	4·258 7310	·392 9087	537	6	·360 1998	·393 0442	525	6	·461 7700	·393 1740	514	6
37	·589 4872	·392 9624	542	5	·698 4283	·393 0967	530	5	·807 4710	·393 2254	519	5
38	·938 4614	·393 0166	547	5	5·055 2844	·393 1498	535	5	5·172 2090	·393 2773	524	5
39	5·306 7534	·393 0713	551	4	·431 8925	·393 2033	539	4	·557 1330	·393 3297	528	4
40	·695 5595	·393 1264	555	3	·829 4749	·393 2572	543	3	·963 4920	·393 3824	531	3
41	6·106 1805	·393 1818	557	3	6·249 3623	·393 3115	545	3	6·392 6458	·393 4355	534	3
42	·540 0352	·393 2376	559	2	·693 0048	·393 3660	547	2	·846 0761	·393 4889	536	2
43	·998 6720	·393 2935	561	1	7·161 9854	·393 4207	548	1	7·325 4006	·393 5424	537	1
44	·483 7836	·393 3496	561	1	·658 0347	·393 4755	549	1	·832 3876	·393 5961	537	0
45	·997 2234	·393 4057			8·183 0474	·393 5304			8·368 9732	·393 6498		

TABLE LIV—(*continued*).

$\phi°$	$r=48$				$r=49$				$r=50$			
	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2	log $F(r, \nu)$	log $H(r, \nu)$	Δ	Δ^2
0	$\bar{1}$·556 2075	0·392 2565	9		$\bar{1}$·551 7763	0·392 3969	9		$\bar{1}$·547 4336	0·392 5316	9	
1	·559 3178	·392 2574	28	18	·554 9527	·392 3978	27	18	·550 6762	·392 5325	26	18
2	·568 6545	·392 2601	46	18	·564 4879	·392 4005	45	18	·560 4099	·392 5352	44	18
3	·584 2348	·392 2647	64	18	·580 3995	·392 4050	63	18	·576 6529	·392 5396	62	18
4	·606 0878	·392 2711	82	18	·602 7172	·392 4113	81	18	·599 4352	·932 5457	79	17
5	·634 2541	·392 2794	100	18	·631 4824	·392 4193	98	18	·628 7993	·392 5536	96	17
6	·668 7863	·392 2894	118	18	·666 7488	·392 4291	116	18	·664 7999	·392 5633	114	17
7	·709 7492	·392 3012	136	18	·708 5826	·392 4407	133	17	·707 5046	·392 5746	131	17
8	·757 2198	·392 3149	154	18	·757 0624	·392 4541	151	17	·756 9936	·392 5877	148	17
9	·811 2881	·392 3302	171	17	·812 2801	·392 4692	168	17	·813 3607	·392 6025	164	17
10	·872 0569	·392 3474	189	17	·874 3406	·392 4859	185	17	·876 7129	·392 6189	181	17
11	·939 6427	·392 3662	206	17	·943 3629	·392 5044	201	17	·947 1718	·392 6370	197	16
12	0·014 1760	·392 3868	222	17	0·019 4803	·392 5245	218	17	0·024 8733	·392 6568	213	16
13	·095 8020	·392 4090	239	17	·102 8409	·392 5463	234	16	·109 9685	·392 6781	229	16
14	·184 6808	·392 4329	255	16	·193 6083	·392 5697	250	16	·202 6245	·392 7010	245	16
15	·280 9888	·392 4584	271	16	·291 9625	·392 5947	266	16	·303 0249	·392 7255	260	15
16	·384 9189	·392 4855	287	16	·398 1005	·392 6213	281	15	·411 3709	·392 7516	275	15
17	·496 6818	·392 5142	302	15	·512 2374	·392 6493	296	15	·527 8818	·392 7791	290	15
18	·616 5066	·392 5444	317	15	·634 6070	·392 6789	310	14	·652 7964	·392 8080	304	14
19	·744 6421	·392 5760	331	15	·765 4636	·392 7099	324	14	·786 3739	·392 8384	318	14
20	·881 3579	·392 6092	345	14	·905 0822	·392 7424	338	14	·928 8954	·392 8702	331	14
21	1·026 9460	·392 6437	359	14	1·053 7610	·392 7762	352	13	1·080 6649	·392 9034	345	14
22	·181 7216	·392 6796	372	13	·211 8218	·392 8114	365	13	·242 0110	·392 9378	357	13
23	·346 0254	·392 7168	385	13	·379 6125	·392 8478	377	12	·413 2886	·392 9736	369	12
24	·520 2250	·392 7553	397	12	·557 5084	·392 8855	389	12	·594 8807	·393 0105	381	12
25	·704 7169	·392 7950	409	12	·745 9141	·392 9244	401	12	·787 2004	·393 0486	393	11
26	·899 9284	·392 8359	420	11	·945 2662	·392 9645	412	11	·990 6930	·393 0879	404	11
27	2·106 3205	·392 8779	431	11	2·156 0351	·393 0057	422	11	2·205 8389	·393 1282	414	10
28	·324 3901	·392 9210	441	10	·378 7283	·393 0479	432	10	·433 1556	·393 1696	424	10
29	·554 6729	·392 9652	451	10	·613 8925	·393 0911	442	10	·673 2014	·393 2120	433	9
30	·797 7467	·393 0103	460	9	·862 1178	·393 1353	451	9	·926 5783	·393 2553	442	9
31	3·054 2350	·393 0563	469	9	3·124 0408	·393 1804	459	8	3·193 9359	·933 2995	450	8
32	·324 8107	·393 1032	477	8	·400 3485	·393 2263	467	8	·475 9753	·393 3445	458	8
33	·610 2007	·393 1509	485	8	·691 7826	·393 2731	474	7	·773 4538	·393 3903	465	7
34	·911 1903	·393 1993	491	7	·999 1454	·393 3205	481	7	4·087 1898	·393 4368	472	7
35	4·228 6293	·393 2485	498	6	4·323 3042	·393 3686	488	6	·418 0685	·393 4840	478	6
36	·563 4374	·393 2982	503	6	·665 1980	·393 4174	493	6	·767 0481	·393 5318	483	5
37	·916 6109	·393 3486	508	5	5·025 8441	·393 4667	498	5	5·135 1668	·393 5801	488	5
38	5·289 2309	·393 3994	513	4	·406 3461	·393 5165	502	4	·523 5508	·393 6289	492	4
39	·682 4708	·393 4507	517	4	·807 9020	·393 5667	506	4	·933 4228	·393 6781	496	4
40	6·097 6055	·393 5024	520	3	6·231 8144	·393 6174	509	3	6·366 1119	·393 7277	499	3
41	·536 0267	·393 5543	522	2	·679 5011	·393 6683	512	3	·823 0651	·393 7776	502	2
42	·999 2449	·393 6066	524	2	7·152 5073	·393 7195	514	2	7·305 8593	·393 8278	504	2
43	7·488 9134	·393 6590	526	1	·652 5197	·393 7708	515	1	·816 2157	·393 8781	505	1
44	8·006 8381	·393 7116	526	1	8·181 3822	·393 8223	516	1	8·356 0160	·393 9286	505	1
45	·554 9966	·393 7642			·741 1137	·393 8739			·927 3206	·393 9791		

TABLE LV.

Miscellaneous Constants.

π	3·141 5926 54
$\log \pi$	·497 1499
$\log 2\pi$	·798 1799
$\log \frac{1}{2\pi}$	$\bar{1}$·201 8201
$\log \frac{1}{\sqrt{2\pi}}$	$\bar{1}$·600 9100
e	2·718 2818 28
$\frac{1}{e}$	·367 8794 41
$\log e$	·434 2944 82
$\log e^{\frac{1}{12}}$	·036 1912 07
$\log \log e$	·637 7799 16

1 centimetre = ·393 70432 ins.
1 inch = 2·539 9772 cm.
1 square cm. = ·155 00309 sq. ins.
1 square inch = 6·451 4842 sq. cms
1 cubic cm. = ·061 025386 cub. ins.
1 cubic inch = 16·386 623 cub. cm.
1 kilogram = 2·204 6212 lbs. avoir.
1 lb. avoir. = ·453 59265 kg.
1 radian = 57·295 7795 degrees.
1 degree = ·017 4532 925 radians.

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